ADAPTIVE TIME FILTRATION OF LIDAR RETURNS

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The paper describes an approach using the apparatus of the Markov filtration of double stochastic Poisson processes. An optimal estimate of the double stochastic Poisson process has been obtained based on the Markov property and the Calman–Bucy filter equations. The Robbins–Monroe procedure is used, which makes it possible to construct the sequence convergent to an unknown ideal value of the parameter.

INTRODUCTION

An extreme spatiotemporal variability of the Earth's atmosphere makes the fields of the atmospheric optical parameters to be random; altitude dependences are spatial realizations of these fields at fixed moments of time, and behavior of the characteristics of a selected volume on a time interval makes their temporal realizations. The lidar measurements of any atmospheric parameter are accompanied by the spatiotemporal smoothing of these realizations that under certain conditions enables one to find their statistical structure.[1] To optimize the processing of sample data for greater efficiency of lidar methods of determination of atmospheric parameters, use of methods of the statistical theory is possible, in particular, the optimal Markov filtration of lidar signals as shown in Refs. 1–3 for the basic modes of their recording.

Optimization in the photon–counting mode has been performed on the basis of equations obtained using the methods of the Markov filtration of continuous processes. A more rigorous approach is connected with the use of the Markov filtration of double stochastic Poisson processes.[4] An idea of application of this apparatus may be fruitful if the two conditions are met.

First, the estimated parameter must be stochastic in the sense that its dependence on time (temporal filtration) or distance (spatial filtration) must be some random process, which has specific properties, namely, Gaussian and Markovian ones. In particular, the temporal dependences of backscattering coefficient fluctuations smoothed in a spatial gate are such processes what is supported by satisfactory exponential approximation of their autocorrelation function.[5]

Second, optimal filtration must be more effective than commonly used nonoptimal processing of lidar signals to the extent sufficient to justify the complication of the processing algorithm and the requirements imposed upon it. So it is necessary to estimate the efficiency of optimal filtration for determining its field of application as well as for checking the efficiency of the algorithm under conditions of closed numerical experiments, for analyzing the sensitivity to inaccurate setting of *a priori* data, and so on. The calculation of the efficiency is also needed for another purpose, for example, for prediction of sounding efficiency under given conditions when developing the lidar and selecting its main parameters as well as for determining the efficiency of operation of particular lidars under various conditions of sounding.

This paper describes an approach related to the use of the Markov filtration of double stochastic Poisson processes.

STATEMENT OF THE PROBLEM AND RELEVANT FORMULAS

Let us assume that the temporal, smoothed in the scattering volume, realizations of the backscattering coefficient $\beta(t; z)$ are Markovian ones, which can be represented as $\beta = \overline{\beta} + \Delta\beta$, where $\overline{\beta}$ and $\Delta\beta$ are the average and β fluctuation values, respectively, t is the time, z is the height. If we consider that the normalized fluctuations $\eta(t; z) = \Delta\beta/\sigma_\beta$, where $\sigma^2_\beta(z)$ is the variance of $\beta(t; z)$, are the Gaussian Markovian process with the exponential autocorrelation function and radius t_c of temporal correlation, we have

$$\beta(t; z) = \overline{\beta} + \sigma_\beta(z)\, \eta(t; z) . \qquad (1)$$

In the photon counting mode box–car integration with the gate duration Δt_g, the intensity $\nu_\Sigma(t; \eta, z)$ of the total Poisson flux of photoelectron numbers $n(t, \Delta t)$ accumulated within intervals $\Delta t \ll t_c$ at the moment t equals

$$\nu_\Sigma(t; \eta, z) = \chi_2[P_s(t; \eta, z) + \chi_1 P_b] / h\nu + i_d / q, \qquad (2)$$

where χ_1 and χ_2 are the loss factors of the receiving optics and quantum efficiency of the photodetector, $h\nu$ is the energy of a radiation quantum, q is the electron charge, i_d is the dark current, P_b is the background power, collected by the receiving aperture,

$$P_s(t; \eta, z) = \chi_1 E_0 S_a (z - z_{lid})^{-2} \frac{c\,\Delta t_c}{2}\, \beta(t; \eta, z) Y^2(z_{lid}, z) f_n \qquad (3)$$

is the realization of a signal component of the backscattered power. The value E_0 in Eq. (3) is the radiation energy per pulse, S_a is the receiving aperture, $Y(z_{lid}, z)$ is the product of (smoothed) functions of aerosol and Rayleigh transmissions, z_{lid} is the height of the lidar location, f_n is the sounding pulse repetition frequency.

For a given realization $P_s(t; \eta, z)$ we have at the photodetector output the unobservable conditionally Poisson signal photoelectron flux $n_s(t; \eta, z)$ with the intensity $\nu_s(t; \eta, z)$ proportional to P_s, and the total flux of signal and dark noise photoelectrons $n(t)$ with the intensity $\nu_\Sigma(t; \eta, z)$ of the type (2). Let us assume that the

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contribution into the fluctuations ΔP_s on the observation interval $[t_0, t_{max}]$ gives random deviations $\Delta\beta$ due to the integral character of fluctuations $Y(z_{lid}, z)$. The fluctuations ΔY with the radius t_{cY} of time correlation ($t_{cY} > t_{max} - t_0 \gg t_{c\beta}$, where $t_{c\beta}$ is the radius of time correlation of β, determined by horizontal scale of optical inhomogeneities and their velocity) together with the other large–scale fluctuations of atmospheric parameters determines the statistical characteristics of the value of $\overline{\nu}_\Sigma$ averaged over the ensemble of fluctuations Δβ which we consider to be the unknown random value. As a result, we have a double stochastic Poisson process with the average intensity *a priori* unknown.

An algorithm for processing $n(t)$ should be synthesized for optimal reconstruction of $\eta(t)$, $\nu_s(t;\eta,z)$, and $\nu_\Sigma(t;\eta,z)$ and simultaneous assessment of the unknown value $\overline{\nu}_\Sigma$.

EQUATION OF FILTRATION AND ADAPTATION

Let us find an algorithm for processing $n(t)$, providing optimal (in terms of the minimum rms error) estimate of $\eta(t)$ on the observation interval $[t_0, t_{max}]$. Substituting Eq. (1) into Eq. (3) and according to Eq. (2) we can write ν_Σ in the form:

$$\nu_\Sigma(t;\eta,z) = \overline{\nu}_\Sigma(z) + \sigma_s(z)\,\eta(t)\,, \quad (4)$$

where $\overline{\nu}_\Sigma(z) = \overline{\nu}_s(z) + \nu_n$, $\overline{\nu}_s(z) = \chi_2 \overline{P}_s/h\nu$, $\nu_n = \chi_1\chi_2 P_b/h\nu + \dfrac{i_d}{q}$ are the values of intensities of the total, signal, and dark noise fluxes of photoelectrons averaged over the ensembles of shot fluctuations of the photodetector and β, $\sigma_s = \overline{\nu}_s m_\beta$, and $m_\beta = \sigma_\beta/\overline{\beta}$.

Using the Markov property of η, one can write the Calman–Bucy filtration equation for its optimal estimate η* when recording the double stochastic Poisson process:

$$d\eta^* = -\frac{1}{t_c}\eta^*\,dt + K\sigma_s/\overline{\nu}_\Sigma[d\,n(t) - \overline{\nu}_\Sigma\,dt - \sigma_s\eta^*\,dt]\,, \quad (5)$$

$$\frac{dK}{dt} = -\frac{2}{t_c}K + \frac{2}{t_c} - \frac{\sigma_s^2}{\overline{\nu}_\Sigma}K^2\,, \quad (6)$$

where $K = \langle(\eta - \eta^*)^2\rangle$ is the *a posteriori* variance of η*, $d\,n(t) = n(t) - n(t + d\,t)$ is the increment of the process $n(t)$ on the interval $[t, t + dt]$. The initial conditions are given at the moment t_0: $\eta^*(t_0) = 0$, $K(t_0) = 1$. The set of equations is solved using the recursion numerical methods (Euler–Cauchy and Runge–Kutta of the fourth order).

Under conditions of *a priori* uncertainty relative to any parameters the application of adaptive processing algorithms is fruitful since it allows us to estimate the unknown values simultaneously. In particular, to estimate average intensity $\overline{\nu}_\Sigma$, unknown due to limited amount of *a priori* information on statistical characteristics of large–scale fluctuations determining the distribution, it is appropriate to use the methods of nonparametric statistics. If we consider the optimal estimate η to be given, obtained from the set of equations (5) and (6), we arrive at the problem on simultaneous estimate of the unknown average against the background of additive Gaussian shot fluctuations of the photodetector. To solve this problem, we use the Robbins–Monroe procedure,[6] allowing the construction of a sequence converging to the unknown ideal value of the parameter according to the recursion formula

$$\overline{n}^*_{j+1} = \overline{n}^*_j + \frac{1}{j+1}[n(t_j + 1) - \sigma_s\eta^* - \overline{n}^*_j]\,, \quad (7)$$

where $t_j = t_0 + (j - 1)\Delta t$, $\overline{n}^*_1 = n_0$, $j = 1$ to M, $M = (t_{max} - t_j)$, n_0 is an arbitrary initial value. The sequence of the estimates $\overline{\nu}^*_j$ of the unknown average intensity $\overline{n}_\Sigma$ is determined as $\overline{\nu}^*_j = \overline{n}^*_j/\Delta t$.

An adaptive algorithm amounts to recursion determination of the estimates η* and $\overline{n}^*_{j+1}$ by solving the set of interrelated equations (5)–(7).

EFFICIENCY OF THE ALGORITHM

The algorithm was analyzed under conditions of a closed numerical experiment including the imitation of $\eta(t)$ giving the form of fluctuations $\Delta\beta(t;z)$, formation of useful $n_s(t;\eta,z) = \nu_s(t;\eta,z)\Delta t$ component and the total $n(t;\eta,z) = \nu(t;\eta,z)\Delta t$ signal photoelectron flux as well as the analysis of spatial and temporal behaviors of indices of efficiency of filtration for different lidar stations.

Imitation of temporal fluctuations of $\beta(t;z)$ was performed according to Eq. (1) by simulating simple Markovian process $\eta(t)$ by the formula[7]

$$\eta(t_i) = \exp(-\Delta t/t_c)\,\eta(t_{i-1}) + \sqrt{1 - \exp(-2\Delta t/t_c)}\,\xi(t_i)\,,$$

where $\xi(t_i)$ are normal random values with zero mean and unit variance.

For simulation of time series of the number of photoelectron counts $n_s(t;\eta,z)$ and $n(t;\eta,z)$ the property of superposition of independent Poisson fluxes[8] is used, which makes it possible to imitate the Poisson fluxes with large input average values using standard procedures and thus to carry out closed numerical experiments for a wide range of sounding altitudes. The time functions $\overline{n}_s(t) = \overline{n}_s + \sigma_s\eta(t)$ serve as the input data to the Poisson operator Π describing the ideal photodetector when obtaining $n_s(t;\eta,z)$, therefore $n_s = \Pi\{\overline{n}_s\}$ is the double stochastic Poisson process. For imitation of the noise component n_n of the input photoelectron mixture $\overline{n}_n$ is used, i.e., $n_n = \Pi\{\overline{n}_n\}$.

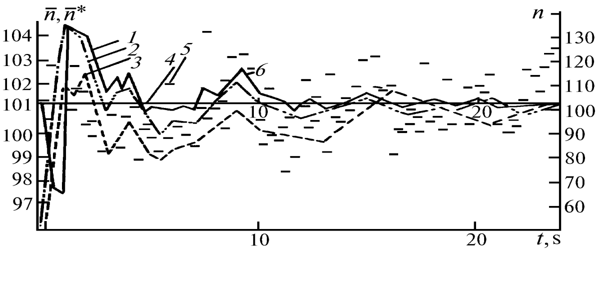

FIG. 1. Reconstruction of unknown time–averaged photoelectron number for $\overline{n}^*(0) = \overline{n}\cdot 1.0$ *(1),* $\overline{n}\cdot 0.7$ *(2), and* $\overline{n}\cdot 0.5$ *(3). Curve 4 is for* $\overline{n}$*, and curve 5 corresponds to initial realization.*

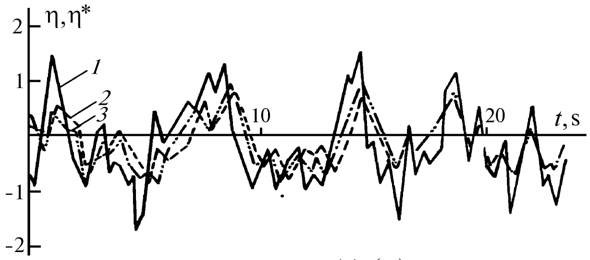

FIG. 2. Comparison of true η(*t*) *(1) and reconstructed values of realization* η(*t*) *and* η*(*t*) *for* $n_0 = \overline{n}\cdot 1.0$ *(2) and* $\overline{n}\cdot 0.5$ *(3).*

Figures 1 and 2 show the results of the numerical experiment. Figure 1 presents a comparison of the true η(*t*) and those reconstructed using Eqs. (5)–(7), η*(*t*), of the relative fluctuations of the backscattering coefficient at different values of the *a priori* uncertainty n_0. In particular, at the *a priori* known $\overline{n}$, corresponding to nonadaptive version of filtration, the estimate of η*(*t*) well follows the fluctuations of η.

Under conditions of adaptation at an essential initial deviation $\overline{n}$ (50%) in the transition mode we observe a marked discrepancy between η* and η. Figure 2 shows the reconstruction of the unknown average by Eq. (7) for different values of *a priori* uncertainty. In particular, from this figure we notice that at the 50% deviation the transition mode slows down that affects the results of reconstruction of η.

CONCLUSION

As a result of our study, the adaptive algorithms of the lidar signal processing in the photon counting recording mode have been found. The algorithm efficiency is demonstrated under conditions of the closed numerical experiment.

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