TOMOGRAPHIC RECONSTRUCTION OF THE ENERGY PROFILE OF AN OPTICAL BEAM

V.N. Sitnikov and L.K. Chistyakova

*Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk*
Received December 22, 1992

In this paper, we propose a tomographic algorithm for reconstruction of the energy profile of an optical beam from the data measured with a linear bolometer. This algorithm is based on a discrete Fourier transform (DFT). When reconstructing the DFT values and making linear interpolation, we used a raster in the form of concentric squares that essentially decrease the error of reconstruction and save a computation time. In a computer experiment, we have estimated the accuracy of the reconstruction method and determined the optimal number of projections. The proposed algorithm has been successfully used for reconstruction of the beam intensity distribution from the experimental data. The results of reconstruction were compared with the data of direct measurements. It is shown that the proposed algorithm allows most fast reconstruction of spatial energy structure of an optical pulse to be done. The accuracy of reconstruction is not worse than that of the existing algorithms when the noise level is no more than 20%.

Application of laser radar technique to the solution of the problems in atmospheric optics calls for measuring the energy distribution over the beam cross section on different sections of the propagation path, because distortions of the beam wavefront are determined by the optical and meteorogical parameters of the atmosphere. The real-–time data on the beam energy profiles can be also used in the systems of adaptive correction of the distortions of the beam wavefront accompanying the laser beam propagation through the ground layer of the atmosphere.

To measure the intensity distribution over the beam cross section, the tomographic methods[1–5] are coming into use. To obtain the projections needed for reconstruction of the intensity distribution, these methods use contact measurement techniques that require insertion of matrix or bolometric receivers into a beam[1,2] and remote techniques harnessing the effects accompanying the propagation of laser radiation through the atmosphere, for example, aerosol scattering[3,4] and sound waves.[5]

It should be noted that in spite of the peculiarities of different techniques of projection recording, all of them use either direct and inverse Fourier transforms or iteration techniques for image reconstruction.

The advantages of the Fourier transform algorithm are the possibility of application of the well–known fast Fourier transform algorithm that can be easy realized in analog form. Moreover, the algorithm allows *a priori* information on the object symmetry to be effectively considered. It is important due to the fact that the intensity distribution over cross sections of various laser beams belongs to the class of integer functions of finite degree or exponential type.[6,7]

The tomographic methods in which all operations on reconstruction of the structure of an object under study are carried out directly in the space of signals (iteration techniques,[8,9] Radon transform techniques[10]), unlike the Fourier methods, have some advantages too and allow us to achieve a high quality of reconstruction for the finite number of initial projections, though they require the increased expenditures of computer time.

Specific choice of the image reconstruction algorithm is determined both by requirements for the method itself (speed of response, resolution, total number of needed projections, and the like) and by its experimental realization including the types of employed recording elements.

This paper presents the algorithm of tomographic reconstruction of the image of a laser beam in its cross section recorded with bolometers whose receiving elements are parallel wires with resistance varying in proportion to the amount of absorbed energy in the specific range of variation of the radiation intensity.

First this class of problems was considered by Efremenko[1] who applied the reconstruction technique harnessing the direct and inverse Fourier transforms. This algorithm yielded good results for the data obtained with the near–zero noise level, whereas the noise level of receiving elements of bolometers may reach 10–20%. Implementation of the algorithm of the inverse Radon transform with approximation of projections by the smoothing splines allowed Pikalov and Preobrazhenskii[11] to obtain the reconstruction results with an error of 20–30% for the noise level up to 5%. Application of the third–degree polynomials in the same algorithm for the approximation of discrete series of obtained projections ensures the image reconstruction with the same accuracy when the noise level achieves 20%.

In general the problem of reconstruction of unknown function $X(u_1, u_2)$ specifying the structure of an object irradiated by a collimated beam is described in the following way. Continuum of the object projections for different angles θ varying in the range $0 \le \theta \le \pi$ is assumed to be given. The projections at the preset θ can be obtained when the radiation source is placed at this angle to the examined object or the object is rotated through the angle θ to the direction of incident radiation. In the orthogonal system (u_1, u_2) when the source radiation is directed perpendicularly to the line making the angle θ with the u_1 axis, the new coordinate system $(\hat{u}_1, \hat{u}_2)$ rotated about the initial coordinate system can be determined as follows:

0235-6880/94/05 325-04 $02.00

$$\hat{u}_1 = u_1 \cos\theta + u_2 \sin\theta ,$$
$$\hat{u}_2 = - u_1 \sin\theta + u_2 \cos\theta, \qquad (1)$$

and the projection of the function $X(\hat{u}_1, \hat{u}_2)$ on the u_1 axis at the angle θ is determined by the expression

$$P_\theta(\hat{u}_1) = \int_{-\infty}^{\infty} X\,(\hat{u}_1 \cos\theta - u_2 \sin\theta,\ u_1 \sin\theta + u_2 \cos\theta)\,d\,\hat{u}_2. \qquad (2)$$

This equation describes a family of linear integrals taken along a number of straight lines parallel to each other and the beam. The reconstruction problem is reduced to the solution of a finite number of equations of the form (2) for different values of the angle θ to obtain the estimate of $X(u_1, u_2)$. Multidimensional discrete Fourier transform is at the same time an exact representation of the Fourier transform of the sequence of finite length as well as the expansion of multidimensional periodic sequence in the Fourier series. Many important properties of the discrete Fourier transform (DFT) follow from duality of the transform. One property is the theorem on projection cross section by which if the desired function $X(u_1, u_2)$ has the Fourier spectrum $X(\omega_1, \omega_2)$, the one–dimensional Fourier spectrum of the function $Px_\theta(u_1)$ will exist, moreover, the spectrum of the Fourier projection at the angle θ will represent a cross section of the two––dimensional Fourier transform $X(u_1, u_2)$. It follows from the theorem that the values of a few projections of an object yield the values of the Fourier transform along the chosen radii in the Fourier plane. So the problem of reconstruction of $X(u_1, u_2)$ estimate is equivalent to the problem of interpolation of the Fourier transform, on the whole, on the basis of these radial cross sections.

All things considered, let us construct an algorithm for reconstruction of radiation intensity distribution over the cross section of a beam in the given measurement plane. The energy distribution in the measurement plane is assumed to be described by the two––dimensional function $P(x, y)$, then the signal measured by each bolometric sensor can be related to the integral of this function. Let the optical radiation be propagated along the OZ axis in the (X, Y, Z) coordinate system affixed to a radiating source. Then a set of signals measured by the bolometers arranged in a grid determines the projection of the function $P(x, y)$ on the OY axis at the angle θ with respect to the OZ axis, while M grids located in the measurement plane $Z = Z'$ and oriented at the angles $\theta_j = \pi j/M$, where $j = 0, 1, \ldots M - 1$, determine M discrete analogs of the projections of the function $P(x, y)$ on the OY axis. The function $P(x, y)$ is reconstructed directly from these projections.

The grid orientation in space is specified in the (X_j, Y_j, Z_j) coordinate system. Then the projection of $P(x, y)$ on the OY_j axis, in accordance with Eq. (2), is determined by the expression

$$AY_j(X_j) = \gamma \int_{-\infty}^{\infty} P\,[X(X_j\, Y_j),\ Y(X_j\, Y_j)]dY_j, \qquad (3)$$

where j is the factor of proportionality, and $AY_j(X_j)$ is the function of the continuous variable X_j.

For the functions $AY_j\,(X_j)$ we have their discrete analogs in the form of a set of signals from all bolometers of each grid. The discrete Fourier transform defined by the expression

$$G(\mathrm{w}_j^k) = \frac{1}{N} \sum_{i=-(N-1)/2}^{(N-1)/2} A(X_j^i)\,\exp\left[-\,i\,\frac{2\,\mathrm{p}\,k\,i}{N}\right], \qquad (4)$$

where N is the number of bolometric receivers of one grid, $-(N-1)/2 \le k \le (N-1)/2$, and $0 \le j \le M$ is taken for each set.

The values of the DFT obtained as a result of transformation of Eq. (4) can be considered as the readings of the Fourier transform taken at the regular polar raster shown in Fig. 1. Since $P(x, y)$ has the finite domain of definition R and limited frequency range, it can be represented by $(N \times N)$ – point discrete Fourier transform. To reconstruct $(N \times N)$ values of the DFT, unlike the method of linear interpolation used in Ref. 1 for the polar raster from two values of polar coordinates nearest to Cartesians reading and defined by circles of smaller and larger radii, a raster in the form of concentric squares is used for interpolation by our algorithm. Such a raster can be obtained by changing the data sampling rate *vs.* the angle θ. The step between the readings is determined by the formula

$$h = 1\ /\ \max(|\cos\theta|,\ |\sin\theta|). \qquad (5)$$

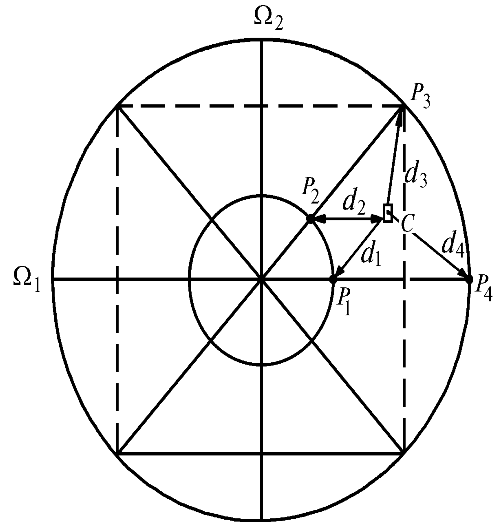

FIG. 1. Illustration of interpolation of the values of Fourier transform of the desired intensity distribution to the nodes of Cartesians' grid: C is the reading at the node of Cartesians' grid; P_1, P_2, P_3, and P_4 *are the readings of polar raster nearest to C ;* d_1, d_2, d_3, *and* d_4 *are the distances from C to* P_1, P_2, P_3, and P_4, *respectively.*

In this case each reading of the DFT is calculated as the weighted mean of the four nearest polar samples whose weights change inversely proportional to the Euclid distance between the points (see Fig. 1)

$$c = p_1\, d_1 + p_2\, d_2 + p_3\, d_3 + p_4\, d_4\ , \qquad (6)$$

where $d_1 + d_2 + d_3 + d_4 = 1$.

For such a raster the interpolation is carried out into rows and columns of rectangular grid of the DFT and is thus one–dimensional. This not only decreases the volume of computations but also substantially decreases the errors in reconstruction of the desired function.[13] The values of unknown function $P(x, y)$ are determined from its Fourier spectrum $P(\omega_1, \omega_2)$ with the use of the inverse Fourier fransform

$$P(X, Y) = \sum_{\omega_1,\, \omega_2=-(N-1)/2}^{\omega_1,\, \omega_2=(N-1)/2} \sum G(\omega_1, \omega_2) \exp \left[i \frac{2\pi}{N} (\omega_1 X + \omega_2 Y) \right]. \quad (7)$$

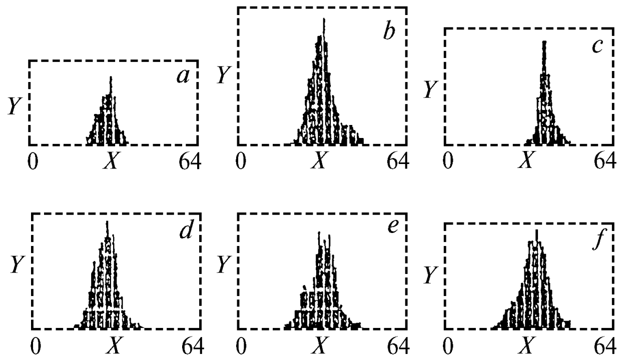

FIG. 2. *Initial graphics of discrete analogs of projections of the function being reconstructed at the angles* θ = 0 *(a),* 30 *(b),* 60 *(c),* 90 *(d),* 120 *(e), and* 150° *(f).*

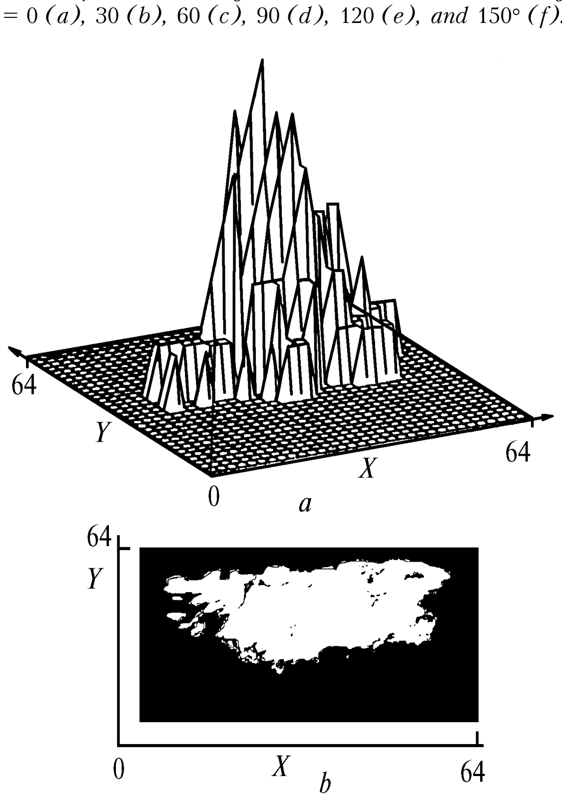

FIG. 3. *Pattern of spatial energy profile of measurable pulse reconstructed from the data of Fig. 2 (a); contour lines of blackening of the photographic material in the plane of measurement of radiation (b).*

The above–described algorithm was used for reconstruction of the intensity distribution from the data measured in the plane of beam cross section by six bolometer grids each containing 64 receiving elements. The grids were placed perpendicular to the beam propagation direction and the receiving elements were oriented at the angles $\theta_j = \pi j/6$, where $j = 1, \ldots 5$, with respect to the axis of radiation propagation. The distributions of integral values of energy recorded by the bolometers for each grid are shown in Fig. 2. The reconstructed two–dimensional function is shown in Fig. 3*a*. Within the limits of the error of direct measurements of energy distribution, the validity of reconstructed profile of the signal under study can be judged from the degree of blackening of photographic material that was placed practically in the same plane with bolometric measurers during the experiment. The blackening contour lines are shown in Fig. 3*b*. It is clear from the comparison of Figs. 3*a* and 3*b* that the profile of horizontal projection of the beam reconstructed in the region occupied by this beam is in good agreement with the profile of heat burn on the material placed in the measurement plane. The same can be concluded about the overall size of the examined profiles along the X and Y axes and about their details.

The computer experimental estimate of the accuracy of the method for the 64 × 64 model matrix which describes the Gaussian distribution and is reconstructed from the six projections shows that the relative error averaged over the whole grid and calculated by the formula

$$\delta = \frac{1}{NM} \frac{1}{f_{\max}} \left(\sum_{X=1}^{N} \sum_{Y=1}^{N} (f(X, Y) - f'(X, Y)) \right), \quad (8)$$

where $f(X, Y)$ are the initial values of the intensity distribution and $f'(X, Y)$ are the reconstructed values of this distribution, is 3%. To estimate the accuracy of the reconstruction algorithm in the presence of noise, the errors in measuring $f(X, Y)$ were determined by the formula

$$A'(X_j^i) = A X_j^i (1 - R a_j^i), \quad (9)$$

where a_j^i are the random variables being normally distributed in the interval between –1 and 1 and R is the parameter determining the measurement accuracy. So for $R = 0.2$ the mean relative error for each value of the reconstructed distribution was 24 – 26%. In the solution of the problems with not very critical requirements for the time of reconstruction of the desired distribution, the algorithms of preliminary filtration[14] may be applied to the initial data. This increases the reconstruction accuracy by 5 – 6%.

It was found by the method of computer experiment that the use of six projections (in the case of 64 receiving elements arranged in a grid) is most optimal for sufficiently correct reconstruction of the initial function $f(X, Y)$. A decrease in the number of projections leads to the sharp growth of the reconstruction error, while its increase insignificantly affects the reconstructed pattern, but sharply increases the volume of computations. This conclusion is in good agreement with the results of theoretical investigations of Ref. 15.

From the preceding, it may be concluded that from all the existing tomographic algorithms, the proposed algorithm allows one to reconstruct most fast the spatial energy structure of optical pulse with the use of optimal combination of the number of initial projections and the number of readings for each projection. It ensures the accuracy of reconstruction of optical radiation structure no worse than that of the data obtained by existing tomographic algorithms when the noise level is up to 20% and can be recommended for use in optical information–measurement systems.

REFERENCES

1. V.V. Efremenko, "*Diagnostics of optical properties of the region of interaction of high–power laser radiation with the atmosphere*," Candidate's Dissertation in Physical and Mathematical Sciences, Moscow (1982).
2. V.V. Vorob'ev, M.E. Gracheva, and A.S. Gurvich, Akust. Zh. **32**, No. 4, 457–461 (1986).
3. V.P. Aksenov and V.V. Pikalov, Kvant. Electron. **17**, No. 2, 167–172 (1990).
4. M.P. Angelov, M.A. Afonin, D.S. Bochkov, et al., Atm. Opt. **2**, No. 5, 384–387 (1989).
5. V.V. Vorob'ev, M.E. Gracheva, A.S. Gurvich, and V.A. Myakinin, Atm. Opt. **2**, No. 7, 593–597 (1989).
6. A.I. Markushevich, *Theory of Analytic Functions*, Part 2 (Nauka, Moscow, 1968), 624 pp.
7. M.A. Efgrafov, *Asymptotic Estimates and Integer Functions* (Nauka, Moscow, 1979), 320 pp.
8. T.S. Mel'nikova and V.V. Pikalov, Teplofiz. Vys. Temp. **22**, 625 (1984).
9. R.V. Shafer, R.M. Mersero, and M.A. Richards, Proc. IEEE, No. 4, 34–55 (1981).
10. B.K.P. Horn, Proc. IEEE **67**, No. 12, 1616–1623 (1979).
11. V.V. Pikalov and N.G. Preobrazhenskii, *Reconstructive Tomography in Gas Dynamics and Plasma Physics* (Nauka, Novosibirsk, 1987).
12. G. Hermen, *Restoration of Images from Projections. Principles of Reconstructive Tomography* [Russian translation] (Mir, Moscow, 1988), 488 pp.
13. D. Dadjion and P.M. Mersero, *Digital Processing of Multidimensional Signals* [Russian translation] (Mir, Moscow, 1988), 625 pp.
14. J.F. Kaiser and W.A. Reead, Rew. Sci. Instrum., No. 11, 1447–1455 (1977).
15. A.D. Kotyuk, A.M. Raitsin, and M.V. Ulanskii, Izmerit. Tekh., No. 11, 52–55 (1987).