FORMATION OF NON-CLASSICAL OPTICAL FIELDS DURING SELF-ACTION

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Methods of suppressing quantum fluctuations and creating squeezed states of light via phase self-modulation are discussed. In contrast to traditional techniques based on parametric processes, those discussed here do not require phase synchronization, important in parametric interactions. Furthermore, the discussed effect leads to the formation of intense optical fields with non-classical properties, i.e., to macroscopic sources of radiation with expressly quantum properties.

INTRODUCTION

The present study presents results from quantum mechanical and semiclassical theories describing phase self-modulation (PSM) of coherent light of the so-called two-photon field. The main attention is paid to analyzing the fluctuations of the quadrature components of the radiation and to the output photon statistics of a nonlinear Mach-Zender interferometer. It is shown that under certain conditions the photons may obey sub-Poisson statistics so that the light field appears to be in a squeezed quantum state.

Interest in radiation with the above non-classical properties stems from the possibilities of applying it to high-precision measurements, spectroscopy, as a means of optical communication, etc. The most typical feature of radiation in that expressly quantum state is that a decrease of photodetection noise below the shot noise level becomes possible. This applies to both direct detection (for photons with sub-Poisson statistics) and to balanced homodyne reception (for light in a squeezed state).

The first study to point to the possibility of the transformation of quantum statistics of radiation during PSM was Ref. 1. Studies that followed[2–7] investigated that process from every angle, including that of cavity feedback. A significant factor in generating radiation with sub-Poisson statistics is interference between initially coherent beams, which have undergone PSM. Below we discuss self-action of coherent radiation in a nonlinear medium, in a nonlinear interferometer, and in a nonlinear cavity.

PSM OF COHERENT RADIATION

First we carry out a quantum-mechanical analysis of the PSM process. Creation and annihilation operators for photons which have passed through a medium with a cubic nonlinearity, $\chi^{(3)}$ have the form

$$b_1^+ = a_1^+ \cdot e^{iKn_1}, \qquad b_1 = e^{-iKn_1} \cdot a_1, \tag{1}$$

where a_1^+ and a_1 are the slowly varying creation and annihilation operators at the entrance to that nonlinear medium, written in the Heisenberg representation; $n_1 = a_1^+ a_1$ is the photon number operator; κ is a nonlinear parameter related to $\chi^{(3)}$ and to the depth of the medium $L(\kappa - \chi^{(3)}L)$. We take the medium nonlinear response to be instantaneous. Note that replacing the operators a_1^+ and a_1 by the complex amplitudes A_1^* and A_1 leads to the classical expressions for PSM.

According to Eq. (1) the photon statistics during PSM remains unchanged: $b_1^+ b_1 = n_1$. However fluctuations of the quadrature components

$$X_1 = (b_1 + b_1^+)/2, \quad Y_1 = (b_1 - b_1^+)/i2 \tag{2}$$

do change. In the case of initially coherent radiation with amplitude α_1 ($a_1|\alpha_1\rangle = \alpha_1|\alpha_1\rangle$), after averaging the operators which have been transformed as a result of PSM over the initial state $|\alpha_1\rangle$, we obtain for the variance of the component X_1 at $\kappa \ll 1$

$$\begin{aligned} \langle(\Delta X_1)^2\rangle &= \langle\alpha_1|(\Delta X_1)^2|\alpha_1\rangle = \langle X_1^2\rangle - \langle X_1\rangle^2 = \\ &= \frac{1}{4}\Big\{1 + 2|\alpha_1|^2\cdot\left[1 - e^{-w_1}\right] + \\ &+ 2|\alpha_1|^2\left[e^{-2w_1}\cos(\kappa + \Phi_1) - e^{-w_1}\cos\Phi_1\right] \simeq \\ &\simeq h\left[1 - 2\Psi_1\sin\Phi_1 + 4\Psi_1^2\sin^2(\Phi_1/2)\right]/4, \end{aligned} \tag{3}$$

where $\omega_1 = (\kappa|\alpha_1|)^2$, $\Phi_1 = 2(\varphi_1 + \psi_1)$, $\varphi_1 = \arg\alpha_1$, $\psi_1 = \kappa|\alpha_1|^2$ is the nonlinear addition to the phase. For radiation of fixed phase the variance (3) is a quasiperiodic function of the nonlinear phase ψ_1.[2] At the same time, if we consider ψ_1 to be prescribed, the function (3) has the minimum

$$\langle(\Delta X_1)^2\rangle_{min} = \frac{1}{4}\left[\sqrt{1+\psi_1^2} - \psi_1^2\right], \quad (4)$$

which is attained at

$$\varphi_{1,opt} = \frac{1}{2}\,\mathrm{arctg}\,\psi_1 - \psi_1.$$

As ψ_1 grows, the value of expression (4) becomes less than 1/4, which corresponds to the initial coherent radiation. In that case the radiation is said to be in a squeezed state. Selecting the optimal phase In the phase plane X_1Y_1 is equivalent to rotating the coordinate axes of the quadrature components, so that the variance of quadrature (3) reaches its minimum.

For $\psi_1 \gg 1$ we have

$$\langle(\Delta X_1)^2\rangle_{min} \approx (4\psi_1)^{-2}. \quad (5)$$

The compression here is inversely proportional to the square of the radiation intensity and the square of the medium depth. Hence deep compression may be obtained over a wide range of variation of the intensity. Note that operators (1) are not reduced to operators of the two-photon coherent state.[8] Thus the PSM phenomenon produces a new class of squeezed states[2] differing from such states found in parametric processes. A positive feature of the implemented PSM process, and hence — of the generated squeezed states. Is the lack of a need to satisfy the phase synchronism condition during such an implementation.

NONLINEAR INTERFEROMETER WITH NON-CORRELATING PHOTON FLUCTUATIONS IN ITS CHANNELS

Consider now a nonlinear interferometer (Fig. 1). The PSM process is realized in each of its arms, e.g., in a fiber light-guide (FLG).

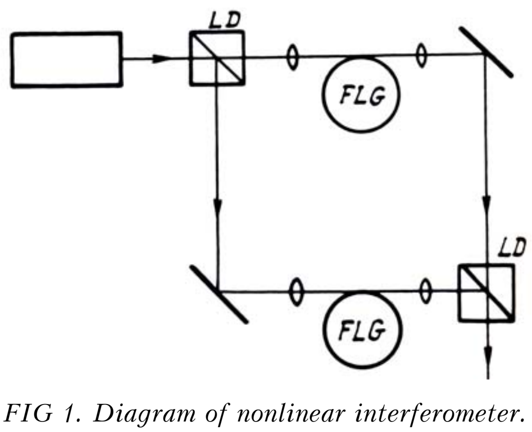

FIG 1. Diagram of nonlinear interferometer.

As was shown above, radiation in such channels is squeezed. Radiation from both channels is mixed by the light-splitting plate (LSP). After mixing, the annihilation operator for the exiting radiation is equal to

$$c = \tau_1 b_1 + \tau_2 b_2, \quad (7)$$

where b_j is the annihilation operator for the jth channel; τ_j is the fraction of radiation in each channel. Note that we assume that $\tau_1^2 + \tau_2^2 = 1$, i.e., there are no losses in the channels.

The photon statistics in the resulting wave appears to be sub-Poisson. Its difference from the pure Poisson statistics is usually characterized by the Fano factor

$$F = \left[\langle(c^+c)^2\rangle - \langle c^+c\rangle^2\right]/\langle c^+c\rangle, \quad (8)$$

which is equal to unity for coherent radiation.

In the considered case we have

$$F = 1 + 4\left[G_1(\psi_1) + G_2(\psi_2)\right]\sin\Phi/(1 + R^2 + 2R\cos\Phi), \quad (9)$$

$$G_1(\psi) = \tau_1^2\psi\,(\psi\sin\Phi - \cos\Phi - R), \quad \Phi = \Phi_1 - \Phi_2,$$

$$G_2(\psi) = \tau_2^2\psi R\,(\psi R\sin\Phi + R\cos\Phi + 1),$$

$$R = \tau_1|\alpha_1|/\tau_2|\alpha_2|$$

The deepest suppression of fluctuations in the number of photons (i.e., the minimal value of F) is reached for the following nonlinear phases:

$$\psi_{1,2} = (\pm R^{\pm} + \cos\Phi)/2\sin\Phi \quad (10)$$

Results of calculations of F for a 50% mixing $\left(\tau_1 = \tau_2 = 1/\sqrt{2}\right)$ and of the values of the nonlinear phases (10) needed in that case are plotted in Fig. 2. It follows from this figure that there are no fundamental limitations on obtaining as low a variance in the number of photons as needed in the interference field. However, the Fano factor is noticeably reduced only for higher nonlinear phase iIncrements. Their signs may then both coincide with and be opposite to that of the initial phase.

From this point of view mixing of squeezed radiation with coherent radiation is just a particular case of the absence of nonlinearity in one of the interferometer channels ($\psi_2 = 0$). The limiting values of the Fano factor then do not differ from those for a scheme with two nonlinear channels; however, here the condition $\tau_1 \gg \tau_2 \approx 0$ must be satisfied, i.e., the realization of such a scheme leads to significant losses of radiation intensity. If on the other hand $\tau_1 = \tau_2 = 1\sqrt{2}$, then fluctuations in the number of photons are suppressed by no more than a factor of 2 ($F \geq 0.5$).

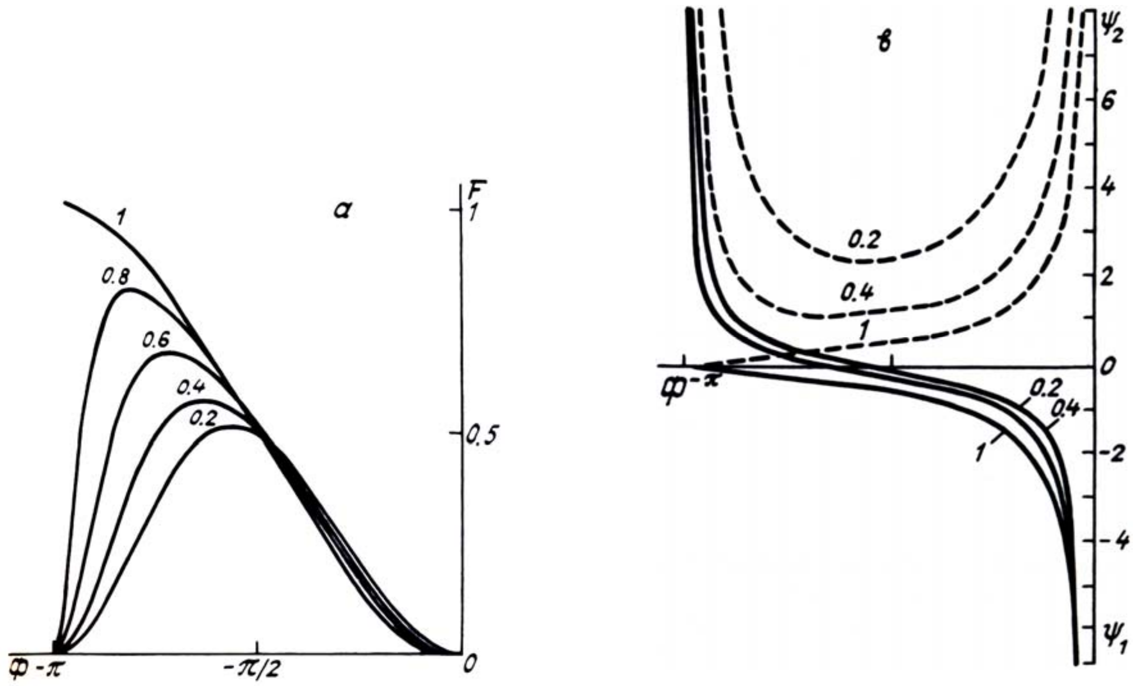

FIG. 2. The Fano factor (a), the nonlinear phase run-ons ψ_1 (solid curves), and ψ_2 (dotted curves) (b) as functions of the phase difference Φ. The numbers above the curves correspond to values of r.

Let us consider, by way of an example, nonlinear interferometer with the following parameters: R = 0.4, $\tau_1 = \tau_2 = 1\sqrt{2}$, $\Phi = -158°$ ($\psi_1 = 0.7$ rad, $\psi_2 = 2.1$ rad). If single-mode quartz fibers (nonlinearity $\tilde{n}_2 = 3.2 \cdot 10^{-16}$ cm^2/W) of 5 μm core diameter and lengths of $L = 62.5$ and 30 m are used as the nonlinear media, the radiation power should be 0.7 and 4.4 W, respectively, at a wave number of $\kappa \approx 10^{-5}$ cm^{-1}. If such conditions are met, a suppression of the variance of the photon fluctuations by a factor of 5 is possible ($F = 0.2$).

Note that the considered nonlinear interferometer has no branching channels, which would have led to significant losses, as, for example, is the case of parametric processes, where more energy must be used to pump the system, and only a small part of it is channeled into the informative signal. In the nonlinear interferometer under consideration the efficiency of transformation of coherent radiation into radiation with sub-Poisson statistics is determined practically only by the interference conditions, the phase difference, and the ratio of intensities, and may reach tens of per cents. Nevertheless, to attain strong suppression of fluctuations large nonlinear phase run-ons are needed (Fig. 2). The practical problem then is that the entrance plate, which splits the initial beam in two in the nonlinear interferometer, is noisy from the quantum point of view. Because of that, the photon fluctuations in various interferometer channels must be noncorrelated. Meanwhile, correlated photon pairs may be generated in the interferometer channels, using the process of degenerate parametric interaction.[9]

NONLINEAR INTERFEROMETER WITH CORRELATED PHOTON FLUCTUATIONS IN ITS CHANNELS

First we demonstrate that interference cannot compensate for noncorrelated fluctuations. We write out the photon annihilation operator corresponding to the exit wave resulting from the interference of two beams with a π/2 phase –difference, as $d = \left(e^{-i\kappa\delta n_1} a_1 + i a_2\right)/\sqrt{2}$, where $\delta n_j = n_j - \langle n_j \rangle$. Consider the quadrature operators of the initial field $x_j = \left(a_j + a_j^+\right)/2$, $y_j = \left(a_j - a_j^+\right)/i2$.

Fluctuations of the photon number at the exit of the nonlinear interferometer are then given by

$$\delta n \approx \left[(\bar{x}_1 - \bar{y}_2)(\delta x_1 - \delta y_2) + (\bar{x}_2 + \bar{y}_1)(\delta x_2 + \delta y_1) - 2\kappa(\bar{x}_1\bar{x}_2 + \bar{y}_1\bar{y}_2)(\bar{x}_1\delta x_1 + \bar{y}_2\delta y_1)\right]; \qquad (11)$$

$$\bar{x}_j = \langle x_j \rangle, \qquad \bar{y}_j = \langle y_j \rangle.$$

We have for independent fluctuations in the arms $\langle(\delta n)^2\rangle = 0$ only if $\langle n \rangle = 0$. Correlation between the quantum states in the different channels can be produced via parametric scattering or by the use of a parametric generator.[9,10] Photons in these processes are generated in pairs. However, in the first of these cases the vacuum fluctuations of $\bar{x}_j = \bar{y}_j = 0$ are amplified, and their stabilization is impossible. Meanwhile, in the second case (degenerate parametric generation of radiation from the vacuum fluctuations) only one pair of quadratures is correlated, e.g., $\langle(\delta x_1 - \delta x_2)^2\rangle$ – 0. For the other pair, in agreement with the uncertainty principle, $\langle(\delta y_1 - \delta y_2)^2\rangle$ – ∞. Then, however, $\bar{x}_1 = \bar{x}_2 = \bar{y}_1 = \bar{y}_2 = 0$, and the intensity fluctuations grow, as described by relations (11).

How now to realize efficient suppression of quantum fluctuations? For such a stabilization of radiation the following conditions must be satisfied:

$$\langle x_1\rangle = \langle x_2\rangle = \langle y_1\rangle = \langle y_2\rangle, \quad \langle(\delta x_1 - \delta x_2)^2\rangle - 0, \tag{12}$$

$$\langle\Psi\rangle = 2\kappa\langle x_1^2\rangle = 1. \tag{12a}$$

Realization of conditions (12) is possible during ordinary and regenerative parametric amplification (PA) of the initial coherent wave.

When degeneration in frequency takes place, and the signal wave and the idler wave are orthogonally polarized (i. e., the "ee" type interaction tends to zero, see Fig. 3), their annihilation operators for non-cavity PA are written in the approximation of a prescribed classical pumping field as

$$a_1 = \mu a_{10} + \nu a^*_{20}, \qquad a_2 = \mu a_{20} + \nu a^*_{10} \tag{13}$$

where the operators a_{j0} correspond to the incident waves, $\mu = \text{ch}G$, $\nu = \text{sh}G$, and G is the increment. In that case we have

$$\bar{x}_{1,2} = \mu\bar{x}_{10,20} + \nu\bar{x}_{20,10}, \quad \bar{y}_{1,2} = \mu\bar{y}_{10,20} - \nu\bar{y}_{20,10},$$

$$\langle(\delta x_1 - \delta x_2)^2\rangle = \frac{1}{2}e^{-2G}. \tag{14}$$

Conditions (12) can also be realized during amplification of a coherent wave, split into waves of ordinary and extraordinary polarization with ratio of amplitudes $\bar{x}_{10} = \bar{x}_{20} = y_{10}\exp^{-2G} = y_{20}\exp^{-2G}$.

Both the separation and combining of orthogonal polarizations in the interferometer are performed by light-splitters (see Fig. 3).

The Fano factor for the exiting radiation is then equal to

$$F = \langle(n - \langle n\rangle)^2\rangle/\langle n\rangle = e^{-2G}. \tag{15}$$

Radiation with parameter relations (12) can also be formed during regenerative PA in a cavity parametric generator with signal injection. Evolution of the eigenvalues α_j for the operators a_j of the signal wave and idler wave and of the complex amplitude α_p of classical pumping in this case obey the equations

$$\dot{\alpha}_{1,2} = \chi\alpha_p\alpha^*_{2,1} - \gamma\alpha_{2,1} + if, \quad \dot{\alpha} = d\alpha/dt,$$

$$\dot{\alpha}_p = -\chi\alpha_1\alpha_2 - \gamma_p\alpha_p + f_{p,1} + if_{p,2}, \tag{16}$$

which hold for interaction in a high-Q cavity. The coefficients γ_j characterize the energy losses, and f and $f_{p1} + f_{p2}$ the external radiation feed. Fluctuation sources are temporally omitted from our equations. Deterministic sources in Eqs. (16) are important for selecting the amplification regime. Note that a similar semiclassical approach was used to describe parametric generation in Ref. 10.

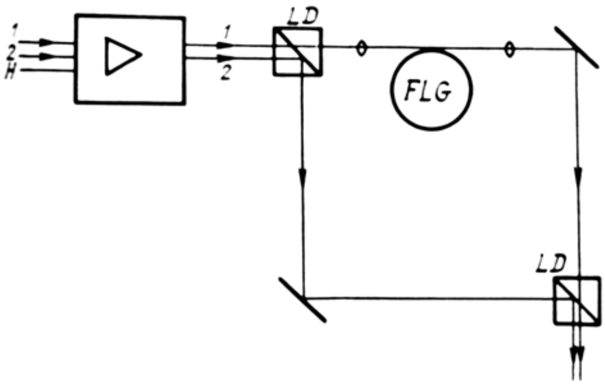

FIG. 3. Diagram of source implementation: 1, 2 — parametrically amplified signal wave and idler wave with mutually orthogonal polarizations; H — pump.

For average stationary values of the sum and difference quadratures (here they are classical) $p = x_1 + x_2$, $q = y_1 + y_2$, $r = x_1 - x_2$, and $s = y_1 - y_2$, satisfying relations (12), we obtain

$$\bar{y}_p = \bar{r} = \bar{s} = 0, \quad \bar{x}_p = \gamma/\chi, \quad \bar{p} = \bar{q} = f/\gamma,$$

$$f_{p,1} = \gamma\gamma_p/\chi, \quad f_{p,2} = \frac{1}{2}\chi(\bar{p})^2. \tag{17}$$

Linearized equations for the quadrature fluctuations, e.g., r, may be written as

$$\delta\dot{r}(t) = -(\chi\bar{x}_p + \gamma)\,\delta r(t) + (1 - \theta^2)^{1/2}R_0(t), \tag{18}$$

or

$$\delta\dot{r}(t) = -(\chi\bar{x}_p - \gamma)\delta r(t) - (1 - \theta^2)^{1/2}R(t), \tag{19}$$

here $R_0(t)$ and $R(t)$ are the entrance and exit fluctuations for the regenerative PA; θ is the amplitude reflectance of the mirrors. The spectral densities of the fluctuations $R(t)$ and $R_0(t)$, $S_R(\Omega)$, and $S_{R,0}(\Omega)$ are used to express the Fano factor of the radiation formed in the interferometer:

$$F = S_R(\Omega)/S_{R,0}(\Omega) = \Omega^2/[\Omega^2 + (2\gamma)^2] \tag{20}$$

It can be seen that at low frequencies the fluctuation spectrum corresponds to sub-Poisson statistics.

We present numerical estimates. According to Eq. (20), $F = 0.5$ at $\Omega_0 = 2\gamma = 2c(1 - \theta)/l$, where l

is the cavity length during PA; for $l = 10$ cm and $\theta = 0.9$, we have $\Omega_0 \approx 5 \cdot 10^8$ Hz. For $\Omega \approx 0.1\Omega_0$ we find that $F \approx 0.01$. However, in the very low frequency region the considered effect of suppression of quantum fluctuations is neutralized by fluctuations in the instrumentation.

In case non-cavity PA is used, the Fano factor is determined by relations (15), and for an increment of $G = 1$ we have $F \approx 0.13$. Recall that condition (12a) must be satisfied in both cases, and that it may be transformed into the form $2k\tilde{n}_2 I_0 L = 1$.

SELF-ACTION IN A PASSIVE CAVITY

Let coherent radiation enter a cavity filled with some cubically nonlinear medium. Instead of the quantum nonlinear equation for the annihilation operator of the radiation we may use an equation for the classical complex amplitude: $\alpha = \mathrm{Tr}(a\rho)$ where Tr is the trace operation and ρ is the density matrix. Thus, we start from a time-dependent equation for α:

$$\dot{\alpha}(t) = i\tilde{\kappa}|\alpha|^2\alpha - \gamma_1\alpha - i\gamma_2\alpha + f_1 + if_2 + \tau\xi_{in}(t). \tag{21}$$

Here $\tilde{\kappa}$ is a coefficient related to the medium cubic nonlinearity and to its length L: $\tilde{\kappa} = c\kappa / L$; γ_1 and γ_2 determine the cavity losses due to partial transparency of its mirrors and to detuning of the radiation from the resonance frequency ω_0 ($\gamma_2 = \omega - \omega_0$); f_1 and f_2 characterize the external radiation feed; $\xi_{in}(t)$ accounts for the input quantum fluctuations; τ is the amplitude transmissivity coefficient of the mirrors. For a high-Q cavity we have $\tau^2 \approx 2\gamma_1 L/c$, where L is the cavity length. We assume for clarity that $f_1 > 0$. The approach outlined below is suitable for high-Q cavities only.

We express $\alpha(t)$ and $\xi_{in}(t)$ in terms of the in quadrature components: $\alpha = x + iY$; $\xi_{in} = x_{in} + iy_{in}$. We then obtain from Eq. (21)

$$\dot{X} = \tilde{\kappa}(X^2 + Y^2)Y - \gamma_1 X + \gamma_2 Y + f_1 + \tau x_{in}(t);$$

$$\dot{Y} = -\tilde{\kappa}(X^2 + Y^2)X - \gamma_1 Y - \gamma_2 X + f_2 + \tau y_{in}(t). \tag{22}$$

The stationarity conditions quadrature values are

$$f_1 = \gamma_1\bar{X} - \gamma_2\bar{Y} - \tilde{\kappa}(\bar{X}^2 + \bar{Y}^2)\bar{Y},$$

$$f_2 = \gamma_1\bar{Y} + \gamma_2\bar{X} + \tilde{\kappa}(\bar{X}^2 + \bar{Y}^2)\bar{X}. \tag{23}$$

The linearized equations for the quadrature fluctuations are

$$\delta\dot{X} = (2\tilde{\kappa}\bar{X}\bar{Y} - \gamma_1)\delta X + \left[\tilde{\kappa}(\bar{X}^2 + 3\bar{Y}^2) + \gamma_2\right]\delta Y + \tau x_{in}(t),$$

$$\delta\dot{Y} = -(2\tilde{\kappa}\bar{X} + \gamma_1)\delta Y - \left[\tilde{\kappa}(3\bar{X}^2 + \bar{Y}^2) + \gamma_2\right]\delta X + \tau y_{in}(t). \tag{24}$$

The expressions including fluctuations at the cavity exit may be obtained using the "time inversion" technique.[11] In that case, instead of Eq. (24) we obtain

$$\delta\dot{X} = (2\tilde{\kappa}\bar{X}\bar{Y} + \gamma_1)\delta X + \left[\tilde{\kappa}(\bar{X}^2 + 3\bar{Y}^2) - \gamma_2\right]\delta Y - \tau x_{out}(t);$$

$$\delta\dot{Y} = -(2\tilde{\kappa}\bar{X}\bar{Y} - \gamma_1)\delta Y - \left[\tilde{\kappa}(3\bar{X}^2 + \bar{Y}^2) - \gamma_2\right]\delta X - \tau y_{out}(t). \tag{25}$$

Transforming to the Fourier spectra in Eqs. (24) and (25) and expressing the fluctuations at the exit by those at the entrance to the cavity, we arrive, after some quite cumbersome transformations, at

$$x_{out}(\Omega) = \left[\nu(\Omega)x_{in}(\Omega) + \eta(\Omega)y_{in}(\Omega)\right]/\mu(\Omega); \tag{26}$$

$$\mu(\Omega) = P_1(\Omega) + P_2(\Omega) + i2\gamma_1(\Omega)(\gamma_1^2 + \gamma_2^2)\bar{X}\bar{Y} + 4\gamma_2\tilde{\kappa}(\gamma_1^2 + \gamma_2^2)(\bar{X}^2 + \bar{Y}^2);$$

$$\nu(\Omega) = Q_1(\Omega) + Q_2(\Omega) + 4\gamma_1\tilde{\kappa}(\gamma_1^2 + \gamma_2^2)\bar{X}\bar{Y} + 2\gamma_2\tilde{\kappa}(\gamma_1^2 + \gamma_2^2)(\bar{X}^2 - \bar{Y}^2);$$

$$\eta(\Omega) = 2(\gamma_1^2 + \gamma_2^2)\left[\gamma_1\tilde{\kappa}(\bar{X}^2 + 3\bar{Y}^2) - \gamma_2(i\Omega - 2\tilde{\kappa}\bar{X}\bar{Y})\right];$$

$$P_{1,2}(\Omega) = \gamma_{1,2}^2\left[\gamma_{1,2}^2 + \gamma_{2,1}^2 - \Omega^2 + 3\tilde{\kappa}^2(\bar{X}^2 + \bar{Y}^2)^2\right];$$

$$Q_{1,2}(\Omega) = \gamma_{1,2}^2\left[\gamma_{1,2}^2 + \gamma_{2,1}^2 + \Omega^2 - 3\tilde{\kappa}^2(\bar{X}^2 + \bar{Y}^2)^2\right].$$

The expression for $y_{out}(\Omega)$ has a similar structure.

If the radiation enters the cavity in a coherent state, its spectral density is $S_{in}^{(x)}(\Omega) = S_{in}^{(y)}(\Omega)$. The ratio of spectra for the component x, for example, is equal to

$$R(\Omega) = S_{out}^{(x)}(\Omega)/S_{in}^{(x)}(\Omega) = \left(|\nu|^2 + |\eta|^2\right)/|\mu|^2. \tag{27}$$

An optimal squeezing of noise at the exit ($R(\Omega) = 0$) is reached when $\nu = \eta = 0$, which is possible at the frequency $\Omega = 0$ if the following conditions are satisfied:

$$\gamma_1 = -2\tilde{\kappa}\bar{X}\bar{Y}, \quad \gamma_2 = \tilde{\kappa}(\bar{X}^2 + 3\bar{Y}^2). \tag{28}$$

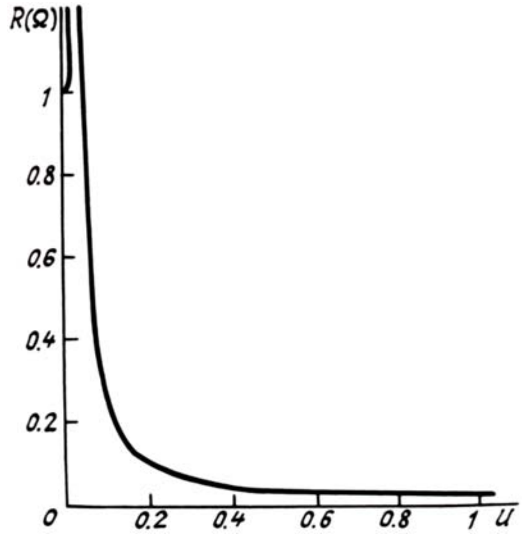

FIG. 4. The squeezing coefficient $R(\Omega) = S_{out}^{(x)}(\Omega) / S_{in}^{(x)}(\Omega)$ *as a function of* $(\tilde{\chi} \cdot \bar{X}^2 / \Omega)^2 u$.

It should also be the case that

$$f_1 = -4\tilde{\kappa}\,\bar{Y}\left(\bar{X}^2 + \bar{Y}^2\right), \quad f_2 = 2\tilde{\kappa}\,\bar{X}\left(\bar{X}^2 + \bar{Y}^2\right). \tag{29}$$

In that case the dependence of the depth of squeezing on the frequency has the form

$$R(\Omega) = \frac{\Omega^2\left(\Omega^2 + 4\gamma_2^2\right)}{\left[\Omega^2 - 8\gamma_2\tilde{\kappa}\left(\bar{X}^2 + \bar{Y}^2\right)\right]^2 + 4\gamma_1^2\Omega^2}. \tag{30}$$

Figure 4 plots $R(\Omega)$ as given by Eq. (30) for $\bar{X} = \bar{Y}$. It can be seen that there exists an upper frequency Ω_{crit}, determined by the condition $R(\Omega_{crit}) = 1$, such that squeezing is impossible above it. For low frequencies we have

$$R(\Omega) = \Omega^2 / \left[4\tilde{\kappa}\left(\bar{X}^2 + \bar{Y}^2\right)\right]^2. \tag{31}$$

Thus, low-frequency fluctuations are efficiently suppressed and deep squeezing is realized. If, on the other hand, $|\bar{X}| \gg |\bar{Y}|$, then radiation exiting the cavity may appear to be sub-Poisson. Let $\omega = \omega_0$ ($\gamma_2 = 0$). Optimal squeezing is not achieved in this case; however, the generation of sub-Poisson statistics in this case is most vivid. According to Eqs. (26) and (27) the exit fluctuations are minimized when

$$\Omega^2 = 3\tilde{\kappa}^2\bar{X}^4 - \gamma_1^2, \quad \bar{Y} = 0. \tag{32}$$

In this case triple squeezing is realized

$$S_{out}^{(x)}(\Omega) = \frac{1}{3} S_{in}^{(x)}(\Omega).$$

The variance of the intensity fluctuations $\langle(\delta I)^2\rangle \approx 4\bar{X}^2 S_{out}^{(x)}(\Omega)$ is also three times smaller than in the coherent state. Thus the photons exiting the nonlinear cavity have sub-Poisson statistics.

In conclusion note again the perspectives of practical implementation of the above techniques for forming nonclassical states of radiation. Their attraction consists in the comparative simplicity and high efficiency of suppresion of quantum fluctuations.

REFERENCES

1. H.H. Ritze and A. Bandilla, Opt. Commun. **29**, 126 (1978), 30, 125 (1979).
2. S. Kielich and R. Tanaś, Monographs in Physics **7**, Quantum Optics, Osselinium, Warsaw (1985), 275 pp.
3. H. Risken and K. Vogel, *Lecture Notes in Physics. Fundamentals of Quantum Optics,* Springer-Verlag, Berlin (1987), 225 pp.
4. C. Gerry and S. Rodrigues, Phys. Rev. A **35**, 4440 (1987).
5. Y. Yamamoto, S. Machida, N. Imoto, et al., J. Opt. Soc. Am. B **4**, No. 10. 1645 (1987).
6. S. Reynaud and E. Giacobino. J. Phys. (Fr.) **49**, Col. C2, Suppl. No. 6. C2-477 (1988).
7. A.V. Belinskiĭ and A.S. Chirkin, Kvant. Electron. **16**, 889 (1989).
8. *Squeezed States of the Electromagnetic Field*, J. Opt. Soc. Am. B **4**, No. 10 (Special issue) (1987).
9. D.N. Klyshko, *Photons and Nonlinear Optics*, Nauka, Moscow (1980).
10. S. Reynaud, C. Fabré, and E. Giacobino, J. Opt. Soc. Am. B **4**, No. 10, 1520 (1987).
11. M. Collet and C. Gardiner, Phys. Rev. A. **30**, 1386 (1984).