SOUNDING OF AIR DENSITY, PRESSURE, AND TEMPERATURE BY A SINGLE–FREQUENCY LIDAR: ANALYSIS OF ERRORS

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The error in reconstructing air density, temperature and pressure from the data of sounding with a single–frequency lidar is analyzed. Two variants of the laser source location in space and on the Earth's surface are considered.

Pressure and temperature are standard meteorological parameters and are measured at the network of aerological stations by means of radiosondes. The highest altitude of radiosonde measurements is about 30 km (Ref. 1). The data on pressure and temperature at the heights above 30 km are obtained by means of meteorological rockets.[2] The systems performing laser techniques of sounding the atmospheric parameters[3] have some possibilities of measuring temperature and pressure at these heights. In parallel with lidar investigations from the Earth's surface, the placement of lidars onboard satellites can be quite effective. In this case the scope of the whole Earth atmosphere by measurements is attractive. Spaceborne lidar measurements have become realistic after the flights of the NASA LITE lidar in 1994 (Ref. 4) and the Russian BALKAN-1 lidar in May, 1995 (Ref. 5).

Laser method of measuring the above-considered meteorological parameters is based on the molecular light scattering phenomenon. When the resonance scattering effect is absent, and the aerosol concentration in the atmosphere is insignificantly low, the backscattering coefficient, $\beta_{\pi m} \gg \beta_{\pi a}$, and the return signal is unambiguously related to the molecular scattering coefficient, that, in its turn, is proportional to the density of the atmosphere. Air temperature and pressure are calculated by means of hydrostatics and state equations from the data on the atmospheric density. The detailed review of papers devoted to the lidar measurements of the scattering coefficients and air pressure can be found in Refs. 2 and 3. The results of temperature measurements in the stratosphere and mesosphere on the basis of the molecular scattering of light are presented in Ref. 6. However, use of the measurement data supposes certain accuracy requirements, which, as for climatology, general and applied meteorology, are reduced to the acceptable error of 5–10% for pressure and 5–25 K for temperature.[2,8] This paper is devoted to the analysis of errors in measuring air density, temperature, and pressure by the laser sounding method from the molecular scattering for spaceborne and ground-based lidars.

When ignoring the aerosol scattering, the solution of lidar equation relative to the molecular scattering coefficient has the from (one can found the general solution of lidar equation in Ref. 9)

a) for a spaceborne lidar

$$\alpha(z) = \frac{S(z)}{S_0}\left\{\alpha_0^{-1} + 2\int\limits_{z_0}^{z}\left[\frac{S(z')}{S_0}\right]dz'\right\}^{-1}; \tag{1}$$

$$S(z) = U(z)(H - z)^2; \tag{2}$$

b) for a ground based lidar

$$\alpha(z) = \frac{S(z)}{S_0}\left\{\alpha_0^{-1} - 2\int\limits_{z_0}^{z}\left[\frac{S(z')}{S_0}\right]dz'\right\}^{-1}; \tag{3}$$

$$S(z) = U(z)\, z^2 . \tag{4}$$

Here $U(z)$ is the return signal from a scattering volume at the height z; z_0 is the height of lidar calibration (the height, from which we want to reconstruct the molecular scattering coefficient profile $\alpha(z)$); $\alpha_0 = \alpha(z_0)$; H is the satellite orbit altitude (the altitude, from which the lidar operates).

Density of the atmosphere $\rho(z)$ is connected with the coefficient of molecular scattering $\alpha(z)$ by the relationship

$$\alpha(z) = \sigma\rho(z), \tag{5}$$

where σ is the molecular scattering coefficient reduced to the unit density.

Thus, if the lidar return signals $U(z)$ have been measured, we obtain the molecular scattering coefficient and then the density of the atmosphere by formulas (1)–(5). Evidently, the height range, in which we want to obtain $\rho(z)$ from optical scattering properties, is completely determined by the molecular scattering.

The relative error in determining the air density δ_ρ is equal to the error in reconstructing $\alpha(z)$

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$$\delta_\rho^2 = \delta_\alpha^2 = \left(\frac{\Delta S}{S}\right)^2 + \left(\frac{\Delta S_0}{S_0}\right)^2 \times$$

$$\times \left[\left(\frac{\alpha(z)}{\alpha_0(S(z)/S_0)}\right)^2 - 2\delta(z - z_0)\,\frac{\alpha(z)}{\alpha_0(S(z)/S_0)}\right] +$$

$$+ \left(\frac{\Delta\alpha_0}{\alpha_0}\right)^2 \left(\frac{\alpha(z)}{\alpha_0(S(z)/S_0)}\right)^2, \tag{6}$$

where

$$\left(\frac{\Delta S}{S}\right)^2 = \left(\frac{\Delta U}{U}\right)^2 = \frac{U(z) + U_\mathrm{b} + U_\mathrm{d}}{nU^2(z)} \tag{7}$$

is the relative error in measuring the return signals written in the Poisson statistics approximation; U_b is the signal due to background radiation; U_d is the dark current signal; n is the number of radiation pulses; δ is the delta function;

$$(\Delta\alpha_0/\alpha_0)^2 = (\Delta P_0/P_0)^2 + (\Delta T_0/T_0)^2 \tag{8}$$

is the relative error in determining α_0 from the radiosonde data on meteorological parameters $P_0 = P(z_0)$ and $T_0 = T(z_0)$; ΔP_0 and ΔT_0 are the errors in measuring pressure and temperature.

In order to obtain the formulas for calculating pressure and temperature from the measured air density $\rho(z)$, it is necessary to use the equations of static and the ideal gas law (see, for example, Ref. 10)

$$\mathrm{d}P = -\,g\rho\mathrm{d}z,\ \rho = P\mu/RT,$$

from which it follows:

$$P(z) = P_0 - \int\limits_{z_0}^{z} g\rho\mathrm{d}z; \tag{9}$$

$$T(z) = \frac{1}{\rho(z)}\left[T_0\,\rho_0 - \frac{\mu}{R}\int\limits_{z_0}^{z} g\rho\mathrm{d}z\right], \tag{10}$$

where $\rho_0 = \rho(z_0)$; R is the universal gas constant; μ is the molecular weight of air; g is the acceleration of gravity equal to[7]

$$g(z) = g_0(\varphi)[R_\mathrm{E}/(R_\mathrm{E} + z)]^2;\ g_0(\varphi) = g_0(1 - a_1\cos 2\varphi),$$

where $g_0 = 980.616$ cm/sec^2 is the acceleration of gravity at the latitude $\varphi = 45°$; $R_\mathrm{E} = 6370$ km is the mean radius of the Earth; φ is the latitude, and $a_1 = 0.0026$.

The errors in determining pressure and temperature are determined by the following formulas:

$$(\Delta P)^2 = (B/\sigma^2) + (\Delta P_0)^2[1 - 2(A/\sigma P_0)]; \tag{11}$$

$$(\Delta T)^2 = \left(\frac{\Delta\alpha}{\alpha}\right)^2 T^2 + B^2\left(\frac{\mu}{R\alpha}\right)^2 + C\,\frac{2T\mu}{R\alpha} + \left(\frac{\Delta\alpha_0}{\alpha_0}\right)^2 \times$$

$$\times\left[T_0^2\left(\frac{\alpha_0}{\alpha}\right)^2 - \frac{2TT_0}{S/S_0} - A\,\frac{2\mu}{R\alpha}\,\frac{\alpha_0\,T_0}{\alpha}\right] +$$

$$+ (\Delta T_0)^2\left[\frac{2T}{T_0}\,\frac{1}{S/S_0} + A\,\frac{2\mu}{R\alpha}\,\frac{\alpha_0}{\alpha\,T_0} - \left(\frac{\alpha_0}{\alpha}\right)^2\right]; \tag{12}$$

$$A = \int\limits_{z_0}^{z} g\alpha\,\frac{\alpha\mathrm{d}z'}{\alpha_0\,(S/S_0)}\,; \tag{13}$$

$$B = A^2\left[\left(\frac{\Delta S_0}{S_0}\right)^2 + \left(\frac{\Delta\alpha_0}{\alpha_0}\right)^2\right]; \tag{14}$$

$$C = A\,\frac{\alpha}{\alpha_0\,(S/S_0)}\left[\left(\frac{\Delta S_0}{S_0}\right)^2 + \left(\frac{\Delta\alpha_0}{\alpha_0}\right)^2\right]. \tag{15}$$

Let us present some results of numerical simulations. Let us make calculations for the parameters of the BALKAN-3 spaceborne lidar[11]: $\lambda = 355$ nm; $E = 0.2$ J is the pulse energy; $f = 50$ Hz is the pulse repetition rate; $\Delta t = 2$ sec is the signal accumulation time; $\Delta z = 3$ km is the spatial resolution; $\eta = 0.1$ is the quantum efficiency of a photoelectric multiplier; NEP = 10^{-15}W$\cdot$Hz$^{-1/2}$, $\theta = 0.5$ mrad is the field of view of the receiving telescope; $\Delta\lambda = 1.5$ nm is the filter bandwidth; and $A_\mathrm{r} = 0.385$ m^2 is the area of the receiving telescope.

The orbit altitude was taken 300 km, and the conditions of sounding were corresponding to the night side of the Earth. Calculation of the lidar supposes the use of aerological data at the altitude $z_0 = 30$ km; $\Delta T_0 = 0.5$ K, $\Delta P_0 = 0.5\cdot 10^{-3}$ atm. The atmosphere was supposed to be cloudless, and the meteorological conditions were corresponding to the mid-latitude summer.

The results of numerical simulations are shown in Figs. 1–3. Figure 1 shows an increase in the relative errors in measuring the air density with increasing height. This increase is comparatively slow up to the height of 50–60 km and essentially more quick at higher altitudes. The error for the spaceborne lidar is a little bit greater than the error for the ground based lidar at all the lidar parameters used. The great sounding range in the case of a spaceborne lidar causes this effect. The increase in the laser pulse energy and in the signal accumulation time leads to a decrease of the error in measuring the air density. In general, the error reaches 10% at the height of 52.5 km for the BALKAN-3 lidar in space and at the height of 62.5 km for the same lidar based on the ground.

The results of estimation of the absolute error in pressure and temperature measurements are shown in Figs 2 and 3, respectively. As follows from Fig. 2, the absolute error in determining pressure sharply increases with the increase of height from 30 to 35–40 km, and then the increase becomes essentially slower, and the error remains constant at the heights above 40–45 km. The values of the error in measuring pressure with a spaceborne lidar is noticeably greater than in the case of a ground based lidar with the same parameters. The increase of the lidar potential due to the increase in the laser pulse energy and the signal accumulation time will lead to a sharp decrease in the error. If one estimates the relative error in determining pressure, one can note that it reaches the value of 10% at the height of 45 km for the BALKAN–3 lidar. In the case of a ground-based lidar, the error of 10% is observed at the height of 55 km. The lidar with an enhanced potential (E = 0.5 J and Δt = 20 sec) has the error of 10% at the height of 65 km.

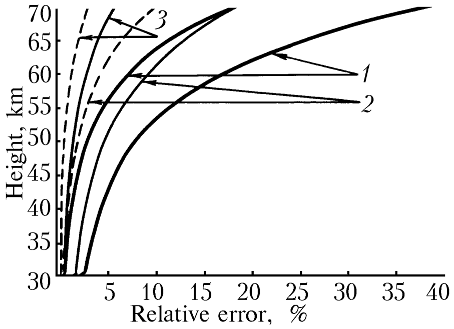

FIG. 1. Vertical profiles of the error in measuring air density by means of lidars with the parameters corresponding to the BALKAN–3 lidar (1); pulse energy is increased up to 0.5 J *(2); the accumulation time is also increased up to* 20 sec *(3). Solid curves are for space version of the lidars, and dashed curves are for the ground-based lidars.*

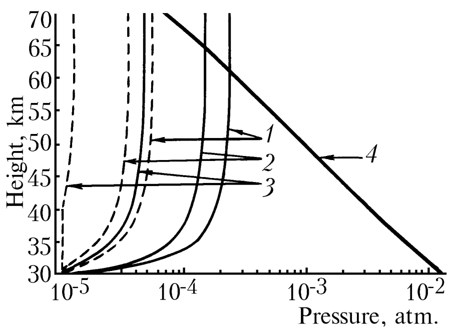

FIG. 2. Absolute error in reconstructing the profile of pressure (4) from data obtained with the lidars whose parameters are the same as in Fig. 1. Solid curves are for space version of the lidars, and dashed curves are for the ground-based lidars.

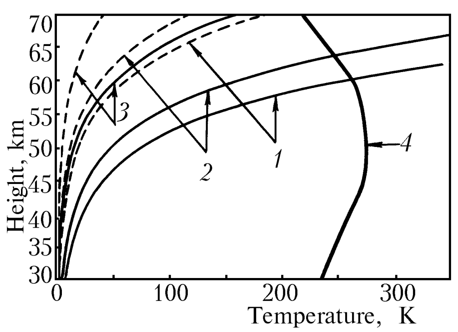

FIG. 3. Absolute error in reconstructing the profile of temperature (4) by means of the lidars whose parameters are the same as in Fig. 1. Solid curves are for spaceborne lidars, and dashed curves are for ground-based lidars.

The errors in lidar measurements of temperature insignificantly increase with height up to 40–45 km. The error sharply increases above this altitude, especially for a spaceborne lidar, and reaches the values close to temperature itself at the heights of 55–60 km. At the same time, the error does not exceed 40 K at the heights up to 70 km for the ground-based lidar of an enhanced potential (E = 0.5 J and Δt = 20 sec). When estimating the relative error in measuring temperature, one should note that the value of 10% is observed at the height of 40 km for the BALKAN–3 lidar in space and at the height of 55 km for its ground-based version. The relative error reaches the values close to 100% at the heights of 50 and 70 km, respectively.

Based on these estimates, one can draw the following conclusions:

1. Reconstruction of the molecular scattering coefficient, temperature, and pressure is more precise when done from the ground surface, than from space. It is connected with a significant increase in the distance (see formula (2)) from the spaceborne apparatus to the atmospheric object under investigation, in comparison with the distance in the case of a ground-based lidar.

2. The increase in energy and signal accumulation time leads to a significant decrease in the error in reconstructing the molecular scattering coefficient, temperature, and pressure.

3. Reconstruction of pressure and temperature is possible with the error no greater than 10% in the height range 30–40 km and with the error up to 100% at the heights up to 60 km (pressure) and 55 km (temperature) at sounding from space by means of a lidar with BALKAN–3 parameters.

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