ANGULAR STRUCTURE OF MULTIPLE LIGHT SCATTERING IN A CLOUDLESS ATMOSPHERE

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Angular distribution of the multiply scattered light is studied based on the results of solving the radiation transfer equation by the method of spherical harmonics for different aerosol optical parameters in the visible wavelength range. Simple approximation formulas are derived that allow one to isolate the aerosol scattering phase function from the brightness phase function observed. Some practical recommendations are given on using them for solving some applied problems.

When solving different problems of ground–based and spaceborne meteorology, it is often necessary to know the aerosol scattering phase function averaged over the entire atmospheric column. Such a scattering phase function under daylight conditions can be obtained from ground–based observations of the cloudless sky brightness in the solar almucantar at different angular distances φ from the sun. The technique for measuring the absolute brightness phase function $f(\varphi)$ is described in detail in Refs. 1–3. The brightness phase function contains the components of single molecular $f_R(\varphi)$ and aerosol $f_D(\varphi)$ scattering, multiple $f_2(\varphi)$ scattering and reflection of light from the underlying surface f_q. One usually assumes the latter component to be independent of the scattering angle (Lambertian surface). The data on aerosol optical properties are in the component $f_D(\varphi)$ normalized by the condition:

$$\tau_D = 2\pi \int_0^{\pi} f_D(\varphi)\sin\varphi d\varphi, \quad (1)$$

where τ_D is the optical thickness due to aerosol scattering. Since often the aim of investigation is to study the variations of aerosol composition of the atmosphere, then the problem is reduced to separation of the scattering phase function observed, $f(\varphi)$, into the aforementioned components:

$$f_D(\varphi) = f(\varphi) - f_R(\varphi) - f_2(\varphi) - f_q. \quad (2)$$

The formula for calculating the Rayleigh scattering phase function $f_R(\varphi)$ is well known:

$$f_R(\varphi) = [3\tau_R/16\pi](1 + \cos^2\varphi), \quad (3)$$

where τ_R is the optical thickness due to molecular scattering. The components $f_2(\varphi)$ and f_q can be determined only by solving the radiation transfer equation. One should note that the rough approximations to calculating the component $f_2(\varphi)$, such as its independence of the scattering angle[1,4] result in large errors in determining $f_2(\varphi)$ at great optical thickness of the atmosphere.[5,6] The most reliable results of solution of this problem were obtained by the Monte–Carlo method.[7] The iteration technique was applied, the observed brightness phase function $f(\varphi)$ was used as the first approximation for taking into account the shape of the single scattering phase function $f_1(\varphi) = f_D(\varphi) + f_R(\varphi)$ in calculation of $f_2(\varphi)$. Evidently, it is not the best choice, because the difference in the shapes of $f_1(\varphi)$ and $f(\varphi)$ is significant, especially at a strong atmospheric turbidity. The aerosol scattering phase functions $f_D(\varphi)/\tau_D$ obtained by authors of Ref. 7 and averaged over 20 days of observation characterize the summer continental aerosol in the South–East Kazakhstan at high atmospheric transmittance. The atmospheric turbidity in other regions of the Earth is usually greater, and undoubtedly the results of Ref. 7 cannot be used in practical calculations.

V.A. Smerkalov[8] proposed a simplified technique for determining $f_D(\varphi)$ from the observed phase function $f(\varphi)$, which is convenient for practical applications and takes into account the dependence of $f_2(\varphi)$ on the scattering angle. However, it is based on the results of numerical solution of the radiation transfer equation[9] obtained in 1958, which do not suppose any clear concept of the spectral composition of the diffuse radiation transmitted. In this paper we present a simple approximation formulas for calculating $f_2(\varphi)$ based on the data of solving the radiation transfer equation by the modified method of spherical harmonics.[10] They correspond to the summer continental aerosol, cover a wide range of the atmospheric turbidity, and are related to three wavelengths of the visible spectral range.

The atmospheric aerosol model that is used for calculating the functions $f(\varphi)$, $f_2(\varphi)$, and $f_q(\varphi)$ is based on the average aerosol scattering phase function at the

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wavelength $\lambda = 0.55$ μm obtained in Ref. 7. In the angular range $2° \le \varphi \le 160°$, it is well approximated by the sum of scattering phase functions corresponding to three groups of particles with lognormal size distribution: $\sigma^2 = 0.3$ and $a = -0.1$ (Aitken nuclei, 15%); $\sigma^2 = 0.4$ and a = 0.4 (submicron fraction, 60%), $\sigma^2 = 0.5$ and $a = 0.8$ (coarse fraction, 25%).[11] Here σ is the radius logarithm variance; $a = \ln \rho_0$; $\rho_0 = 2\pi r_0/\lambda$; r_0 is the geometrical mean radius of particles. The contribution of each fraction into the aerosol optical thickness is given in parentheses. The refractive index was taken to be 1.5. One can set the spectral behavior of $f_D(\lambda)$ and $\tau_D(\lambda)$ by means of the tables from Ref. 12. The absolute brightness phase function $f(\varphi)$ is usually measured at the values of the solar zenith angle $65°< Z_0 < 80°$. The left limit is caused by the fact that the scattering angle φ is related to Z_0 and azimuth ψ by the relationship:

$$\cos\varphi = \cos^2 Z_0 + \sin Z_0 \cos\psi. \tag{4}$$

If $\psi = 180°$, then $\varphi = 2Z_0$. So at $Z_0 \le 45°$ the back part of the observed brightness phase function is completely undetermined. This results in not only the loss of the information on the aerosol optical properties, but yet in large errors in calculating the value

$$\tau_n = 2\pi \int_0^\pi f(\varphi)\sin\varphi d\varphi, \tag{5}$$

used for all subsequent constructions.[13] The right limit $Z_0 \le 80°$ is connected with two circumstances. First, it is the fast increase of the component $f_2(\varphi)$ with a subsequent increase of Z_0, especially at strong atmospheric turbidity, that leads to a sharp decrease in the accuracy of determining $f_D(\varphi)$ by means of the difference relationship (2). Second, the applicability of the plane-parallel model of the atmosphere for calculating $f_2(\varphi)$ is approximately limited by the value Z_0 set. For greater Z_0 it is necessary to take into account the spherisity of atmospheric layers, what makes additional difficulties.[14]

Let us pay attention to the fact that the scattering phase function summed up over all scattering layers for the solar almucantar does not depend on the stratification.[2] That is why we, in our calculations, used the homogeneous model of the atmosphere. The component f_q was calculated in the Lambertian approximation based on the data on the spectral albedo of the grass q, taking into account its weak dependence on Z_0 (Ref. 15). The number of spherical harmonics N used for solving the radiation transfer equation, was determined by the condition of $f(\varphi)$ being invariant in the limits of 0.5% at their subsequent increase. Usually N was more than 50. Calculations of $f(\varphi)$ were done for three wavelengths, 0.40, 0.55 and 0.65 μm, and for three values of the turbidity factor

$$T = (\tau_R + \tau_D)/\tau_R \tag{6}$$

at $\lambda = 0.55$ μm: T = 2, 3, and 4. The step in scattering angle was 1° in the range $0° \le \varphi \le 10°$; 5° for $10° \le \varphi \le 30°$; and 10° for $30° \le \varphi \le 2Z_0$. The values $\sec Z_0$ were 2.86, 3.64, 4.35, and 5.00. The results of calculations of the brightness phase function $f(\varphi)$ and all its components $f_R(\varphi)$, $f_D(\varphi)$, $f_2(\varphi)$, and f_q for $\lambda = 0.55$, $T = 3$, sec $Z_0 = 2.86$, and $q = 0.10$ are shown in Fig. 1. It is seen from the figure that the multiple scattering phase function $f_2(\varphi)$ is elongated along the radiation incidence direction, has a small aureole at $\varphi < 10°$ and weakly pronounced minimum at $\varphi \approx 110°$. The aerosol component $f_D(\varphi)$ exceeds the component $f_2(\varphi)$ only in the angular range $\varphi \le 50°$ in the green wavelength range at a medium turbidity of the atmosphere ($T = 3$). The general information on the contribution of $f_D(\varphi)$ into $f(\varphi)$ as a function of λ, φ, T, and sec Z_0 is given in Table I.

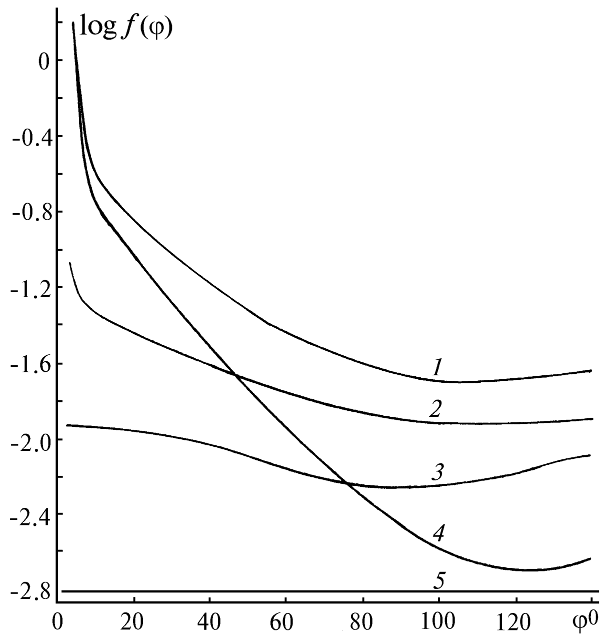

FIG. 1. Brightness phase function (1) and its components caused by multiple (2), single Rayleigh (3), and single aerosol (4) scattering, as well as the reflection of light from the underlying surface (5).

TABLE I. Contribution of aerosol component into the brightness phase function $f_D(\varphi)/f(\varphi)$.

λ, μm	0.40	0.55				0.65
T	3	2	3		4	3
$\sec Z_0$	3.64	3.64	2.86	5.00	3.64	3.64
2	0.87	0.95	0.94	0.92	0.91	0.94
5	0.66	0.84	0.83	0.78	0.76	0.84
10	0.54	0.74	0.75	0.68	0.67	0.76
30	0.29	0.53	0.56	0.49	0.48	0.61
60	0.12	0.27	0.31	0.26	0.27	0.38
90	0.05	0.13	0.16	0.14	0.14	0.22
120	0.03	0.07	0.10	0.08	0.09	0.14

If one changes two parameters and fix the other, one can obtain that the ratio $f_D(\varphi)/f(\varphi)$ decreases with decreasing wavelength λ, increasing solar zenith angle Z_0, and increasing scattering angle φ. The increase of the atmospheric turbidity weakly changes the contribution of $f_D(\varphi)$ into $f(\varphi)$, because the decreasing role of single molecular scattering is compensated by an increase in the portion of multiply scattered radiation.

The data in Table I allow one to distinctly judge about the effect of the errors in measuring $f(\varphi)$ on the accuracy of the aerosol component $f_D(\varphi)$ to be isolated from. If the relative error in measuring the brightness phase function $f(\varphi)$ is 3–4% (Refs. 1–3), then the back part ($\varphi > 90°$) of the aerosol scattering phase function $f_D(\varphi)$ in the wavelength range $\lambda = 0.4$ μm is completely undetermined. The accuracy at these φ in red and green wavelength ranges is not greater than 20–40% even supposing that there is no error in setting the components $f_R(\varphi)$, $f_2(\varphi)$, and f_q in the relationship (2). Obviously, the last fact is not realistic. The accuracy of isolation of $f_D(\varphi)$ from $f(\varphi)$ sharply decreases in the case of observation of $f(\varphi)$ from the snow cover due to the increase in the component f_q.

All the aforementioned is an evidence of the fact that one should impose strict demands on the accuracy of representing the multiple scattering phase function $f_2(\varphi)$ (the contribution of which into $f(\varphi)$ dominates in the wide range of the parameters λ, T, Z_0, and φ) by any approximation formula for engineering calculations. The exception is the range of small scattering angles, where the sky brightness is mainly caused by the single aerosol scattering. Calculations of the function $f_2(\varphi)$ for other parameters by means of the method of spherical harmonics, analogous to that shown in Fig. 1, showed their principle similarity in many aspects. This allows us to represent the full set of curves by the modified Krat formula[3]

$$f_2(\varphi) = \tau_2\{k\exp(-\pi r\varphi/180) + s[1 + t\cos^2(\varphi - 20)]\}, \quad (7)$$

where

$$\tau_2 = \tau_n - (\tau_R + \tau_D) - 4\pi f_q; \quad (8)$$

$$f_q = \varkappa f_n(60°), \quad (9)$$

and the angles φ are set in degrees. The parameters ϰ, τ_2, k, r, s, and t are given in Table II. The example of approximation of the multiple scattering phase function by the relationship (7) using the data of Table II is shown in Fig. 2. It is seen that the approximation formula describes the scattering phase function $f_2(\varphi)$ quite well, except for the range of small angles φ, that, according to the aforementioned, has not principal meaning for its use in the relationship (2). One can judge on the errors in this approach, from Table III.

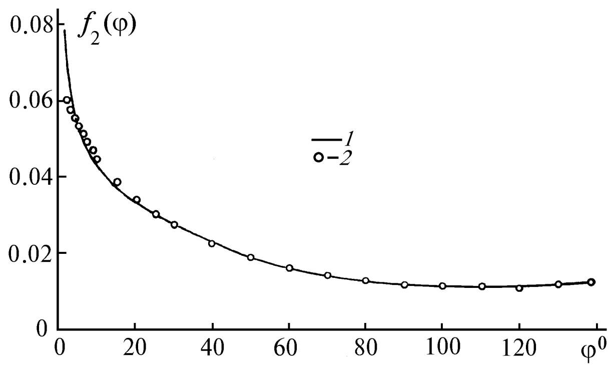

FIG. 2. Multiple scattering phase function (1) and the result of its approximation (2) by the relationship (7). Parameters are the same as in Fig. 1.

TABLE II. Parameters of the scattering phase function $f_2(\varphi)$.

λ.	T	2				3				4			
μm	secZ_0	2.86	3.64	4.35	5.00	2.86	3.64	4.35	5.00	2.86	3.64	4.35	5.00
	ϰ	0.022	0.021	0.022	0.023	0.022	0.021	0.022	0.023	0.022	0.021	0.022	0.023
	τ_2	0.478	0.533	0.586	0.646	0.816	0.952	1.090	1.242	1.294	1.576	1.884	2.242
0.40	k	0.019	0.025	0.028	0.030	0.071	0.073	0.075	0.078	0.105	0.106	0.107	0.108
	s	0.076	0.075	0.075	0.075	0.072	0.072	0.072	0.072	0.070	0.070	0.070	0.069
	r	6.17				4.35				3.28			
	t	0.257				0.283				0.255			
	ϰ	0.043	0.039	0.036	0.034	0.043	0.039	0.036	0.034	0.043	0.039	0.036	0.034
	τ_2	0.084	0.094	0.100	0.106	0.196	0.227	0.249	0.269	0.368	0.435	0.491	0.543
0.55	k	0.150	0.157	0.169	0.186	0.247	0.273	0.285	0.287	0.287	0.317	0.326	0.337
	s	0.066	0.065	0.064	0.063	0.059	0.057	0.056	0.056	0.055	0.054	0.054	0.052
	r	3.48				2.86				2.52			
	t	0.476				0.421				0.293			
	ϰ	0.049	0.045	0.041	0.038	0.049	0.045	0.041	0.038	0.049	0.045	0.041	0.038
	τ_2	0.043	0.050	0.053	0.057	0.118	0.139	0.153	0.166	0.236	0.283	0.317	0.349
0.65	k	0.232	0.252	0.270	0.283	0.350	0.368	0.375	0.387	0.387	0.410	0.410	0.427
	s	0.060	0.058	0.057	0.055	0.052	0.050	0.049	0.048	0.048	0.047	0.046	0.045
	r	2.87				2.75				2.67			
	t	0.625				0.537				0.423			

TABLE III. Errors (%) in $f_D(\varphi)$ determined by the relationships (2) and (7).

$\varphi°$	λ, μm									
	0.40				0.55		0.65			
	T									
	2		4		3		2		4	
	secZ_0									
	2.86	5.00	2.86	5.00	2.86	5.00	2.86	5.00	2.86	5.00
2	−1	−1	−2	−4	−1	−2	2	−3	0	−3
5	0	0	1	2	1	0	0	0	0	0
10	1	0	2	4	1	1	2	2	1	1
30	5	11	−2	−4	−1	−1	−4	−4	1	1
60	40	74	16	13	4	1	−7	−10	6	6
90	65	>100	40	20	17	14	10	6	8	10
120	>100	6	−12	−75	5	−1	17	16	−10	−9

The spectral region near $\lambda = 0.4$ μm at $\varphi > 60°$ is presented poorer than others. According to Table I, the contribution of the component $f_D(\varphi)$ into $f(\varphi)$ is so insignificant here, that its obtaining from the experimental data is impossible due to the measurement errors. Use of the relationship (7) in all other cases is quite acceptable.

Let us present for a conclusion some practical recommendations on the use of the results presented. Let us assume that the absolute brightness phase function $f_D(\varphi)$ was measured at the sea level under continental conditions over the area with green cover. The aerosol scattering optical thickness τ_D corresponding to the solar zenith angle Z_0 is calculated by the technique described in Refs. 11 and 13. The multiple scattering phase function typical for the continental aerosol is determined by formulas (5)–(9) and the interpolated data from Table II. The relationship (2) allows us to obtain the aerosol scattering phase function $f_D(\varphi)$ with the accuracy acceptable for many engineering calculations. If corresponding software for solving the radiation transfer equation is available, this scattering phase function can be considered as first approximation in the iteration process, the scheme of which is given, for example, in Ref. 7.

It is especially expedient to use the data on $f_2(\varphi)$for solving the direct problem, i.e. the calculation of the brightness distribution over the cloudless sky in summer. This case requires only one parameter to be set *a priori*, i.e., the aerosol optical thickness.

REFERENCES

1. E.V. Pyaskovskaya-Fesenkova, *The Study of Light Scattering in the Atmosphere* (Nauka, Moscow, 1957), 219 pp.
2. G.Sh. Livshits, Light *Scattering in the Atmosphere,* Part 1 (Nauka, Alma-Ata, 1965), 177 pp.
3. A.I. Ivanov, G.Sh. Livshits, et al., *Light Scattering in the Atmosphere,* Part 2 (Nauka, Alma–Ata, 1968), 116 pp.
4. V.V. Sobolev, *Radiation Transfer in the Atmospheres of Stars and Planets* (Academy of Sciences, Moscow, 1956), 391 pp.
5. E.M. Feigelson, Izv. Akad. Nauk SSSR, ser. Geofiz., No. 10, 1222–1232 (1958).
6. V.E. Pavlov, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana **13**, No. 1, 94–97 (1977).
7. V.S. Antyufeev, A.I. Ivanov, et al., Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana **16**, No. 2, 146–154 (1980).
8. V.A. Smerkalov, Trudy IPG, No. 17, 194–202 (1973).
9. E.M. Feigelson, M.S. Malkevich, et al., *Calculation of the Light Brightness in the Atmosphere* (Academy of Sciences, Moscow, 1958), 104 pp.
10. T.Z. Muldashev and U.M. Sultanganzin, Zh. Vychislit. Matem. Matem. Fiz. 26, No. 6, 888–893 (1986).
11. T.Z. Muldashev, V.E. Pavlov, et al., Atm. Opt. **2**, No. 11, 959–963 (1989).
12. E.G. Yanovitskii and Z.O. Dumanskii., *Tables on Light Scattering by Polydisperse System of Spherical Particles* (Naukova Dumka, Kiev, 1972), 123 pp.
13. T.Z. Muldashev, V.E. Pavlov, et al., Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana **27**, No. 8, 831–841 (1991).
14. G.I. Marchuk, G.A. Mikhailov, et al., *Monte Carlo Method in Atmospheric Optics* (Nauka, Novosibirsk, 1976), 283 pp.
15. K.Ya. Kondrat'ev, ed., *Radiative Characteristics of the Atmosphere and the Earth's Surface* (Gidrometeoizdat, Leningrad, 1969), 345 pp.