Adaptive correction of a focused beam under conditions of strong intensity fluctuations

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A comparison of the efficiency of phase correction under conditions of strong intensity fluctuations for a plane wave and a focused Gaussian beam is being done. Different algorithms to reconstruct the phase of an optical wave are investigated. Peculiarities of two-color adaptive correction are discussed.

Introduction

During a long time, the terms "phase correction" and "wave-front correction" were considered as interchangeable concepts, and the terms "phase corrector" and "wave-front corrector" were considered as synonyms. Quite often, the adaptive correction was interpreted as a straightening of wave front, be it referred to receiving a distorted wave. In case of adaptive focusing of beams, the correction was considered as a pre-distortion of the wave front.

On the other hand, a more rigorous mathematical consideration[1] within the frames of wave optics describes the beam focusing or image formation as adding partial waves with the allowance for their phases. From this point of view, an adaptive element phases the partial waves and provides for maximum intensity in the focus of a system.

Under general conditions, if the wave front is a sufficiently smooth surface, both of the approaches are practically equivalent. However, when the condition of wave-front smoothness is violated, the situation changes. This occurs, for example, in the turbulent atmosphere when the intensity fluctuations caused by turbulent fluctuations of the refractive index are sufficiently strong.

Wave-front dislocations

It is known that the wave-front dislocations at the points where instantaneous value of the intensity equals zero occur at the distances of optical wave propagation through a random medium approximately equal to the diffraction length $L_d = kr_0^2$, where r_0 is the coherence radius of a plane wave; k is the wave number of radiation.

In the presence of such points, it is impossible to describe the wave front of a reference wave as a smooth simply connected surface. In this connection, the adaptive systems with deformable mirrors become less efficient.

At the same time, numerical experiments[2] with the model of a composite phase corrector showed that its efficiency practically does not change when operated under conditions of strong intensity fluctuations. These results were presented earlier in Refs. 2 and 3. The calculations were performed for a plane wave, with the adaptive system operated as a receiver.

Plane wave

Before considering the results obtained for an adaptive system to focus wave beams, i.e., the system operating in the transmission mode, we would like to remind most important results obtained in Refs. 2 and 3 for a plane wave.

It was revealed in these studies that the efficiency of an adaptive system with a composite corrector is the same both under conditions of weak and strong intensity fluctuations in the atmosphere. First we considered this conclusion as a paradox because we expected that the presence of singular points in the phase and the wave-front discontinuities would require the use of an adaptive corrector with a larger number of components. However, nothing of this kind has happened. Moreover, from the standpoint of the approach treating an adaptive system as a system that phases partial waves this is just what to be expected.

Really, the transformation of phase distortions into the amplitude ones in passing from a short path to an equivalent long path does not, at all, cause a decrease in the size of the coherence area, moreover, the contrary situation may happen, that this can cause an increase in that size. Therefore, having a segmented mirror with the size of an element equal to the coherence radius we can make phasing-in of these areas and, hereby, provide a coherent summation of waves in the telescope focus.

From this point of view, it is easy to explain another one result[2,3] for a plane wave, namely, the fact that the dependence of the efficiency of an adaptive system on the delay in correction circuit does not

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practically depend on the change in the turbulence mode from weak to strong intensity fluctuations.

Bounded Gaussian beams and a "perfect corrector"

The purpose of our study was to investigate a more interesting type of waves, namely, the focused Gaussian beams. We would like to address the question on whether or not it would be possible to obtain that high quality of correction for the beam as for a plane wave. Moreover, in this particular case, the adaptive system operates in the transmission mode that also can yield different results.

It is known that an ideally "perfect" adaptive system is capable of providing high-quality correction of focused beams as was shown in Refs. 4 and 5. "Perfect" means that the phase corrector has an infinitesimal size of elements, and the boundary conditions describing the field at the emitting aperture of the adaptive system have the following form:

$$U(\rho) = A_0(\rho) \exp [- \arg u(\rho)].$$

The phase corrector has a finite size of elements, which equals, in our numerical experiments, to the Fried radius of coherence for the plane wave.

Let us consider first the results for a perfect sensor and a corrector. In so doing, it is interesting to compare the efficiency of adaptive correction for the initial plane wave and a focused beam. The calculated result is shown in Fig. 1.

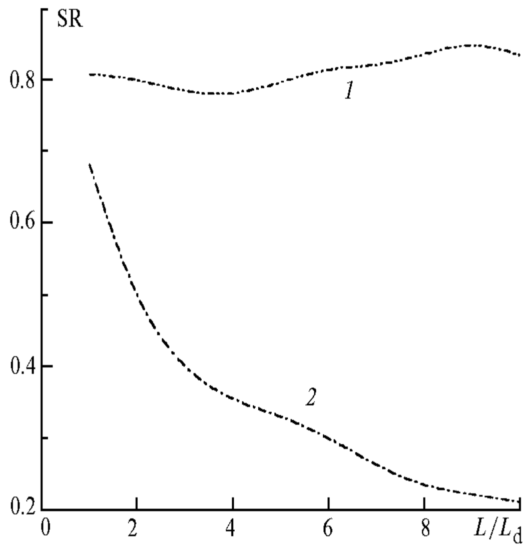

Fig. 1. Dependence of the parameter SR on the normalized path length $D/r_0 = 10$ for a plane wave (*1*) and for a focused Gaussian beam (*2*).

In both of these cases an optical system operates in the transmission mode, i.e., the emitted wave is first modulated by the adaptive phase corrector, and after that the wave propagates through inhomogeneities of the refractive index. Thus, the adaptive correction is introduced in optical wave as a pre-distortion of the initial radiation. For a focused Gaussian beam the mean radiation intensity in a focus is a measure of the correction quality, and for a plane wave the measure of the correction quality is the wave intensity that is measured in the far zone, i.e., in a focus of a lens placed at the far end of the path in the plane $z = L$. This approximately corresponds to a broad collimated beam or a broad beam focused far beyond the layer of randomly-inhomogeneous medium.

It follows form Fig. 1 that the result obtained for a focused beam and a plane wave is essentially different. In numerical experiment, changing the path length L in the range from 1/10 to 10 L_d did not reveal any essential reduction in the correction efficiency for the case of a plane wave. Different result was obtained for the Gaussian beam. Already at $L = 2L_d$ the intensity in focus is reduces twice, for $L = 5L_d$ it reduces three times, and for $L = 7L_d$ it drops by four times, as compared with the diffraction-limited value. From this we draw a conclusion that there exists a principle limitation on the purely phase correction for the turbulent divergence of a focused beam. It turns out that it is impossible to compensate completely the turbulent effects on the long paths, no matter what adaptive system is used. The table presents the values L_d, achievable values of the Strehl ratio (SR) for $r_0 = 10$ cm, $\lambda = 0.5$ μm, and the above mentioned values of the path lengths.

L, km	$L = L_d = 125$	$L = 2L_d$	$L = 5L_d$	$L = 7L_d$
SR	0.68	0.48	0.33	0.25

The calculation used the aperture size $D = 10r_0$, i.e., for $D >> r_0$. For $D > 10r_0$ one can expect approximately the same dependence of the SR parameter on L/L_d, at least, for $0.1 < SR < 1$.

Correction of a non-vortex phase

Consider now the other version, namely, the correction for the non-vortex component of phase distortions only. This variant corresponds to the adaptive system of a traditional type with a deformable mirror and a sensor that uses standard algorithm to reconstruct a phase from its differences. In Refs. 2 and 3 we have presented the result obtained for a plane wave, now it is interesting to compare it with the results calculated for a focused beam. These are presented in Fig. 2. It is a little bit unexpected, from the first sight that the efficiency of the system with a plane wave (curve *2*) decreases more rapidly than that for the focused beam (curve *1*). On the contrary, in the case presented in Fig. 1, the intensity of the focused beam experienced a more rapid fall off. However, it can easily be explained if one takes into account that in the system with a focused beam the reference radiation is a diverging wave. The matter is that in a diverging wave the intensity fluctuations develop more slowly than in a plane wave. It is well seen if one compares the expressions for scintillation indices of a plane wave

$$\beta_0^2 = 1.24\ C_n^2\ k^{7/6}\ L^{11/6}$$

and of a diverging spherical wave

$$\beta_0^2 = 0.42\ C_n^2\ k^{7/6}\ L^{11/6}.$$

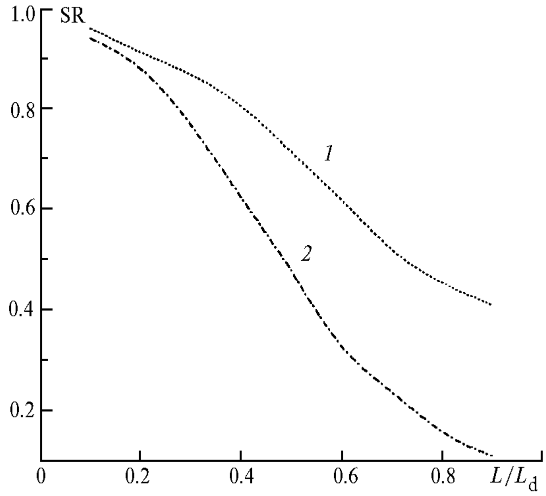

Fig. 2. Dependence of the parameter SR on the normalized path length in correcting for the "non-vortex part" of phase distortions. Complex amplitude sensor uses the algorithm based on the solution of UNE, $d << r_0$.

It follows from these formulas, that the given value of scintillation index will occur in the diverging wave on the path, which is almost twice as long at equal values of $C_n^2\ k^{7/6}$. Therefore, at the identical length L the number of phase dislocations in the diverging spherical reference wave will be smaller, and the correction efficiency higher.

Thus, comparing the efficiency of adaptive correction of a plane wave and a focused beam, we have revealed that in correcting for all aberrations (including the phase dislocations) the efficiency of correction at focus decreases, for a focused beam, with the increasing path length more rapidly than for a plane wave, and at the correction for smoothed (non-vortex) part of the phase aberrations only we have the reverse situation. Note that the scales of path lengths in this case differ almost by the factor of 10. Note also that in both of the cases the spatial resolution of adaptive system is supposed to be infinite, i.e., we assumed that the size of elements of the sensor and corrector d is much less than the Fried coherence parameter r_0.

Effect of the size of wave-front sensor element

Consider now the efficiency of an adaptive system with the finite size of its elements. For certainty, we suppose $d = r_0$. As shown in Refs. 2 and 3, for a plane wave such spatial resolution (i.e., $d = r_0$) of the adaptive correction is quite enough both in the range of weak and strong intensity fluctuations. Let us now address the question on whether or not the same occurs for a focused beam.

Let us remind that in Refs. 2 and 3 the phase at a subaperture element with the size d is determined through the mean complex amplitude by the following formula:

$$\varphi_{ij} = \arg(\bar{U});\quad \bar{U} = \frac{1}{d^2} \iint_d U(x,\ y)\ \mathrm{d}x\mathrm{d}y.$$

Actually, we change places of the operation of averaging over the area and the operation of calculating the arctangent (more exactly, calculation of the principal value of its argument). Hereby we avoid troubles connected with the problem on determining a continuous phase over the whole aperture in the presence of phase dislocations. At the same time, the question remains on what is the type of optical sensor able to carry out such measurement. We propose to suspend solving this problem for a while. Note only that using such model of adaptive system, we do not answer the question on physical implementation of this model, but in reward we succeed in keeping, in such a mathematical model, the parameter characterizing the spatial resolution of adaptive correction, namely, the size of the area d. In our case, this area is simultaneously a subaperture of the sensor and an element of the composite corrector.

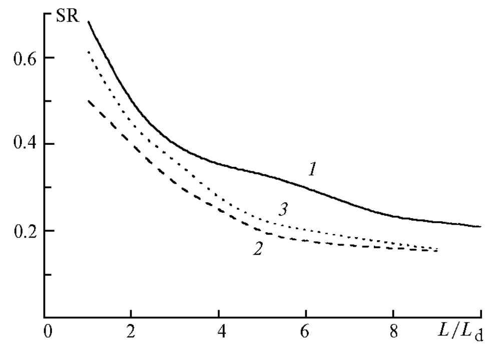

Fig. 3. Dependence of the parameter SR on the normalized path length for correction with the adaptive system having a finite-size element. Complex amplitude sensor uses the algorithm based on the solution of MNE, (*1*) $d << r_0$; (*2*) $d = r_0$, the correction of "mean" phase and tilt; (*3*) the correction of "mean" phase only.

Let us consider the numerical results of simulation. Figure 3 presents three curves: one for the infinite spatial resolution (when the size of the area $d = 0$) and two others for $d = r_0$. In one of the latter cases, only mean phase is corrected, and in the other the mean phase and local tilt of the wave front are being corrected. It is clear from the comparison of curves *1–3* that, on the whole, the difference between these three variants is not of principle importance. Although the efficiency of the adaptive system with the infinite spatial resolution is higher the path length is more important.

Two-color adaptive system

A problem on non-monochromatic adaptation acquires the absolutely new character in view of the possible presence of phase dislocations. The problem arises owing to the necessity of scaling the measured phase aberrations at the wavelength of the reference radiation λ_r and at the wavelength λ of radiation to be corrected.[1] If the correction at the wavelength λ_r which equals to $\varphi_r + 2\pi n$ is determined, then even in the case of the absence of intensity fluctuations the principle uncertainty of the $2\pi n$ term essentially effects the result of phase correction.

Really, if a segmented mirror introduces additional optical path length difference determined as

$$\Delta l = (\lambda_r/2\pi)\,(\varphi_r + 2\pi n),$$

then at the other wavelength we obtain a change of phase, which equals to

$$\varphi = (\lambda_r/\lambda)\,(\varphi_r + 2\pi n).$$

Note, that the values φ, φ_r, and n are the functions of transverse coordinates (x, y), which are omitted for brevity. If, for example, the relation $\lambda_r/\lambda = 1/2$, then the phase difference at the edges of the wave front discontinuity, which equals 2π for the reference wave, takes the value π for the wave being corrected, i.e., the oscillations which should be added in phase will be added in antiphase. Thus, the uncertainty in the term $2\pi n$, which matters nothing at the wavelength λ_r can cause drastic consequences in passing to another wavelength.

As a result, it turns out that already the statement of the problem itself on the use of reference radiation with different wavelength ought to be related to the algorithm of the wave-front sensor operation. In a two-color adaptive system, it is more logical to measure exactly the wave front aberrations that characterize fluctuations of the difference Δl between the optical path lengths, not the fluctuations of the phase difference. However, we cannot measure directly the optical path length, while calculating Δl as a product of the subaperture size d and the measured local tilt s yields a large error under conditions of strong intensity fluctuations. Even if we determine a wave-front surface of the reference wave, the presence of dislocation points and wave-front discontinuities we again shall face exactly the same problem.

Since the problem of two-color correction is too complex being also inherently a multifactor task, we will not complicate it by introducing additional spatial scale – the beam size.

Let us consider the results of numerical experiment with a plane wave for several variants of the system. The variants differ between each other by the measured value. It can be either a local tilt or the "mean" complex amplitude. The second distinction consists in the reconstruction algorithm used, which predetermines the method of identifying the term $2\pi n$ (n is an integer number).

Let the wave-front sensor calculates the mean complex amplitude at every subaperture $\bar{U}_{i,j}$, then the corresponding phase (the argument of a complex number), and the matrix of phase differences between the adjacent subapertures

$$\varphi_{ij} = \arg(\bar{U}_{i,j});\ \Delta^x_{ij} = \varphi_{i+1j} - \varphi_{ij};\ \Delta^y_{ij} = \varphi_{ij+1} - \varphi_{ij}.$$

Determining the phase differences seems to be an unnecessary step, since our task is calculating the phase. However, we have, for certain reasons, to attribute the value $2\pi n_{ij}$ to every ijth subaperture, and it is just for this reason that we calculate the phase differences and process the obtained array Δ_{ij} with the phase reconstruction algorithm.

The main drawback of this approach is that one can miss an integer number of wavelengths even in the absence of intensity fluctuations, since the function arg returns the values into the interval $[0, 2\pi]$ that corresponds to $[0, \lambda]$ in terms of the optical path length. However, one cannot be sure that the path-length difference between the beams separated by a distance d exceeds λ. If, as it is in our case, $d = r_0$, then the structure function at such separation equals to $D(\rho = d) = 6.88(d/r_0)^{5/3} = 6.88$ rad. Correspondingly, the root-mean-square value of fluctuations of the path-length difference, which is expressed in terms of wavelengths, equals to $6.88^{1/2}/2\pi = 0.4\lambda$. Thus, the situations are quite possible, when the optical path-length difference equals to 1.1λ, for example, and the phase difference sensor gives a value equaled to $0.1\cdot 2\pi = 0.628$ rad.

The only method to detect that actual optical path-length difference was larger then the wavelength is to decrease the size of sensor elements d, for example, by 2 to 3 times and to sum the obtained phase differences. Another one method could be the measurement of a local tilt of a wave front and to multiply it by the size d. However, the latter method gives an error that rapidly increases in the range of strong intensity fluctuations.

To illustrate these considerations, let us discuss the results of numerical simulation which are presented in Figs. 4 and 5. All these results were obtained for a plane wave and the following relations between the size of a focusing lens D, coherence radius r_0, and the subaperture size d: $D/r_0 = 10$, $d = r_0$. Ten random screens simulated the randomly inhomogeneous medium, and the radiation intensity at the lens' focal plane was averaged over 10 random samplings.

The path length was taken to be $L/L_d = 0.01$ (see Fig. 4), i.e., it was assumed that the intensity fluctuations at the exit from the random medium are practically absent. We have varied the wavelength of reference radiation λ_r. (Note that the wavelength enters into the formulas for L_d and r_0. Because now we consider, in our problem, radiation at two different wavelengths, it is necessary to choose a radiation with the concrete wavelength to be used for normalizing parameters of the problem). We used a fixed wavelength λ of the corrected radiation in normalizing. Therefore, the normalized path length L/L_d and the normalized aperture size D/r_0 for the reference radiation with the wavelength λ_r will be different. In the description of numerical results, we always give the values corresponding to the initial wavelength λ.

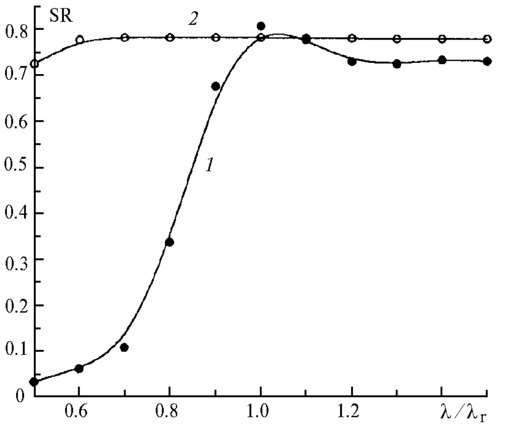

Fig. 4. Dependence of the parameter SR on the normalized wavelength of reference radiation for two types of sensor: "mean" phase sensor (*1*) and local tilt sensor (*2*).

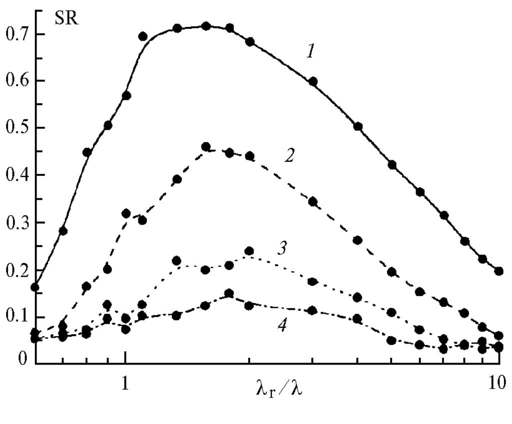

Fig. 5. Dependence of the normalized intensity (parameter SR) on the wavelength of reference radiation. Value of the normalized path length L/kr_0^2 (normalized to the wavelength of corrected radiation) is: 0.25 (*1*); 0.5 (*2*); 0.75 (*3*); 1.0 (*4*).

Two curves in Fig. 4 show the dependence of normalized intensity at the lens focus (the parameter SR) on the ratio λ_r/λ. The curves are different only by the method used to determine the array of the phase differences $\Delta\varphi_{ij}$. In one case (curve *1*) the phase difference was determined through the argument of the mean complex amplitude U, and in the other case (curve *2*) the phase difference was determined as a product of the local tilt and the subaperture size (the local tilt also has been calculated by making use of U and the gradient of U).[2,3] In both of these cases, the algorithm was used based on solving the modified normal equation (MNE), which was proposed in Refs. 6 and 7 and used earlier in Refs. 2 and 3. In this case it yields the same result as the algorithm based on solution of unmodified normal equation (UNE), since the intensity fluctuations are negligible ($L/L_d = 0.01 << 1$).

As was expected, the direct measurement of the phase difference for scaling to larger wavelengths causes a rapid fall off of the correction efficiency. Already at $\lambda_r = 0.8\lambda$ the parameter SR falls twice, and, when λ_r decreases down to 0.5λ, its value falls simply catastrophically. At the same time the second variant of the wave-front sensor that uses the values of local tilts operates practically with the same quality over the whole range of calculations for $0.5\lambda < \lambda_r < 1.5\lambda$. A certain decrease in the parameter SR for $\lambda_r < \lambda$ is explained by a small rise of the intensity fluctuations of the reference radiation when the wavelength λ_r decreases.

It is interesting to identify the limits, within which the correction efficiency for the system with the local tilt sensor keeps unchanged. Let us consider longer paths. We set the limits of the normalized lengths to be $L/L_d = 0.25, ..., 1.0$ and will vary the reference wavelength λ_r within a more wide range from 0.5 to 10λ. The results of numerical simulation are presented in Fig. 5. It is clear from this figure that even at equal wavelengths, i.e., at $\lambda_r = \lambda$ the normalized intensity of a focal spot is less then the diffraction-limited value. It is caused by that, in the system with a local tilt sensor, the error of estimation of the phase difference increases rapidly with the rise of intensity fluctuations.

When the wavelength of reference radiation decreases, its intensity fluctuations become stronger and correspondingly the correction efficiency falls. When the wavelength of reference radiation λ_r changes from λ to 0.7λ the parameter SR decreases almost twice. An increase in the wavelength of reference radiation causes, first, a certain growth of the SR for λ_r changed from λ to 2λ, and in passing to a yet longer wavelength of reference radiation the adaptive correction efficiency again decreases, but the decrease is sufficiently slow. The parameter SR decreases approximately twice when λ_r increases to 6λ, and it falls three times when λ_r increases up to 8λ.

Thus, the use of radiation with longer wavelengths as the reference one causes a small change in the correction efficiency for $\lambda_r = 1, ..., 3\lambda$, which slowly decreases with the further increasing wavelength.

It seems to be impossible to overcome this situation in any other way, since it is not connected with the intensity fluctuations (they decrease with the growth of λ) or with unsuccessful selection of the distortion sensor. Simply, this is connected with the diffraction. With the growth of wavelength the phase distortions more and more quickly become the amplitude ones and this does not cause the growth of the intensity fluctuations by only one reason: that the phase distortions themselves decrease at the increasing wavelength.

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