SYNTHESIS OF AN ADAPTIVE OPTICAL SYSTEM FOR APERTURE SOUNDING

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Received December 11, 1990

An adaptive optical system for aperture sounding is synthesized. The problem of synthesis of such a system is reduced to that of optimal nonlinear filtering and is solved based on the method of invariant embedding. The resulting relations are derived for the operation algorithm of the adaptive optical system.

At present the development of efficient operation algorithms of the adaptive optical systems (AOS's) is an important scientific and technical problem. It was shown in Refs. 1 and 2 that such algorithms for phase–conjugated AOS's may be synthesized employing the theory of optimal linear filtering. In addition, the adaptive algorithm synthesized in Ref. 2 permits one to compensate partly for the effect of the *a priori* uncertainty in the statistical characteristics of the phase fluctuations on the operating efficiency of a phase–conjugated AOS.

The present paper analyzes the operation algorithm of an aperture–sounding AOS. In contrast to phase–conjugated AOS's, the aperture–sounding AOS's are known to operate on the basis of optimizing against certain complex quality functionals, which depend on the design of the AOS, the given physical requirements, and technical capabilities of the system.[3] In addition, the method of aperture sounding is based on the direct measurements of the parameter being optimized, while the needed phase corrections (the controllable values) $\alpha(kT)$, usually nonlinearly related to the measured quantities, are found only indirectly. Thus the method may be extensively used in adaptive focusing systems, in image forming systems, and those forming beams of prescribed shapes. They all differ in their quality functionals being optimized. However, their common feature is that the vector $\alpha(kT)$ of the parameters being monitored in the process of operation of the aperture–sounding AOS (here and below we consider the system operating in discrete time) is nonlinearly related to the valid signal $\mathbf{u}_s[\alpha(kT), kT]$. When the input field of the system $\mathbf{u}_{inp}(\alpha, kT)$ is formed, noise is additively superimposed on the valid signal, so that

$$\mathbf{u}_{inp}(\mathbf{a}, kT) = \mathbf{u}_s[\mathbf{a}(kT), kT] + \mathbf{n}(kT) , \tag{1}$$

where T is the sampling period, $\mathbf{n}(kT)$ is the discrete white noise with zero mean and correlation matrix $M\{\mathbf{n}(iT)\mathbf{n}^T(jT)\} = R\delta_{ij}$, R is the symmetric nonnegative matrix, and $M\{.\}$ is the operator of mathematical expectation.

As a figure of merit of operation of the aperture–sounding AOS, we can choose the functional J, which estimates the degree of "similarity" of the detected field $\mathbf{u}_{inp}(\alpha, kT + T)$ and the (prescribed) field under study,

$$\mathbf{u}_{pr}(\alpha, kT + T) = \mathbf{u}_s\left[\hat{\alpha}(kT + T/kT), kT + T\right] ,$$

and has the form

$$J = \frac{1}{2}\sum_{k=0}^{l-1}\{\mathbf{u}_{inp}(\alpha, kT+T) - \mathbf{u}_s[\hat{\alpha}(kT+T/kT), kT+T]\}^T R^{-1} \times$$

$$\times \{\mathbf{u}_{inp}(\alpha, kT + T) - \mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]\} , \tag{2}$$

where $\hat{\alpha}(kT + T/kT)$ is the extrapolated estimate of the parameter of the information–bearing signal (its wavefront tilt).

In that case, the problem of synthesis of an aperture–sounding AOS as a nonlinear system can be generally formulated in the following way: for the prescribed parameters of the valid signal $\mathbf{u}_s[\alpha(kT), kT]$ and noise $\mathbf{n}(kT)$ we must determine such physically possible transformations of the input field of the system $\mathbf{u}_{inp}(\alpha, kT)$ recorded starting from time k_0T, which would yield an estimate $\hat{\alpha}(kT)$ of the valid signal parameter which minimizes the value of figure of merit (2) at each moment $k \ge k_0$. It is also assumed that the parameter being estimated can be described by the equation of the form

$$\alpha(kT + T) = F[\alpha(kT), kT] + \Gamma[\alpha(kT), kT]\xi(kT) , \tag{3}$$

where $F[\alpha(kT), kT]$ and $\Gamma[\alpha(kT), kT]$ are the nonlinear functions; $\xi(kT)$ is the discrete white noise with zero mean and covariation matrix $M\{\xi(iT)\xi^T(jT)\} = Q\delta_{ij}$, Q is the symmetric nonnegative matrix, and $M\{\mathbf{n}(iT)\xi^T(jT)\} = 0$.

The formulated problem belongs to the class of the problems of optimal nonlinear filtering. We consider its solution based on the method of invariant embedding.[4] That method is used for solving the two–point boundary problem of determining either the difference equation or the coefficients of that equation which describes the physical phenomena in our AOS on the basis of some prescribed criterion (functional (2) in our case). This method is conceptually simple and highly flexible, which was confirmed in Ref. 2, in which an adaptive algorithm for filtering the signals in a phase–conjugation AOS is synthesized with the help of that technique.

Let us find the estimate $\hat{\alpha}(kT)$, which minimizes the functional (2). For this purpose let us consider the Hamiltonian[4]

$$Z[\alpha(kT), \lambda(kT + T)] = \frac{1}{2}\{\mathbf{u}_{inp}(\alpha, kT + T) -$$

$$- \mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]\}^T R^{-1}\{\mathbf{u}_{inp}(\alpha, kT + T) -$$

$$- \mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]\} + \lambda^T(kT + T)\alpha(kT) ,$$

where $\lambda(kT + T)$ are undetermined factors. In our case the canonical equations for $\lambda(kT + T)$ and $\alpha(kT)$ have the form

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$$\alpha(kT + T) = \frac{\partial Z[\alpha(kT), \lambda(kT + T)]}{\partial \lambda(kT + T)},$$

$$\lambda(kT) = \frac{\partial Z[\alpha(kT), \lambda(kT + T)]}{\partial \alpha(kT)}$$

with the boundary conditions at the ends of the interval $[0, eT]$

$$\alpha(0) = \alpha_0, \ \ \lambda(eT) = 0 \ .$$

Thus, the problem of minimizing functional (2) is reduced to the two–point boundary problem. Its solution by the method of invariant embedding[4] gives the following recursion equations, optimal within the quasioptical approximation, which describe the operation of the aperture–sounding AOS:

$$\hat{\alpha}(kT + T) = \hat{\alpha}(kT + T/kT) + K(kT + T) \times$$

$$\times \frac{\partial \mathbf{u}_s^T[\hat{\alpha}(kT + T/kT), kT + T]}{\partial \hat{\alpha}(kT + T/kT)} R^{-1}\{\mathbf{u}_{inp}(\alpha, kT + T) -$$

$$- \mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]\} , \qquad (4)$$

$$\hat{\alpha}(kT + T/kT) = F\,[\hat{\alpha}(kT), kT] , \qquad (5)$$

and

$$K^{-1}(kT + T) = \left\{ \frac{\partial F[\hat{\alpha}(kT), kT]}{\partial \hat{\alpha}(kT)} K(kT) \frac{\partial F^T[\hat{\alpha}(kT), kT]}{\partial \hat{\alpha}(kT)} + \right.$$

$$\left. + \Gamma[\hat{\alpha}(kT), kT]\, Q\Gamma^T[\hat{\alpha}(kT), kT] \right\}^{-1} +$$

$$+ \frac{\partial \mathbf{u}_s^T[\hat{\alpha}(kT+T/kT), kT+T]}{\partial \hat{\alpha}(kT+T/kT)} R^{-1} \frac{\partial \mathbf{u}_s[\hat{\alpha}(kT+T/kT), kT+T]}{\partial \hat{\alpha}(kT+T/kT)} -$$

$$- \{\mathbf{u}_{inp}(\alpha, kT + T) - \mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]\} \times$$

$$\times R^{-1} \frac{\partial}{\partial \hat{\alpha}(kT + T/kT)} \frac{\partial \mathbf{u}_s^T[\hat{\alpha}(kT + T/kT), kT + T]}{\partial \hat{\alpha}(kT + T/kT)} , \qquad (6)$$

with the initial conditions ($k = 0$)

$$\hat{\alpha}(0) = m_\alpha; \ \ K(0) = d_\alpha, \qquad (7)$$

where m_α and d_α are the mathematical expectation and the variance of the parameter of valid signal.

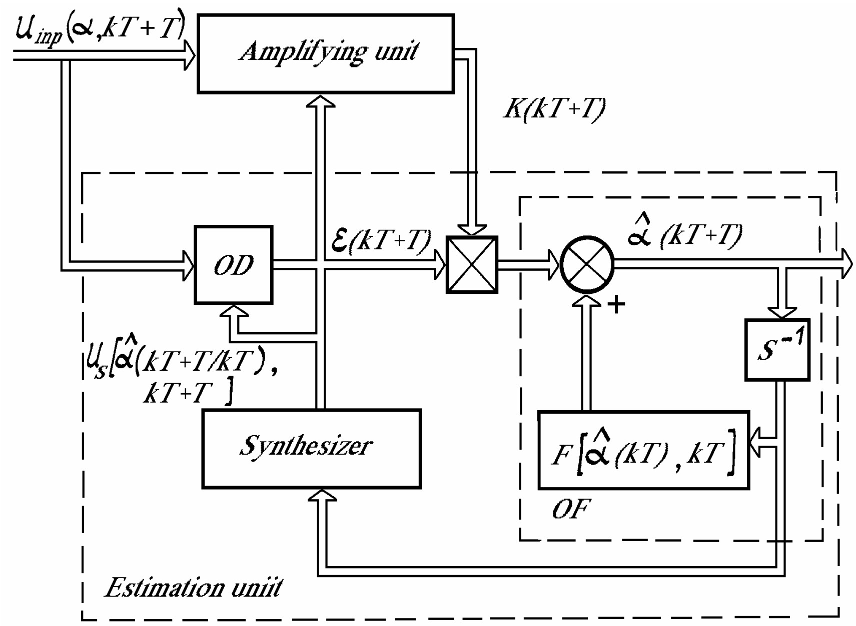

FIG. 1. Block diagram of the aperture–sounding AOS.

In accordance with Eqs. (4)–(6), the block diagram of the aperture–sounding AOS can be represented in the form shown in Fig. 1. The block diagram includes two units. The input field $\mathbf{u}_{inp}(\alpha, kT + T)$ is fed to both inputs. The current estimate $\hat{\alpha}(kT + T)$ is formed at the output from the first described by Eq. (4) and designated as an estimation unit. According to Eq. (4), the estimation unit includes an optimal discriminator (OD) a smoothing optimal filter (OF) and a synthesizer. They are the nonlinear elements. The output signal from the discriminator

$$\varepsilon(kT + T) = \frac{\partial \mathbf{u}_s^T[\hat{\alpha}(kT + T/kT), kT + T]}{\partial \hat{\alpha}(kT + T/kT)} \times$$

$$\times R^{-1} \{\mathbf{u}_{inp}(\alpha, kT + T) - \mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]\}$$

characterizes the difference between the field at the input of the system and the field under study. The structure of the smoothing optimal filter is described by Eqs. (4) and (5). The synthesizer appears to be the major part of the aperture–sounding system, with the help of which the emitted field $\mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]$ is controlled and the feedback loop of the AOS is closed on the basis of estimates $\hat{\alpha}(kT)$

The amplification of the signal from the estimation unit with the gain $K(kT + T)$ which characterizes the accuracy of the estimation is realized by the second amplifying unit. The structure of this unit is described by Eq. (6) and is not shown to simplify the figure.

In practice one may always identify such an interval of variation of the evaluated parameter (kT), for nonlinear and discrete process (3) may be linearized. Equation (3) then takes the form

$$\alpha(kT + T) = F\alpha(kT) + \Gamma\xi(kT) , \tag{8}$$

where F and Γ are the matrices, accounting for the statistical properties of the processes $\alpha(kT)$ and $\xi(kT)$.

In this case, the recursion equations (5) and (6) describing the operation of the aperture–sounding AOS, take the form

$$\hat{\alpha}(kT + T/kT) = F\hat{\alpha}(kT) \tag{9}$$

and

$$K^{-1}(kT + T) = \{FK(kT)F^T + \Gamma Q\Gamma^T\}^{-1} +$$

$$+\frac{\partial \mathbf{u}_s^T[\hat{\alpha}(kT+T/kT), kT+T]}{\partial\hat{\alpha}(kT+T/kT)} R^{-1} \frac{\partial \mathbf{u}_s[\hat{\alpha}(kT+T/kT), kT+T]}{\partial\hat{\alpha}(kT+T/kT)} -$$

$$- \{\mathbf{u}_{inp}(\alpha, kT + T) - \mathbf{u}_s[\hat{\alpha}(kT + T/kT), kT + T]\} \times$$

$$\times R^{-1} \frac{\partial}{\partial\hat{\alpha}(kT + T/kT)} \frac{\partial \mathbf{u}_s^T[\hat{\alpha}(kT + T/kT), kT + T]}{\partial\hat{\alpha}(kT + T/kT)} . \tag{10}$$

Then in accordance with Eqs. (4), (9), and (10), the block diagram of the aperture–sounding AOS (see Fig. 1) is simplified as follows: the smoothing optimal filter in the estimation unit becomes linear (and, generally speaking, nonstationary) while the nonlinear transformations are performed by the optimal discriminator only.

The advantage of the described method consists in the fact that the optimal discriminator and the optimal filter are synthesized separately. In addition, the optimal discriminator remains unchanged when the spectral characteristics $\alpha(kT)$ change, so that only a new smoothing optimal filter is sought.

The approach permits one to employ the apparatus of the theory of the optimal nonlinear filtering to design AOS's for various applications which use the method of aperture sounding.

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