EFFICIENCY OF PROCESSING LIDAR SOUNDING DATA

A.I. Isakova and G.M. Igonin

Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk
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Some results of application of the efficient Markovian filtration algorithm to the processing of the lidar sounding data are presented. The range of heights is determined, where the filtration makes it possible to reach the necessary accuracy of reconstruction. The methods of lidar sounding data processing using the Marcovian filtration and moving smoothing are compared. Markovian filtration is shown to provide essentially better accuracy of the backscattering coefficient $\beta(t)$ *estimation.*

A wide class of atmospheric optical experiments consists in measurement of spatial or temporal realizations of the random fields of optical parameters of the atmosphere. We aware of the experiments that use optical sounding in different schemes with aerosol scattering for measuring the extinction coefficients, differential scattering, concentration, etc.

Lidar measurement of the optical parameters is accompanied by smoothing of these samples in space and time, that allows one to set their statistical structure.[1] Then it is possible to apply the optimal Markovian filtration of the lidar signals for efficient processing of the experimental data. The necessary requirement to the parameter to be estimated is its randomness, i.e. its dependence on time or distance should be a sample of random process, which has the Gaussian and Markovian properties. In particular, the spatial and temporal dependences of air temperature, density, and pressure, fluctuations of aerosol backscattering coefficient and other parameters smoothed by the lidar pulse are such processes. Optimal filtration should be much more efficient that nonoptimal signal processing in order to justify the complication of calculations and the requirements imposed on it. Calculation of efficiency is necessary for other purposes, for example, for prediction of sounding efficiency within the range of conditions set when creating the lidar, in particular, when selecting its principal parameters.

The efficiency of the lidar signal filtration and, hence, the efficiency of sounding of the given atmospheric parameter is described by different characteristics. The simplest characteristic is the time dependence of *a posteriori* variance $D(\beta_a^*)$, or the dependence $K(t)$, where $K(t) = D(\beta_a^*)/D(\beta_a)$ is the ratio of *a posteriori* and *a priori* variances at the time moment t. If one takes the *a priori* mean value $\overline{\beta}_a$ as an estimate of the sample $\beta_a(t)$, then the estimate of the sample in the ensemble of fluctuations $\beta_a(t)$ has the variance $\sigma^2(\beta_a)$. So $K(t)$ sets a local benefit of filtration in comparison with this *a priori* “estimate”: the less is K, the greater is the benefit. For example, one can take $[K(t)]^{-1/2}$ as a local efficiency of filtration at the time moment t. Global criterion of the filtration efficiency on the set interval $[t_0, t_m]$ is given by the formula

$$W = \left[\frac{1}{[t_m - t_0]} \int\limits_{t_0}^{t_m} K(t) \mathrm{d}t \right]^{-1/2} . \tag{1}$$

As is shown below, $K(t)$ for the temporal filtration under the stationary conditions ($D(\beta_a) = \text{const}$) quickly decreases to the value $\bar{K}$, and then becomes constant. So, if $(t_m - t_0)$ is much greater than the establishment interval Δt_e, then $W = 1/\sqrt{\bar{K}}$. Thus, from the standpoint of both local and global criteria, the most important values that determine the filtration efficiency at $(t_m - t_0) \gg \Delta t_e$ are $\bar{K}(t)$ and Δt_e.

The equation for $K(t)$ at $t \in [t_0, t_m]$ has the form

$$\frac{\mathrm{d}K}{\mathrm{d}t} = -\frac{2}{t_c} K + \frac{2}{t_c} - \frac{\bar{\nu}_s^2 \, m_\beta^2}{\bar{\nu}} K^2 \tag{2}$$

with the initial condition $K(t_0) = 1$. Here t_c is the temporal correlation radius of the process $\beta_a(t, z)$; $\bar{\nu}_s$ and $\bar{\nu}$ are the signal and total densities of photoelectron fluxes, respectively; $m_\beta = \sigma(\beta_a)/\overline{\beta}_a$ is the *a priori* relative mean square filtration of the process $\beta_a(t, z)$ or its modulation depth. The value $Q = \bar{\nu}_s^2 \, m_\beta^2 t_c/\bar{\nu}$ is the generalized signal–to–noise ratio. The importance of

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this value is that the lidar signal filtration is the most efficient at $Q \gg 1$.

The expression (2) does not depend on the sampled data and can be solved *a priori*, that makes it possible to predict the filtration efficiency under the proposed conditions of sounding and to determine the height range, where the filtration allows us to reach the necessary accuracy of reconstruction. The temporal behavior of $K(t, z)$ under the same conditions of sounding is shown in Fig. 1 taking into account different values of power potential of lidar stations (χ_0 is the product of loss coefficient and quantum efficiency; f_n is the laser pulse repetition rate; E is the energy emitted by the pulse transmitter; λ_0 is the working wavelength; and $S_{\rm an}$ is the effective area of the receiving antenna).

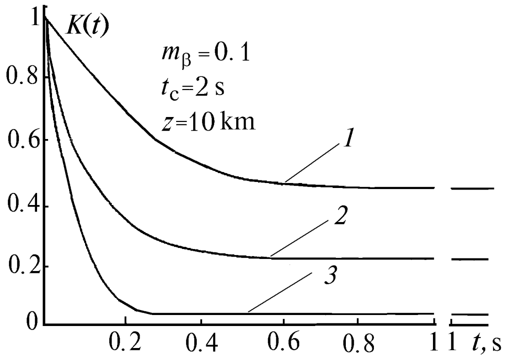

FIG. 1. K(t) behavior for the lidars, the energy potential of which is given in the table.

TABLE.

No.	χ_0	f_n, Hz	E, J	λ_0, μm	$S_{\rm an}$, m^2
1	$1.5\cdot10^{-4}$	$3\cdot10^{3}$	$0.2\cdot10^{-3}$	0.532	0.75
2	0.01	12.5	$10\cdot10^{-3}$	0.532	0.1963
3	0.016	12.5	$10\cdot10^{-3}$	0.532	0.785

The view of temporal behavior is the same for all lidars: $K(t, z)$ comparatively quickly decreases from the initial value $K(t_0, z) = 1$ to the stationary value $\bar{K}(z)$ in the established filtration regime. This value essentially depends on the power potential at a given height. The establishment time is $\Delta t_{\rm e} \ll t_{\rm c}/Q$. The solution corresponding to the stationary filtration period (after the transition regime) can be found from the equation $QK^2 + K - 1 = 0$, derived from Eq. (2) under the condition $dK/dt = 0$. The dependences $\bar{K}(z)$ that allow one to estimate the possibilities of sounding at different heights are shown in Fig. 2.

The best possibilities of realizing the Markovian filtration algorithm has the lidar No. 3 in the meaning of prediction of the efficiency in the entire range of sounding heights.

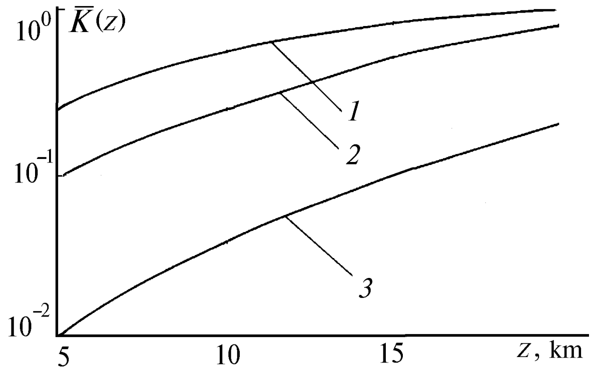

FIG. 2. Vertical dependences of the stationary values of the relative variance for the same lidars.

The results of comparison of the methods of lidar sounding data processing using the Markovian filtration and moving smoothing are presented below. Simple estimate in the form of moving smoothing by the least squares method (on the rectangle window of the duration $T = M\Delta t_{\rm d}$) has the form $\hat{\beta}_{i+2} = 1/M \sum_{j=i}^{i+M} \hat{\beta}_j$, ($\Delta t_{\rm d}$ is the discretization interval). Let us determine the variance of this estimate in assumption of exponential correlation. Using Refs. 2 and 3, the variance of such estimate is obtained in the following form:

$$D(\hat{\beta}_i) = \frac{D_0}{A^2M} + \frac{2\xi D_0}{A^2M^2}\left[\frac{(M-1)e^{\gamma} - M + e^{-(M-1)\gamma}}{(e^{\gamma}-1)^2}\right]. \quad (3)$$

The filtration benefit is demonstrated by the ratio $\psi = D(\hat{\beta}_{\rm a})/D(\beta^*_{\rm a})$. The dependences $\psi(\xi, M)$ calculated for $0.1 \le \xi \le 0.9$ and M = 3, 5, 7, 9, 11 assuming $t_{\rm c} = T$ are shown in Fig. 3; A and γ are calibration constants.

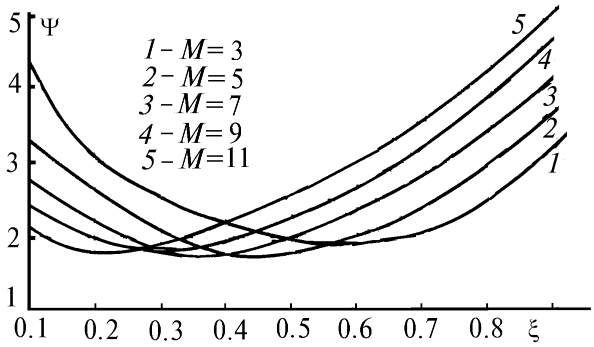

FIG. 3. Estimate of the benefit of the Markovian filtration algorithm in the accuracy of estimating the sample β(t).

It is seen that the use of a Markovian filtration algorithm provides the essential benefit in comparison with the moving smoothing method in the accuracy of estimation of the sample $\beta(t)$.

REFERENCES

1. G.I. Glazov, Gr.N. Glazov and G.M. Igonin, Avtometria, No. 5, 46–51 (1985).
2. J. Bendat and A. Piersol, *Measurement and Analysis of Random Processes* [Russian translation] (Mir, Moscow, 1971), 408 pp.
3. M.B. Nevel'son and R.Z. Khas'minskii, *Stochastic Approximation and Recurrent Estimation* (Nauka, Moscow, 1972), 304 pp.
4. Yu.G. Pollyak, in: *Probability Simulation on Computers* (Sov. Radio, Moscow, 1971), pp. 188–189.
5. A.I. Isakova and G.M. Igonin, Atmos. Oceanic Opt. **8**, No. 5, 407–409 (1995).