APPLICATION OF THE RADIATION TRANSFER EQUATION FOR DESCRIBING WIND REFRACTION OF PARTIALLY COHERENT BEAMS

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The thermal self-action of partially coherent beams in a moving medium is described based on the radiation transfer equation. The solution of this equation in the ray approximation permits reducing the problem of the self-action to the solution of a system of ordinary differential equations. Differences in the self-action of coherent and partially coherent beams are discussed. Calculations of the coherence radius over the beam cross section are presented. It is shown on the basis of this approach that in the geometric-optics approximation the coherence factor is invariant along any geometric ray.

Researchers are becoming increasingly more interested in the application of the radiation transfer equation (RTE) for describing the self-action of partially coherent radiation.[1,2] The solution of the RTE by means of the ray approximation in the case of stationary thermal defocusing was reported in Ref. 1. In this paper the RTE is solved for a nonlinear medium with wind refraction when the problem is not axially symmetric.

The use of the RTE or the equivalent closed equation for the second-order coherence function for describing the self-action of partially coherent beams is predicated on the possibility of separating the correlation functions of the field and the dielectric constant of the medium, which is a functional of the radiation intensity. This separation can be strictly performed only for Gaussian random fields. We shall use the approach of Ref. 3 to determine the restrictions on the characteristics of the radiation and the medium under which the nonlinear interaction of' the radiation with the medium does not affect the starting Gaussian statistics of the field. We shall assume that the nonlinear Interaction of the field with induced fluctuations of the dielectric constant of the medium occurs over a characteristic distance that is much larger than the region of longitudinal correlation of the field. In this case the additional correlation between the random radiation field and the dielectric constant field will be small and the starting Gaussian statistics will be conserved if

$$P \ll P_{cr}\left(\frac{\tau_v}{\tau_0}\right)^{1/2} \quad \text{when } t_0 > \tau_v,$$
$$P \ll P_{cr}\left(\frac{\tau_v^2}{\tau_0 t_0}\right)^{1/2} \quad \text{when } t_0 < \tau_v, \qquad (1)$$

where P is the beam power; $P_{cr} = \dfrac{\pi n_0 \rho C_p a_0}{a k^2 r_{k0}^2 |dn/dT|}$, P_{cr} is the critical power above which the self-action of a partially coherent beam occurs; v is the wind velocity; n_0, ρ, C_p, and α are the refractive index, density, isobaric heat capacity, and the absorption coefficient of the medium, respectively; a_0 and r_{c0} are the width and coherence radius of the beam; $k = 2\pi/\lambda$ is the wave number; τ_0 is the coherence time of the radiation; t_0 is the duration of the radiation pulse; and, $\tau_v = a_0/v$ is the time of flight of a particle of the medium across the transverse cross section of the beam.

Thus as the coherence time decreases the region of applicability of the RTE increases significantly. In particular, the beam power can significantly exceed the critical power and the propagation distance can be much greater than the nonlinear refraction length

$$L_R^2 = \frac{\sqrt{\pi} n_0 \rho C_p v a_0^3}{\alpha |dn/dT| P}.$$

The solution of the RTE

$$\left\{\frac{\partial}{\partial z} + \kappa\nabla_R + \frac{1}{2}\nabla_R\tilde{\varepsilon}(z,\mathbf{R})\nabla_\kappa\right\} J(z,\mathbf{R},\kappa) = 0 \qquad (2)$$

for the intensity

$$W(z,\mathbf{R}) = E_r\iint_{-\infty}^{\infty} d\kappa J(z',\mathbf{R},\kappa) = E_r\iint_{-\infty}^{\infty} d\kappa J_0(z=0,\mathbf{R}(z=0),\kappa(z=0)) \qquad (3)$$

is constructed along the characteristic $r(z)$, which satisfies the equation

$$\frac{d^2\tilde{\mathbf{r}}(z)}{dz^2} = \frac{1}{2}\nabla_R\tilde{\varepsilon}(z,\tilde{\mathbf{r}}(z)) \qquad (4)$$

with the initial conditions given in the emission plane $\tilde{\mathbf{r}}(z=0)=\mathbf{r}_0$, $d\tilde{\mathbf{r}}(z=0)/dz=\lambda$.

To find the solution of the RTE in the ray approximation[1] the system of equations (2)–(4) is supplemented by an equation for the variation

$$\delta\tilde{R}(z)=(\delta\tilde{X}(z),\delta\tilde{Y}(z));$$

$$\frac{d^2\delta\tilde{R}(z)}{dz^2} = \frac{1}{2}(\delta\tilde{R}(z)\nabla_R)\nabla_R\,\tilde{\varepsilon}(z,\tilde{\mathbf{r}}(z)) \qquad (5)$$

with the initial conditions given in the reception plane $\delta\tilde{\mathbf{R}}(z=z')=0$; $d\delta\tilde{\mathbf{R}}(z=z')/dz=\delta\kappa=(\delta\kappa_x,\delta\kappa_y)$. Replacing the second-order vector differential equation (5) by a system of four first-order scalar equations the solution of (5) can be represented in the form

$$\psi(z)=u(z)\psi(z'), \qquad (6)$$

where

$$\psi(z)=\begin{bmatrix}\delta\tilde{X}(z)\\ \delta\tilde{Y}(z)\\ \delta\tilde{\kappa}_x(z)\\ \delta\tilde{\kappa}_y(z)\end{bmatrix}, \qquad \psi(z)=\begin{bmatrix}0\\ 0\\ \delta\kappa_x\\ \delta\kappa_y\end{bmatrix},$$

and $u(z)$ is the Green matrix of Eq. (5) in the reception plane $u(z=z')=I$. For partially coherent beams with the initial brightness distribution[4]

$$J_0(\mathbf{R},\kappa) = \frac{1}{\pi}\frac{k^2a_0^2}{1+a_0^2/r_{k0}^2}\exp\left\{-\frac{R^2}{a_0^2}-\frac{k^2a_0^2(\kappa-R/F)^2}{1+a_0^2/r_{k0}^2}\right\}. \qquad (7)$$

The integrand in (3) may be written in the form

$$J_0=(\tilde{\mathbf{r}}(0)+\delta\tilde{R}(0,\delta\kappa_x,\delta\kappa_y),E_r^{1/2}\delta\tilde{\kappa}(0,\delta\kappa_x,\delta\kappa_y)) =$$

$$= \exp(-S(x_0,y_0,\delta\kappa_x,\delta\kappa_y)), \qquad (8)$$

where the quadratic form $S(\mathbf{x}_0, \mathbf{y}_0, \delta\chi_x, \delta\chi_y)$ is defined, using (5), as follows:

$$S(x_0,y_0,\delta\kappa_x,\delta\kappa_y) = x_0^2+y_0^2+\delta\kappa_x^2[u_{13}^2(0)+u_{23}^2(0)+$$

$$+E_r(u_{33}^2(0)+u_{43}^2(0)]+2\delta\kappa_x(x_0u_{13}(0)+y_0u_{23}(0))+$$

$$+\delta x_y^2+[u_{14}^2(0)+u_{24}^2(0)+E_r(u_{34}^2(0)+u_{44}^2(0))]+$$

$$+2\delta\kappa_y(x_0u_{14}(0)+y_0u_{24}(0))+2\delta\kappa_x\delta\kappa_y[u_{13}(0)u_{14}(0)+$$

$$+u_{23}(0)u_{24}(0)+E_r(u_{33}(0)u_{34}(0)+u_{43}(0)u_{44}(0))], \qquad (9)$$

where u_{ij} are elements of the matrix $u(z)$ $(i, j = 1 \ldots 4)$. The integral in (3) with the integrand (8) can be easily integrated and the expression for the intensity in the ray approximation assumes the form

$$W(z',\mathbf{R}) = \frac{1}{\Delta}\exp\left\{-\frac{1}{\Delta^2}[x_0^2(\Delta_1^2+\beta(\Delta_2^2+\Delta_3^2))+\right.$$

$$\left.+y_0^2(\Delta_1^2+\beta(\Delta_4^2+\Delta_5^2)-2\beta x_0y_0(\Delta_2\Delta_4+\Delta_3\Delta_5)]\right\}, \qquad (10)$$

where

$$\beta = E_r^{-1} = \frac{L_R^2}{L_D^2}, \qquad L_D = ka_0^2(1+a_0^2/r_{k0}^2)^{-1/2};$$

$$\Delta^2 = \Delta_1^2+\beta(\Delta_2^2+\Delta_3^2+\Delta_4^2+\Delta_5^2)+\beta^2\Delta_0^2;$$

$$\Delta_1=v_{11}(z')v_{22}(z')-v_{12}(z')v_{21}(z');$$

$$\Delta_2=v_{14}(z')v_{21}(z')-v_{11}(z')v_{24}(z');$$

$$\Delta_3=v_{11}(z')v_{23}(z')-v_{21}(z')v_{13}(z');$$

$$\Delta_4=v_{12}(z')v_{24}(z')-v_{14}(z')v_{22}(z');$$

$$\Delta_5=v_{13}(z')v_{22}(z')-v_{12}(z')v_{23}(z');$$

$$\Delta_6=v_{14}(z')v_{23}(z')-v_{13}(z')v_{24}(z').$$

The intensity at the point of reception of the radiation in (10) is determined in terms of the elements of the Green matrix of Eq. (5) $V(z)$ with boundary conditions given in the emission plane: $V(z = 0) = I$.

Under conditions when the nonlinear distortions are strong ($E_r \to \infty$) expression (10) for the intensity assumes a simple form

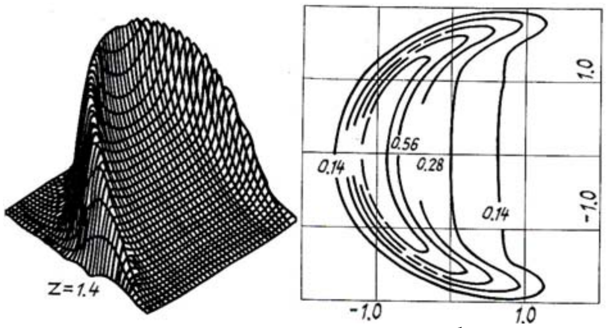
$$W(z',R) = \frac{1}{\Delta_1}\exp(-x_0^2-y_0^2). \qquad (11)$$

It is not difficult to show[5] that in this case the solution of the RTF transforms into the geometric optics solution of a parabolic equation.

Thus the solution of the problem of the the self-action of partially coherent beams in the ray approximation reduces to the simultaneous solution of Eq. (4) for the characteristic $\tilde{\mathbf{r}}(z)$ and Eq. (5) for the elements of the matrix $V(z)$. These equations are closed by a material equation in order to determine the perturbation of the dielectric constant of the medium $\tilde{\varepsilon}(z, \mathbf{R})$. The results of investigations of the self-action of the quasicontinuous partially coherent radiation in a medium with the nonlinear wind refraction are presented below.

For a homogeneous medium with constant wind velocity directed perpendicular to the direction of propagation of the beam , the solution of the problem of the self-action will be determined by the nonlinear refraction length L_R refraction parameter E_r.

The aberrational distortions of the beam subjected to thermal self-action in a moving medium are illustrated in Fig. 1. The results of the calculations of the profiles of the intensity of the starting Gaussian beam for different values of the nonlinear refraction parameter are presented in Fig. 2. As the value of the nonlinear refraction parameter is increased the profiles of the beam intensity converge to a limit determined by geometric optics. The maximum difference, the reasons for which will be discussed below, is observed in the region of local aberrational focusing of the beam.



a b

FIG. 1. The aberrational structure of a Gaussian beam obtained in the geometric optics approximation (a) and isoplets for the intensity of the same beam (b).

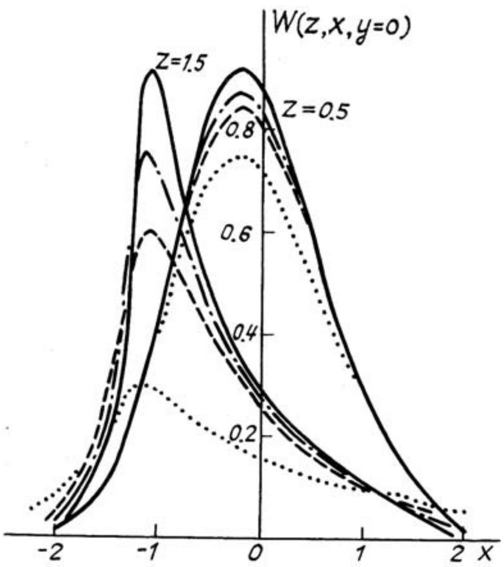


FIG. 2. The change in the profiles of the intensity of a Gaussian beam as a function of distance for the nonlinear refraction parameter E_r = 1 (dotted curves), E_r = 4 (dashed curves), E_r = 10 (dot-dashed curves), and in the geometric — optics approximation.

It follows from the relation between the brightness of the radiation and the coherence function of the field that the solution of the RTF should describe the change in the statistical properties of the field. The most important characteristic of statistically inhomogeneous radiation is the spatial coherence radius defined as the characteristic scale along the difference coordinate over which the spatial coherence function changes. Since we are discussing partially coherent beams with a starting brightness distribution of the form (7) the normalized value of the coherence radius will be inversely proportional to the effective angular width of the brightness body in (8) and (9).

In the ray approximation the cross section of the brightness body is an ellipse, whose dimensions and orientation Eire determined by the coefficients of the quadratic terms. As a result the scales of the coherence function along the difference coordinate, which determine the coherence radius, will also be different. The anisotropy of the statistical properties of the coherence function along the difference coordinate will be described by the coherence ellipse (by analogy to the polarization ellipse) defined as follows:

$$a_{11}(z',R) = \frac{1}{2}\frac{\partial^2 S(x_0, y_0, \delta\kappa_x, \delta\kappa_y)}{\partial(E_r^{1/2}\delta\kappa_x)^2};$$

$$a_{22}(z',R) = \frac{1}{2}\frac{\partial^2 S(x_0, y_0, \delta\kappa_x, \delta\kappa_y)}{\partial(E_r^{1/2}\delta\kappa_y)^2}; \qquad (12)$$

$$a_{12}(z',R) = \frac{1}{2}\frac{\partial^2 S(x_0, y_0, \delta\kappa_x, \delta\kappa_y)}{\partial(E_r^{1/2}\delta\kappa_x)^2\partial(E_r^{1/2}\delta\kappa_y)^2};$$

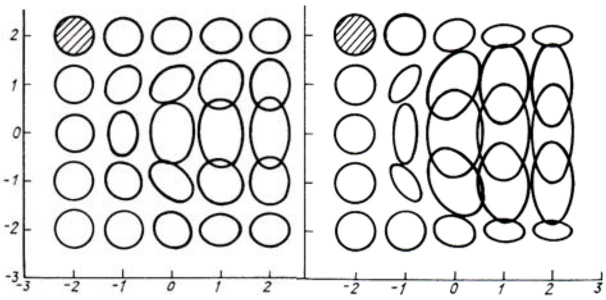


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FIG. 3. Transformations of the coherence ellipse over the Gaussian beam cross-section calculated in the geometric — optics approximation at the distances z = 1 (a) and z = 1.5 (b). The cross-hatched part of the figure shows the undisturbed normalized coherence ellipse in the starting plane.

Transforming to a coordinate system tied to the princpal axes of the coherence ellipse we obtain the following expressions for the coherence radii

$$r_{kx}^2(z',R) = \frac{1}{2}\,[a_{11}(z',R)+a_{22}(z',R)-$$

$$-\sqrt{(a_{11}(z',\mathbf{R})+a_{22}(z',\mathbf{R}))^2-4\Delta^2}];$$

$$r^2_{ky}(z',\mathbf{R}) = \frac{1}{2}\,[a_{11}(z',\mathbf{R})+a_{22}(z',\mathbf{R})+$$

$$+\sqrt{(a_{11}(z',\mathbf{R})+a_{22}(z',\mathbf{R}))^2-4\Delta^2}]\ . \tag{13}$$

The results of calculations of the coherence radii represented in Fig. 3 clearly demonstrate that the anisotropy of the coherent properties of the radiation is significant. For this reason investigations of the behavior of the coherence radius which on the beam axes only [6–7] or in one direction are by no means complete. The coefficients (12)which determine the coherence effective ellipse give more complete information about the change in the coherence radii.

The ratio of the coherence area to the beam area $C_1 = r_c^2 / a^2$, or the product of the coherence area $S_c = z_{cx} \times r_{cy}$ and the intensity on the beam axis[6,8] $C_2 = S_c W$ can be used as a quantitative measure of the coherence of the radiation. It is easy to see that both definitions of the coherence factor are identical. The change in the coherence area $S_c(z, (\tilde{\mathbf{r}}))$ along a geometric ray $\tilde{\mathbf{r}}(z)$ is given by expression

$$S_c(z,\tilde{\mathbf{r}}(z)) = r_{cx}(z,\tilde{\mathbf{r}}(z))r_{cy}(z,\tilde{\mathbf{r}}(z)) = \Delta.$$

In the geometric-optics approximation $(E_r \to \infty)$ $\Delta = \Delta_1$. From here, using expression (11) for the radiation intensity, it follows that in the geometric-optics approximation the coherence factor is invariant along any geometric ray $\tilde{\mathbf{r}}(z)$

$$C(z,\tilde{\mathbf{r}}(z)) = C(0,\tilde{\mathbf{r}}(0)).$$

Taking diffraction into account increases the coherence factor along the geometric-optics rays over the entire cross section of the beam with the exception of the region near the axis. In Refs. 6–8 it was shown that the coherence factor on the beam axis can decrease as well as increase. The change in the coherence factor in these cases is obviously associated with the fact that the self-action of the fluctuations of the field through the induced fluctuations of the medium is taken into account. Our calculations are based on the RTE, whose application in self-action problems is predicated on the assumption that the induced fluctuations of the dielectric constant of the medium do not significantly affect the statistical properties of the radiation.

It is of interest to investigate the differences between the self-action of coherent and partially coherent beams. The nonlinear refraction length of such beams does not depend on the degree of coherence and will be identical for beams with equal power and size. The nonlinear refraction parameters of coherent and incoherent beams will be equal, if their diffraction lengths are equal, which is possible only for beams with different wavelengths. The diffraction divergences of such beams are equal, and in this sense, one can say that the comparison of the self-action of coherent and Incoherent beams presented in Fig. 4 refers to beams with the same diffraction divergence.

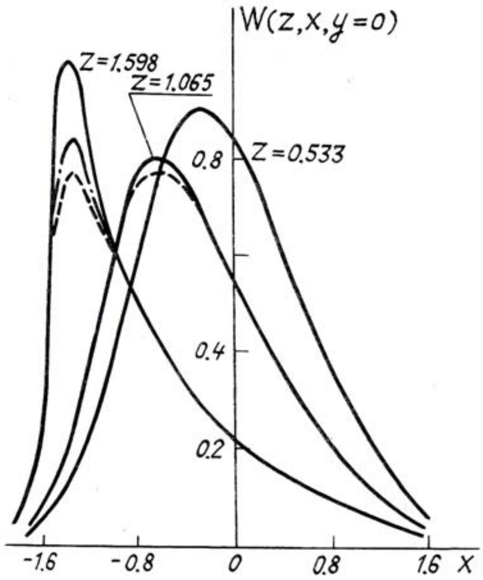

FIG. 4. Comparison of the calculations of the self-action of partially coherent radiation (solid curves) and coherent radiation: the results of Ref. 10 (E_r = 123) and Ref. 11 (E_r = 246, dot-dashed curves.).

It is obvious from the figure that if the intensity profiles are sufficiently smooth, the solution for a coherent beam will be close to that for a partially coherent beam. As the aberrational distortions, which increase the inhomogeneity of the beam in the region of maximum intensity, grow the contribution of diffraction to the formation of the intensity profile will increasingly determine the degree of coherence of the radiation. In addition diffraction is manifested more strongly for coherent radiation than for partially coherent radiation. To explain this difference we shall write down an expression for the effective size of a partially coherent beam

$$S_{pc}(z) = z^2 W_0^{-1}\iint\limits_{-\infty}^{\infty} dR[-\nabla^2_\rho|\Gamma_2(z,\mathbf{R},\rho=0)|+$$

$$+ W(\mathbf{R})(\nabla_\perp\varphi-\overline{\nabla_\perp\varphi})]^2$$

and a coherent beam, derived in Ref. 9,

$$S_c(z) = z^2 W_0^{-1}\iint\limits_{-\infty}^{\infty} d\mathbf{R} \qquad [(\nabla_\perp A)^2 A^2(\nabla_\perp\varphi-\overline{\nabla_\perp\varphi})^2].$$

Here A and φ are the amplitude and phase of the wave.

These expressions are distinguished by the first terms in the integrands. In the case of coherent radiation the broadening of the beam is determined by the gradient of the intensity while the broadening of the incoherent beam is determined by the change in the coherent properties. Analysis of Figs. 2 and 3 shows that at distances of the order of 1 or 2 of refraction lengths the normalized coherence radius changes by a factor of 1.5–2. At the same time the gradients of the intensity can increase by an order of magnitude. The smoother change in the coherence function along the difference coordinate explains the fact that as the nonlinear refraction parameter E_r increases the intensity at the aberrational maximum grows more rapidly for partially coherent radiation, since diffraction slows the growth of the intensity less in this case than for coherent radiation.

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