Physical model for retrieving integral parameters of the troposphere from satellite IR measurements

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An analytical model for retrieving integral parameters of the troposphere (TIPs) from multichannel IR measurements is presented, which could equally suit measurements in the transmission windows and in the sensing channels with strong absorption. Use of the model for retrieving four TIPs, i.e., surface temperature, lapse rate, integral content, and exponential scale (height) of water vapor vertical distribution is discussed in application to interpretation of data acquired from geostationary satellites. The model is shown to be useful in atmospheric correction of ultra high-resolution IR imagery, height assignment of cloud motion winds, as well as in tropospheric temperature and moisture sensing problems.

Introduction

The problem of atmospheric correction, which is very urgent in determining sea surface temperature (SST) with ultra high resolution, has been the primary motivation of this study. The problem consists in contradiction between requirements, as the satellite radiometers have high spatial resolution only in one IR channel in a sufficiently wide spectral range (for example, 60-m resolution for a Landsat-7/ETM+ in the 6th channel covering the wavelength range from 10.4 to 12.5 μm), while the atmospheric correction requires several channels. The approach to the problem solution, proposed in Ref. 1, reduces the problem to the imagery recalibration to data from a multi-channel radiometer operated in a close spectral range with the following pixel correction by a MCSST-type procedure (multi-channel SST is the common technique for the popular NOAA/AVHRR radiometer). In addition to difficulties with synchronizing the observations of a region from several satellites, such an approach uses regression SST algorithms that describe complicated nonlinear processes of radiation transfer through the atmosphere unsatisfactorily.

Existing approaches to developing SST algorithms rely either on the problem linearization under the assumption of weak absorption and following fitting of additive coefficients or on the regularization of the multifactor radiation transfer models calculated using aerological profiles, the so-called theoretical algorithms. In the first case, a model is incapable of adequately incorporating all possible observation conditions, e.g. Arctic and tropical air conditions or different angles of scanning. In the second case, a necessary cutting down of the number of unknowns is being done at the expense of additional data related to standard situations. Finally, this gives the same results. In practice, the theoretical algorithms are always worse than empirical regressions.[2,3] Again, nonlinear regression relationships[4] and nonregression algorithms[5] with empirical multiplicative deductions turn out to be no better than linear algorithms.[3]

In solving inverse problems of remote sensing (RS), two interrelated troubles arise, namely: modeled processes are multifactor and nonlinear ones while their solutions are to be obtained over a limited number of observations under variable conditions. Nonlinearity of the processes becomes apparent through multiplicative influence of atmospheric factors. This complicates the derivation of an adequate physical model and multiplicity of the factors complicates solution of the inverse problems. Statement of the problem in terms of integral parameters (TIPs) of the troposphere, which are characteristic of vertical atmospheric profiles and of independent meteorological interest, can be promising. Note, that TIPs determination is not reduced to derivation of temperature and humidity profiles, which is the traditional problem of satellite atmospheric sensing, since data on these profiles are redundant and the corresponding inverse problem is ill-posed. On the contrary, the problem of TIPs derivation will be well posed if the parameters are among the factors influencing the radiation transfer. Moreover, TIPs imply certain "specialization" of the measurement channels and use of external (additional) data sources removes restrictions on the model realization both by use of data from different satellites and by problem division into different scales that provides for accuracy increase by means of measurement statistics (atmospheric scale is about 100 km). In so doing, the physical model, assuming optimally chosen measurement channels will serve the data interface for different radiometers.

In the literature there are only a few statements of the remote sensing problem in terms of individual TIP and they basically relate to microwave range (for instance, Refs. 6 and 7), where the use of weak absorption approximation is admissible, which allows one to neglect the pressure (height) dependence of the absorption coefficients. In this paper, an approach

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free of this admission is proposed to solving the inverse problem of combined TIPs derivation from multi-channel IR measurements. This approach extends the proposed[8] physical model of atmospheric absorption to the case of several spectral ranges. The main advantage of the new model is its capability of describing both weak and strong absorption equally well. This is especially important for IR range, where these two extreme cases are observed even in the so-called transmission windows (3.5–4 and 10.5–12.5 μm).

Derivation of the model

The radiation transfer equation for cloudless plane-parallel and horizontally homogeneous atmosphere, written for the path at the zenith angle θ under the assumption of locally thermodynamic equilibrium and neglecting scattering, gives the following value of radiation signal[8] measured with a satellite-borne radiometer

$$R_\lambda(\theta) = \varepsilon_\lambda(\theta)B_\lambda(T_0)\tau_\lambda(\theta) + I_\lambda^{\uparrow}(\theta) + [1 - \varepsilon_\lambda(\theta)]I_\lambda^{\downarrow}(\theta)\tau_\lambda(\theta), \quad (1)$$

where λ is the spectral range indicator (wavelength); ε_λ is the emissivity of the sea surface in the given spectral range; T_0 is the surface temperature; B_λ is the Planck's function; $I_\lambda^{\uparrow}(\theta)$ describes the upward and $[1 - \varepsilon_\lambda(\theta)] \times I_\lambda^{\downarrow}(\theta)\tau_\lambda(\theta)$ the fraction of downward going atmospheric radiation reflected from surface. Vertical inhomogeneity of the atmosphere is described by the height integrals:

$$I_\lambda^{\uparrow}(\theta) = \int_0^{z_s} B_\lambda[T(z)]\,d\tau_\lambda(z, z_s, \theta),$$
$$I_\lambda^{\downarrow}(\theta) = \int_{z_s}^{0} B_\lambda[T(z)]\,d\tau_\lambda(z, 0, \theta), \quad (2)$$

where $T(z)$ is the vertical profile of air temperature; $\tau_\lambda(z, z_s, \theta)$ and $\tau_\lambda(z, 0, \theta)$ are the atmospheric layer transmissions along θ-direction between the height z up to the satellite height z_s and down to the ocean surface, respectively. The physical model in terms of efficient TIPs will be sought by parameterization of the transmission functions $\tau_\lambda(z, z_s, \theta)$ and $\tau_\lambda(z, 0, \theta)$ using the typical (exponential) form of vertical distribution of the absorbing gas pressure and concentration but not specific standard or statistical variables. In this study, transmission function is represented by a superposition of exponents

$$\tau_\lambda(z, z_s, \theta) = \exp[-u_\lambda \exp(z/h_\lambda)m], \quad (3)$$

where u_λ and h_λ are the height-independent parameter functions of TIPs. Such a parameterization follows from the standard expression[8–10]:

$$\tau_\lambda(z, z_s, \theta) = \exp[-u_\lambda(z)m],$$

where $u_\lambda(z) = \int_z^{z_s} k_\lambda(z)\rho(z)\,dz$ is the absorption along vertical direction; $m = \sec\theta$ is the air mass (path length); $k_\lambda(z)$ is the absorption coefficient; $\rho(z)$ is the absorbing gas density distribution under the assumption of exponential height behavior of $k_\lambda(z)$ and $\rho(z)$. Grounds for the assumption are given below.

The absorption coefficient $k_\lambda(z)$ depends on the absorber temperature, pressure, and concentration. Molecular absorption in spectral line wings prevails in bands used for remote sensing of lower atmosphere (< 50 km). In the case of isolated lines the dependence $k_\lambda(z) \sim f(\lambda)p(z)/T(z)$ (Ref. 10) is obtained using the Van Vlackk–Weiskopff law; then it is extended to the case of wide spectral ranges by use of the universal "square root" law $k_\lambda \sim (\rho p)^{1/2} \sim (pp)^{1/2} = p$.[9] In tropospheric sensing, the temperature dependence of absorption coefficients can be neglected as the error $\delta k_\lambda = \delta T/T^2$, introduced by the variation $\delta T = 30$ K, is less than 0.05% at the temperature $T = 250$ K. The situation with the pressure dependence of absorption coefficients is quite different: the error $\delta k_\lambda = \delta p$ attains 40% at the variation $\delta p = 0.4$ atm about the average value $p = 0.6$ atm. The vertical distribution of pressure and concentration of uniformly mixed gases (UMG), mainly, CO_2 in the IR and O_2 in the microwave ranges, to a high accuracy, follows the barometric law

$$p(z) = p_0 \exp(-z/h_0)$$
$$\text{and } \rho(z) = -\partial p(z)/\partial z = (p_0/h_0)\exp(-z/h_0)$$

with the parameter of exponential scale $h_0 \approx 7.8$ km and the standard surface pressure $p_0 = 1$ atm. Again $u_\lambda(z) \sim \exp(-2z/h_0)$, or $h_\lambda = h_0/2$ for the absorption by UMG. It is just this form of parameterization $u_\lambda(z)$, which is used for thermal sensing of the atmosphere in a wide range of absorption by CO_2 (13 to 15 μm).[10]

Absorption of radiation by water vapor (composite absorption) in parallel with that by UMG plays an important role in IR transmission windows. Water vapor concentration in the atmosphere strongly varies but cumulative content curves $w(z)$ in the layer from z to z_s have a universal form for different seasons and regions:

$$w(z) = w_0 \exp(-z/h_{H_2O})$$

with $w_0 = w(0)$ and the scale parameter h_{H_2O} referred to as an exponential height of water vapor. Then

$$\rho_{H_2O}(z) = -\partial w(z)/\partial z = w_0/h_{H_2O} \exp(-z/h_{H_2O}).$$

According to Ref. 11 (Fig. 1.5) $h_{H_2O} \approx 2.4$ km but possible variations from 1.5 to 3 km are to be taken into account.

Two different mechanisms contribute to absorption of IR radiation by water vapor, i.e., 1) linear absorption in wings of H_2O spectral lines – the constituent proportional to air pressure and

2) quadratic absorption – the constituent proportional to water vapor density. The contribution from the second mechanism prevails in a humid air.

Unfortunately, there is a significant uncertainty in the parameterization of water vapor absorption that possesses a huge number of spectral lines that are distributed over the IR and microwave ranges in quite a complicated way. In particular, there is no uniqueness in Van Vlackk–Weiskopff and square root laws interpretations when describing two absorption mechanisms (linear and quadratic relative to humidity). However, this question can be addressed from the standpoint of dimension analysis, which does not require examination of microphysics of H_2O absorption processes. Then quadratic character of integral absorption

$$u_\lambda = u_\lambda(0) = k_\lambda^{(0)} + k_\lambda^{(1)} w_0 + k_\lambda^{(2)} w_0^2$$

with the indices 0, 1, and 2, respectively, for UMG, linear, and quadratic relative humidity constituents completely defines the way of combining exponents:

$$k_\lambda^{(1)}(z)\rho(z) \sim w_0 \exp(-z/h_0)\exp(-z/h_{H_2O}),$$

$$k_\lambda^{(2)}(z)\rho(z) \sim w_0^2 \exp(-2z/h_{H_2O}).$$

Taking into account the earlier obtained UMG absorption parameterization, the vertical absorption $u_\lambda(z)$ from Eq. (3) can be written as a sum (association) of exponents:

$$u_\lambda(z) = \sum_i u_\lambda^{(i)} = \sum_i k_\lambda^{(i)} w_0^i \exp(-z/h^{(i)}),$$

where $h_\lambda^{(0)} = h_0/2$,

$$h_\lambda^{(1)} = h_{H_2O} h_0/(h_{H_2O} + h_0), \quad h_\lambda^{(2)} = h_{H_2O}/2,$$

and $k_\lambda^{(i)}$ are the weighting coefficients determining the contribution of each of the absorption constituent.

Unfortunately, it is impossible to obtain the above weighting coefficients with the help of our phenomenological approach (this is its disadvantage). However, these can be obtained based on laboratory data or precision microphysical computations as well as by direct empirical fitting using *in-situ* (satellite and aerological) measurement data. Actually, determination of the weighting coefficients (calibration) is a separate and quite a complicated problem. They depend on the instrumental function (spectral filter) of a radiometer used and, hence, practical solution of the inverse problems requires that each of the channels used must be calibrated. For the purposes of this work, order-of-magnitude estimations of the weighting coefficients are quite sufficient.

These estimations for atmospheric windows in the IR have been obtained by empirical fitting using averaged experimentally measured angular patterns of the outgoing radiation (3rd, 4th, and 5th channels of the NOAA/AVHRR radiometer) in humid (over Philippine Sea) and dry (Sea of Okhotsk) atmosphere (Ref. 8, Fig. 2.28*b*). Note, that all the values are normalized by the surface pressure ($p_0 = 1$ atm) and, therefore, in solving the altitude problem (e.g., determination of cloud temperature), the weighting coefficients should be recalculated using the barometric law ($k_\lambda^{(0)} \sim p^2$, $k_\lambda^{(1)} \sim p$).

Another way of estimating the weighting coefficients is the use of normalized weighting functions $\partial\tau_{H_2O}(z)/\partial z$ of the water vapor channel of a GMS-5 radiometer (the 6.3–6.7 μm range of absorption by H_2O) obtained using the MODTRAN-3 model for the dry tropical atmospheric profile ($w_0 = 2$ g/cm^2, $h_{H_2O} = 2.3$ km is the air downgliding zone) and the humid one ($w_0 = 4$ g/cm^2, $h_{H_2O} = 2.5$ km is the air upgliding zone) from the TIGR-2 database.[12] In so doing, it was assumed that absorption by UMG and the quadratic constituent of H_2O absorption can be neglected ($k_{H_2O}^{(0)} = k_{H_2O}^{(2)} = 0$), while the values of h_{H_2O} were taken based on indirect information from the information source. Under these assumptions, the value $k_{H_2O}^{(1)} = 5$ gave an excellent agreement between the heights of maximum of the weighting function $\partial\tau_{H_2O}(z)/\partial z$.

To substantiate the parameterization (3), it remained only to check the capability of approximation (association) of different-scale exponents by one exponent with some effective parameters. In our experiments, $\sum_i k_\lambda^{(i)} w_0^i \exp(-z/h^{(i)})$ were χ^2-approximated, using the ORIGIN package, by the exponent $u_\lambda \exp(-z/h_\lambda)$ with the weighting coefficients from the below Table.

Table. Weighting coefficients ($u_\lambda = k_\lambda^{(0)} + k_\lambda^{(1)} w_0 + k_\lambda^{(2)} w_0^2$, w_0, g/cm^2) for 3rd, 4th, and 5th channels of an NOAA/AVHRR radiometer

Wavelength λ, μm	$k_\lambda^{(0)}$	$k_\lambda^{(1)}$	$k_\lambda^{(2)}$
3.7	0.05	0.03	0.003
11	0.015	0.035	0.033
12	0.006	0.06	0.05

The value χ^2 turned out to be quite small (on the order of 10^{-4}) in wide ranges of w_0 (from 0 to 6 g/cm^2) and h_{H_2O} (from 1.8 to 3 km) values. This allowed the conclusion on correctness of the parameterization (3) to be drawn. Fitting results h_λ are presented in Fig. 1.

The final stage in the model derivation is a linear approximation of the radiation temperature profile

$$B_\lambda[T(z)] = B_\lambda(T_0) - \gamma_\lambda z,$$

where $\gamma_\lambda = [B_\lambda(T_{z^*}) - B_\lambda(T_0)]/z^*$ and temperature at some specific height z^* (usually 5–6 km) is set by the temperature height antigradient Γ: $T_{z^*} = T_0 - \Gamma z$. Such an approximation works in the troposphere since practically there is no water vapor in the higher layers and the atmosphere there is transparent for IR radiation. On the other hand, the statistical analysis of sensing data over the North Atlantic[13] confirms linearity of the temperature profile up to the tropopause and, besides, the closeness of the surface temperature to SST that can serve a reference point in the linear approximation of the profile $B_\lambda[T(z)]$.

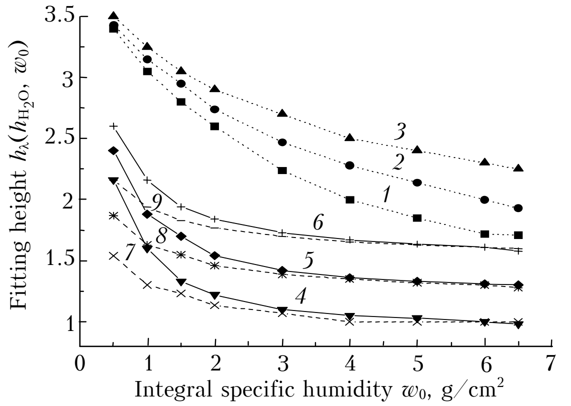

Fig. 1. The reduced absorption height h_λ for IR atmospheric transmission windows as a function of the integral water vapor content w_0 and of the water vapor height scale (exponential height) h_{H_2O}: curves *1*, *2*, *3* ($h_{3,7}$) correspond to 1.8, 2.4, and 3 km; curves *4*, *5*, *6* (h_{11}) and *7*, *8*, *9* (h_{12}) correspond to similar values as for the curves *1*–*3*.

The assumptions accepted result in the physical model whose derivation and the definition of special functions Ein and E_1n are given in the Appendix:

$$R_\lambda = B_\lambda(T_0)(1 - \tau_\lambda^2\rho_\lambda) - \\ - \gamma_\lambda h_\lambda[Ein(u_\lambda m) + \tau_\lambda^2\rho_\lambda E_1 n(u_\lambda m)], \quad (4)$$

where $\rho_\lambda = 1 - \varepsilon_\lambda$ is the reflection coefficient (~10^{-2} in the IR range) and the sea surface emissivity ε_λ is known for the zenith angles $\theta < 70°$ with good accuracy and it weakly depends on the wind strength.[14] It is a characteristic feature of the model that radiation R_λ measured with a satellite radiometer linearly depends on h_λ; this is rather unexpected fact that emphasizes importance of an accurate parameterization of the vertical distribution of absorption. Having preliminarily calibrated the weighting coefficients $k_\lambda^{(i)}$, the effective absorption u_λ is the following known (quadratic) function

$$u_\lambda = k_\lambda^{(0)} + k_\lambda^{(1)} w_0 + k_\lambda^{(2)} w_0^2,$$

while the above obtained approximation of the reduced height h_λ makes it a function of h_{H_2O} and w_0. For practical use, the parameter h_λ can be directly defined as a function of w_0 and h_{H_2O} for a characteristic height z^*, for example

$$h_\lambda = z^*/\ln[(1/u_\lambda)\Sigma_i k_\lambda^{(i)} w_0 \exp(-z^*/h^{(i)})].$$

Hence, the recorded radiation R_λ depends only on four TIPs (T_0, Γ, w_0, h_{H_2O}) characteristic of tropospheric profiles of temperature and humidity.

Comparative analysis of models

Curves of radiation temperature deficit $T_\lambda - T_0$ as functions of the path length m (Fig. 2*a*), calculated for spectral range near 11 μm, can be used for limb analysis (by m) of existing methods and algorithms for TIPs derivation by use of criteria of adequate nonlinearity modeling and allowing for multifactor nature of the processes of radiation transfer through the atmosphere.

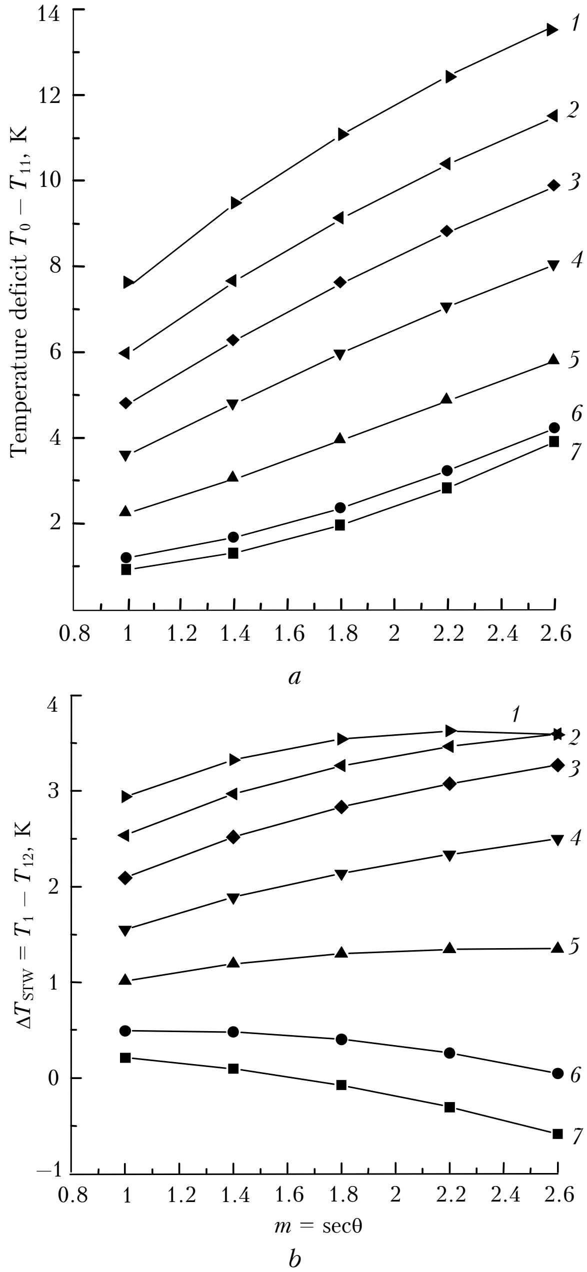

Fig. 2. Limb analysis of MCSST: radiation temperature deficit for $\lambda = 11$ μm (*a*) and temperature difference in channels of a split transmission window ΔT_{STW} (*b*) as a function of the path length m at different w_0 values (fixed $h_{H_2O} = 2.4$ km and $\Gamma = 6.5$ K/km): $w_0 = 6$ (*1*); 5 (*2*); 4 (*3*); 2 (*5*); 1 (*6*), and 0.5 g/cm^2 (*7*).

In particular, the nonlinear dependence of temperature deficit on the path length is evidently the reason why the double-angle method (extrapolation to zero air mass, $m = 0$) of the TIPs determination doesn't work.[15] As shown in Ref. 8, correct determination of the TIPs in a single spectral range requires at least three-angle measurements.

In deriving the physical model, we did not limit the value of the effective absorption of radiation by the atmosphere. Compare the model (4) with the radiophysical algorithms[6,7] by their asymptotic behavior under strong absorption. In our designations, the IR-adapted model[6] is written as follows

$$R_\lambda = B_\lambda(T_0)\left(1-\tau_\lambda^2\rho_\lambda\right) - \gamma_\lambda h_{H_2O}(1-\tau_{H_2O})\left(\tau_{UMG} + \tau_\lambda^2\rho_\lambda\right).$$

Our model describes the loss of transparency at large angles and humidity ($\tau_{H_2O} \approx 0$) as the logarithmic growth:

$$R_\lambda \approx B_\lambda(T_0) - \gamma_\lambda h_\lambda[\ln(u_\lambda m) + 0.58],$$

while Ref. 6 interprets it like constant saturation $R \approx B_\lambda(T_0) - \gamma_\lambda h_{H_2O}\tau_{UMG}$. Since experimental data[8] does not reveal constant saturation even in the tropical atmosphere, our model is evidently preferable at least for the IR range. On the other hand, recently proposed radiophysical model of the cloudy atmosphere[7] takes, for the situations considered above, the following form

$$R_\lambda \approx B_\lambda(T_0) - \gamma_\lambda h_\lambda u_\lambda(1 - k u_\lambda)$$

and thus it agrees with the model of spectral-angular algorithm of the TIPs determination[16] accurate to the coefficient k (0.25 and 0.26, respectively). Both of these models work well under moderate absorption (in the IR and microwave windows even in the tropics), but at stronger effective absorption ($u_\lambda > 1/k$) the temperature deficit becomes a surplus and makes the models useless in the ranges where H_2O and CO_2 absorb.

Finally, the derived model allows qualitative description of numerous linear and nonlinear regressive MCSST algorithms based on the assumption of sufficiency of two channels within the "split transmission window" (STW), i.e., 11 and 12 μm, to eliminate temperature deficit (atmospheric correction). All the existing versions of this algorithm are built up using the basic model in the form

$$T_0 - T_{11} = \gamma\Delta T_{STW},$$

where $\Delta T_{STW} = T_{11} - T_{12}$, $T_\lambda = B_\lambda^{-1}(R_\lambda)$, and γ is a constant. Since this model, theoretically justified only for weak absorption, turned out to be insufficiently accurate in practice, some attempts to modify it have been undertaken. Principal modifications reduced to introducing additive corrections for the angular dependence (limb correction), linear MCSST algorithms.

Some authors substitute the constant γ by an empirical function (nonlinear MCSST algorithms). In this case, nonlinearity of implicit w_0-dependence of ΔT_{STW}, set by the empirical function γ, is assumed. Figure 2*b* presents the limb analysis for this case. Since the shape of the curves is w_0-depended, the existence of a universal correlation between the temperature difference in STW channels and the temperature deficit in the 11 μm channel is evidently of low probability. In other words, a one-to-one transformation of the curves in Fig. 2*b* to those in Fig. 2*a* seems to be impossible. Moreover, even an explicit form of the curves in Fig. 2*b* (known w_0) reflects only a part of multifactor influence of the atmosphere as these are obtained at a fixed value of the cumulative parameter $t^* \approx 11$ K $(B[t^*] = \gamma_\lambda h_\lambda)$. Figure 3 shows the γ variations at admissible variations of this parameter, absolutely uncontrollable by the MCSST algorithms.

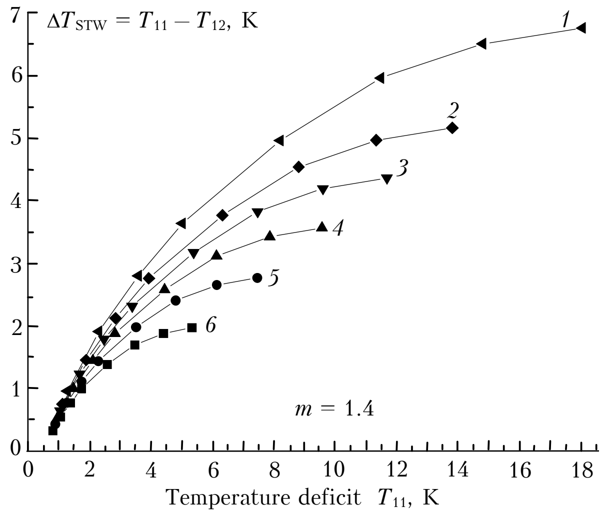

Fig. 3. Influence of uncontrollable TIPs on errors of MCSST algorithms when w_0 varies from 0.5 to 5 g/cm^2 and the cumulative parameter $t^* = B^{-1}[\gamma_\lambda h_\lambda]$ varies from 5 to 15 K: t = 15 (*1*); 13 (*2*); 11 (*3*); 9 (*4*), 7 (*5*), and 5 (*6*).

The following conclusion can be drawn from the qualitative analysis performed: multifactor nature of the processes modeled requires that all of the four TIPs must be determined and, hence, a nonlinear physical model is needed, capable of working under conditions of different absorption strength.

Conclusion

Let us show possibilities of determining all the four TIPs, by means of the physical model derived, from data of geostationary meteorological satellites with STW, H_2O, and CO_2 channels (e.g. GOES and MSG). Tropospheric model (4) works for channels whose weighting function $\partial\tau_\lambda/\partial z$ is mostly in the troposphere, where temperature varies linearly with height, in particular, for STW, H_2O channel, and "low" channels in the CO_2 absorption channel (12.5–13.5 μm). The possibility of separating multiplicative variables Γ and h_{H_2O} in H_2O and CO_2 channels follows from the fact that they are characterized by "logarithmic growth" of R_λ (their absorption coefficients $k_\lambda^{(1)}$ and $k_\lambda^{(0)}$ are about 100 times larger than for STW channels) and, hence, weakly depend on w_0. Taking into account that h_λ has known value $h_0/2$ in CO_2 channel; this channel can be specialized for use in determining the Γ parameter. Then, the H_2O channel can be used for determining h_{H_2O} and STW channels for determining w_0 and T_0.

Some prospects of using the derived model in solving the inverse problems of remote sensing of the Earth's and sea surfaces follow from model analysis. First, since all the TIPs have atmospheric scales, the accuracy of TIPs determination can be essentially increased by spatially smoothing measured data. This allows one to separate cloudiness filtration from surface temperature determination and to use data of different spatial resolution, e.g. combining different data sources (radiometers). Second, the derived model can be useful for solving a number of problems, among which there are determination of parameters of clouds and altitude of cloud wind, as well as of the tropospheric temperature and humidity sensing (as the initial approximation or for limb correction).

Thus, from the positions of this study, it is impossible to separate the account of angular dependence from determination of the corresponding TIPs. Therefore, limb correction of AMSU microwave radiometer measurements seems to be not quite adequate, at least in the form proposed in Ref. 17, while correct solution for cloudy atmosphere is possible only using six TIPs (four cloudless and two for cloudy conditions). Simple arithmetic suggests the possibility of their determination using data from only three channels (e.g. two STW channels and the H_2O channel) available practically at all geostationary satellites. Brief prerequisites are as follows. Cloudy atmosphere always requires a pair of cloudy and clear-sky measurements in the IR range. As a rule, such pairs are obtained by means of isolating separate modes in histograms or extrapolation in sets of pixels with partial covering. In any case, it is possible to talk about doubling of channels number sufficient to determine all six TIPs. Adaptation of the model to the microwave range (but not to the visible one due to neglect of scattering) is not problematic, but detailed consideration of the question is beyond the scope of this study.

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Appendix

Derivation of analytical representation of the tropospheric model (spectral indicator λ and angle θ are omitted)

For analytical representation of the first integral in Eq. (2), we isolate the isothermal part

$$I^{\uparrow} = B(T_0)[1-\exp(-um)] - \gamma\int_0^{z_s} z\,\mathrm{d}\tau(z,z_s).$$

Integrating the gradient part by parts, we obtain

$$\int_0^{z_s} z\,\mathrm{d}\tau(z,z_s) = \int_0^{z_s}[1-\tau(z,z_s,\theta)]\,\mathrm{d}z - \int_0^{z_s}\mathrm{d}[1-\tau(z,z_s)]z,$$

where the last term is zero since $\tau(z_s, z_s) = 1$ for the upper integration limit while $z = 0$ for the lower one. Change of variables $y = \mathrm{e}^{-z/h}$ (for passage to ordinate exponent in τ) allows the first integral to be expressed (using the change $\mathrm{d}z = -h\mathrm{d}y/y$ and $t = umy$; where t varies from um to 0) through an additional integral exponential function (e.g. Ref. 18):

$$\int_0^{z_s}[1-\tau(z,z_s)]\,\mathrm{d}z = hEin(um),$$

where

$$Ein(x) = \int_0^x \frac{1-\mathrm{e}^{-t}}{t}\mathrm{d}t = E_1(x) + \ln x + c, \quad c \approx 0.5772$$

is the Euler constant. (For $E_1(x)$ calculation, good approximations and software implementations exist in different libraries). Finally, for the first integral, we obtain

$$I^{\uparrow} = B(T_0)[1-\exp(-um)] - \gamma hEin(um).$$

The second integral (2) can be transformed to the form of the first one by splitting the multiplicative total atmospheric transmission into two parts $\tau = \tau(z, z_s)\tau(z, 0)$ and taking τ outside the integral:

$$I^{\downarrow} = \tau\int_0^{z_s} B[T(z)]\,\mathrm{d}\frac{1}{\tau(z,z_s)}.$$

As $\tau(z, z_s) = \exp(-t)$, we have

$$I^{\downarrow} = B(T_0)(1-\tau) - \gamma h\tau\int_0^{um}[\exp(t)-1]\,\mathrm{d}t/t$$

or using the special function

$$E_1n(x) = \int_0^x \frac{\mathrm{e}^t - 1}{t}\mathrm{d}t = E_1(x) - \ln x - c,$$

we obtain

$$I^{\downarrow} = B(T_0)(1-\tau) - \gamma h\tau E_1n(um).$$

Hence, the final expression for radiation measured from a satellite in any IR range considered has the form (4).

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