# Methods and models for assessment of ecological risks

**V.V. Penenko and E.A. Tsvetova**

*Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of the Russian Academy of Sciences, Novosibirsk*

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A technique is proposed for deterministic and deterministic-stochastic assessment of ecological risks for the climatic system of receptor regions under the anthropogenic impact. It is based on a combination of methods of direct and inverse modeling, the sensitivity theory, and the mathematical risk theory. The structure of algorithms for its realization is described.

## Introduction

The practice of assessment of ecological perspectives of industrial regions stimulates the development of new approaches to construction of complex models for studying the behavior of the climatic system and changes in the atmospheric quality. The models are usually developed in two directions. The first one is in modification of basic models for more adequate and detailed description of the processes proceeding under joint impact of natural and anthropogenic factors. The second direction involves development of new methods of mathematical modeling based on the principles of joint use of models and observations under actual conditions.

Recent ecological investigations actively use the concept of ecological risk/ fragility of territories with respect to anthropogenic impact. The problem of risk assessment became especially urgent in connection with the Chernobyl accident in 1986. Such accidents show the necessity to develop constructive methods for prediction and measurement of risks/fragility for various climatic and ecological situations under conditions of both ordinary and extraordinary anthropogenic load.

In this paper, we present a structure of mathematical models oriented at their use for risk/fragility assessment. Basic models directly account for the anthropogenic impact through parametric description of heat, moisture, and pollution sources and landscape at vast territories. The results of anthropogenic impact may show themselves indirectly through mechanisms of transformation and interaction of multicomponent substances in the gas and aerosol states. We propose a technique for deterministic and deterministic-stochastic assessment of ecological risks for receptor regions based on direct and inverse modeling, the model sensitivity theory, and the mathematical risk theory.

## Models of processes

The processes of our interest are described by models of hydrothermodynamics in a climatic system, models of transport and transformation of moisture and chemically and optically active pollutants in the gas and aerosol states. Source functions in the models parametrically account for the effect of natural and anthropogenic factors. To consider jointly the models of processes and a monitoring system in order to organize the interaction between them in the direct coupling and feedback modes, we assume that all elements of the system (i.e., models and observations) can include uncertainties and errors. In this case, it is natural to formulate the problem of constructing algorithms for realization of direct coupling and feedback based on conditions of minimization of some measure of uncertainties and errors.

The general structure of models for this class of problems can be written in the following operator form:

$$L(\boldsymbol{\varphi}) = B\,\frac{\partial \boldsymbol{\varphi}}{\partial t} + G(\boldsymbol{\varphi}, \mathbf{Y}) - \mathbf{f} - \mathbf{r} = 0, \qquad (1)$$

where $B$ is a diagonal matrix; $G(\boldsymbol{\varphi}, \mathbf{Y})$ is a nonlinear matrix differential operator, whose basic element is an advective-diffusion operator acting on different components of the state function; $\mathbf{f}$ is the source function; $\mathbf{r}$ are model errors; $\mathbf{Y}$ is the vector of model parameters belonging to the domain of acceptable values $R(D_t)$.

In this paper, we will not describe in detail all elements of the system and methods for algorithm constructions. Their various aspects are considered in Refs. 1–10. Among all models of the climatic system, we consider only the models directly connected with the processes of heat conduction and transport of optically and chemically active substances in the atmosphere:

$$L\boldsymbol{\varphi} \equiv \frac{\partial \pi \varphi_i}{\partial t} + \operatorname{div} \pi\,(\varphi_i\,\mathbf{u} - \mu_i \operatorname{grad} \varphi_i) +$$

$$+ \pi(H\boldsymbol{\varphi})_i - \pi\,[f_i\,(\mathbf{x}, t) + r_i] = 0, \quad i = \overline{1, n}. \qquad (2)$$

Here $\boldsymbol{\varphi} = \{\varphi_i(\mathbf{x}, t),\ i = \overline{1, n}\} \in Q(D_t)$ is the vector function of state; its components $\varphi_i$ describe the potential temperature, mixture ratios for characteristics of atmospheric humidity (water vapor, cloud liquid water, rain water), the concentrations of pollutants in

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the gas and aerosol state; $D_t$ is the domain of variability of spatial coordinates and time; $\mathbf{f} = \{f_i(\mathbf{x},t),\ i = \overline{1, n}\}$ is the function of heat, moisture, and pollutant sources; $r_i$ are the functions describing uncertainties and errors of models; $\mathbf{u} = (u_1, u_2, u_3)$ is the velocity vector; $\mu_i = (\mu_1, \mu_2, \mu_3)$ are the coefficients of turbulent exchange for some substance $\varphi_i$ in the coordinate direction $\mathbf{x} = \{x_i\}$, $i = \overline{1, 3}$; $H(\boldsymbol{\varphi})$ is the nonlinear matrix operator describing the local processes of transformation of the corresponding substances. The functions $\mathbf{u}$, $\mu_i$, and $f_i$ and the input data on the initial and boundary conditions are included in the set of components of the parameter vector $\mathbf{Y}$. The structure of regions and the coordinate system are described in detail in Ref. 9. The form of the function $\pi$ is determined by the structure of the vertical coordinate in the domain $D_t$.

To take into account the processes of moisture transformation, we use the appropriate modifications of parameterization schemes by analogy with the models described in Ref. 11. The operators of chemical transformation of pollutants are described pithily and constructively using an automated system for constructing kinetic models of atmospheric chemistry.[8]

If a model accounts for processes of aerosol formation and transformation, then one more variable, namely, the particle size is added, and the transformation operator has an integro-differential structure.[12]

Let us present the system of equations describing transformation and interaction of multicomponent aerosols, which is used in the presented system of models:

$$\frac{\partial \varphi_i(q)}{\partial t} = \frac{1}{2}\int_0^q \left[\sum_{k=1}^{M} \gamma_{ik}\varphi_k(q_1) \times \left(\sum_{m=1}^{M} \alpha_{km} K(q-q_1)\varphi_m(q-q_1)\right)\right] \mathrm{d}q_1 -$$

$$-\varphi_i(q)\int_0^{q_M} K(q,q_1)\left(\sum_{k=1}^{M}\beta_{ik}\varphi_k(q_1)\right)\mathrm{d}q_1 - \frac{\partial}{\partial q}[r_i\varphi_i(q)] +$$

$$+\frac{\partial^2}{\partial q^2}\left[\nu_i\varphi_i(q)\right] - R_i\varphi_i(q) + Q_i(q,t),\quad i = \overline{1, M};\quad M \geq 1,$$

where $\boldsymbol{\varphi} = \{\varphi_i(q, t),\ i = \overline{1, M}\}$ is the particle concentration in the volume $[q, q + \delta q]$, the spatial coordinates are involved parametrically; $K(q, q_1)$ is the "coagulation nucleus"; $r_i(q, t)$ is the rate of condensation growth and evaporation; $\nu_i(q, t)$ is the coefficient of diffusion change of particles; $R_i(q, t)$ is the particle removal rate; $Q_i(q, t)$ is the particle production rate; $\gamma_{ik}$, $\alpha_{ik}$, and $\beta_{ik}$, $\{i, k, m = \overline{1, M}\}$ are the parameters of group interaction of particles. The processes of dry and wet deposition of particles are taken into account in the transport operator in the model (2).

The initial conditions at $t = 0$ and the model parameters can be written as follows:

$$\boldsymbol{\varphi}^0 = \boldsymbol{\varphi}_a^0 + \xi(\mathbf{x}),\quad \mathbf{Y} = \mathbf{Y}_a + \zeta(\mathbf{x}, t), \tag{3}$$

where $\boldsymbol{\varphi}_a^0$ and $\mathbf{Y}_a$ are the given *a priori* estimates of the initial fields $\boldsymbol{\varphi}^0$ and the parameter vector $\mathbf{Y}$; $\xi(\mathbf{x})$ and $\zeta(\mathbf{x}, t)$ are the errors and uncertainties of the initial fields and parameters.

If it is assumed that the model and initial data are exact, then the terms including errors can be omitted. The boundary conditions closing the model follow from the physical content of the problem under study. In the operator form they can be presented as

$$[R_b(\boldsymbol{\varphi})]_i - g_i = 0,\quad i = \overline{1, n}, \tag{4}$$

where $R_b$ are the operators of boundary conditions; $g_i$ are source functions at the boundaries $\Omega_t$ of the domain $D_t$.

Write the variational formulation of the model (1)–(4) as:

$$I(\boldsymbol{\varphi}, \mathbf{Y}, \boldsymbol{\varphi}^*) = \int_{D_t} [L(\boldsymbol{\varphi}), \boldsymbol{\varphi}^*)]\ \mathrm{d}D\ \mathrm{d}t = 0, \tag{5}$$

where $\boldsymbol{\varphi}^* \in Q^*(D_t)$, the space conjugate to $Q(D_t)$.

The integral identity (5) is constructed with allowance for the boundary and initial conditions so that at the substitution $\boldsymbol{\varphi}^* = \boldsymbol{\varphi}$ the functional (5) gives the energy balance of the studied system.[3]

After all the needed transformations in Eq. (5) for the model (2)–(4), we finally obtain the integral identity in the form

$$I(\boldsymbol{\varphi}, \mathbf{Y}, \boldsymbol{\varphi}^*) \equiv$$

$$\equiv \sum_{i=1}^{n}\left\{(\Lambda\varphi,\varphi^*)_i + \int_{D_t}(H(\boldsymbol{\varphi})_i - f_i - r_i)\varphi_i^*\,\pi\mathrm{d}D\mathrm{d}t\right\} = 0. \tag{6}$$

Here

$$(\Lambda\varphi, \varphi^*)_i \equiv$$

$$\equiv \left\{\int_{D_t}\left\{0.5\left[(\varphi^*\frac{\partial\pi\varphi}{\partial t} - \varphi\frac{\partial\pi\varphi^*}{\partial t}) + (\varphi^*\,\mathrm{div}\,\pi\varphi\mathbf{u} - \varphi\,\mathrm{div}\,\pi\varphi^*\mathbf{u})\right] + \right.\right.$$

$$\left. + \pi\mu\,\mathrm{grad}\,\varphi\,\mathrm{grad}\,\varphi^*\right\}\mathrm{d}D\mathrm{d}t + \int_D 0.5\,\varphi\varphi^*\pi\ \mathrm{d}D\Big|_0^{\overline{t}} + \tag{7}$$

$$+ \int_{\Omega_t}(0.5\varphi u_n - \mu\frac{\partial\varphi}{\partial n})\varphi^*\pi\,\mathrm{d}\Omega\,\mathrm{d}t +$$

$$\left. + \int_{\Omega_t}((R_b\varphi) - q)\varphi^*\pi\,\mathrm{d}\Omega\,\mathrm{d}t\right\}_i;$$

$u_n$ is the velocity component normal to the boundary.

Note that the operators $R_b$ in the boundary conditions (4) should be described in such a way as to

exclude the terms with the normal derivative of the state function in two last integrals in Eq. (7).

The variational formulation (5)–(7) is used for construction of discrete model approximations. For this purpose, a grid domain $D_t^h$ is introduced in $D_t$ and discrete analogs $Q^h(D_t^h)$, $Q^{h*}(D_t^h)$, $R^h(D_n^h)$ of the corresponding functional spaces are determined in it. Then Eq. (5) is approximated by a summatory analog

$$I^h(\boldsymbol{\varphi}, \mathbf{Y}, \boldsymbol{\varphi}^*) = 0, \quad (8)$$

$$\boldsymbol{\varphi} \in Q^h(D_t^h),\ \boldsymbol{\varphi}^* \in Q^{h*}(D_t^h),\ \mathbf{Y} \in R^h(D_n^h).$$

The superscript $h$ denotes the discrete analog of the corresponding object.

Numerical schemes for the model (1) follow from the conditions of stationarity of the functional $I^h(\boldsymbol{\varphi},\mathbf{Y},\boldsymbol{\varphi}^*)$ at points of the grid domain $D_t^h$ at arbitrary and independent variations of the grid components $\boldsymbol{\varphi}^* \in Q^{h*}(D_t^h)$ for the main problem and at variations $\boldsymbol{\varphi} \in Q^h(D_t^h)$ for conjugate problems.[3]

Practical implementation of the basic models of processes is now well-proven.[1–4] We use the variational organization of models in order to use them as a basis for construction of methods for solution of problems of the higher system level that are connected with ecological safety and control for the quality of nature medium, as well as assimilation of observations for monitoring and prediction tasks.[5–7,13–15] These methods include, as basic elements, the algorithms for calculating the functions of sensitivity to variations of input data, parameters, and sources. The latter, in turn, demands solution of some direct and conjugate problems. All these algorithmic constructions are based on the corresponding variational principles, the key role in which is played by the integral identity of the form (5)–(7).

## Some definitions and estimates of ecological risks

Consider one of typical problems of ecological prediction and design, namely, assessment of the ecological risk and fragility of territories with respect to the anthropogenic impact. Problems of this class are solved with the use of combined methods of direct and inverse modeling and methods of the sensitivity theory as applied to models and generalized characteristics of environmental quality. Let us describe briefly the main idea and the algorithmic scheme for its implementation.

First of all, determine a set of assessment characteristics in the form of functionals

$$\Phi_k(\boldsymbol{\varphi}) = \int_{D_t} F_k(\boldsymbol{\varphi})\, \chi_k(\mathbf{x},t)\, \mathrm{d}D\mathrm{d}t,\quad k = 1, \ldots, K, \quad (9)$$

where $F_k(\boldsymbol{\varphi})$ are given-form functions definite and differentiable at the set of values of the state function; $\chi_k(\mathbf{x}, t) \geq 0$ are weighting functions; $\chi_k \in Q^*(D_t)$ and $\chi_k(\mathbf{x}, t)\, \mathrm{d}D\mathrm{d}t$ are the corresponding Radon and Dirac measures in $D_t^h$ (Ref. 16). The part of the region, in which the weighting function has nonzero values, will be called the receptor region. With the appropriate choice of the functions $F_k(\boldsymbol{\varphi})$ and $\chi_k$ in Eq. (9), the generalized characteristics of the system behavior, ecological restrictions on the environment, results of different-type observations, control and quality criteria of models, etc. can be described by functionals.[5,14]

For this set of functionals, we construct the main equation of the sensitivity theory that relates variations $\delta\Phi_k(\boldsymbol{\varphi})$ to variations of the model parameters[7]:

$$\delta\Phi_k^h(\boldsymbol{\varphi}) \equiv \frac{\partial}{\partial\alpha} I^h(\boldsymbol{\varphi},\mathbf{Y} + \alpha\delta\mathbf{Y},\ \boldsymbol{\varphi}_k^*)\big|_{\alpha=0} \equiv$$

$$\equiv (\boldsymbol{\Gamma}_k,\ \delta\mathbf{Y}) \equiv \sum_{i=1}^{n} \Gamma_{ki}\, \delta Y_i, \quad (10)$$

$$\boldsymbol{\Gamma}_k = \frac{\partial}{\partial\delta\mathbf{Y}} \left( \frac{\partial}{\partial\alpha} I^h(\boldsymbol{\varphi},\mathbf{Y} + \alpha\delta\mathbf{Y},\ \boldsymbol{\varphi}_k^*)\big|_{\alpha=0} \right), \quad (11)$$

where $\alpha$ is the real parameter; $\delta\mathbf{Y} = \{\delta Y_i\}$ are variations of parameters, $k = \overline{1, K}$, $i = \overline{1, N}$; $\boldsymbol{\varphi}$ is the solution of the main problem, and $\boldsymbol{\varphi}_k^*$ is the solution of the conjugate problem corresponding to the functional $\Phi_k^h(\boldsymbol{\varphi})$, which follows from the conditions of stationarity of the summatory functional from Eq. (8); $\boldsymbol{\Gamma}_k = \{\Gamma_{ki}\}$ are the sensitivity functions. The algorithms for construction of the main equation and the sensitivity functions in the considered class of problems are described in Refs. 5, 7, 9, and 13.

A particular attention should be paid to equations including heat, moisture, and pollution sources in the system (10). Factors at source variations are the corresponding sensitivity functions. They are the measures of the direct effect of source variations on variations of the functional (in linear problems, the effect of sources themselves on the functional). However, these terms do not describe completely the effect of sources on the functional. There is some indirect contribution, which is described by the joint effect of other terms with the sensitivity functions, whose computational formulae include components of the state function. For example, they are terms including variations of the turbulence coefficients, functions of the near-surface pressure, geopotential, parameters of the pollutant transformation operator, etc.

The functions of sensitivity of functionals (9) to source variations are determined in the domain $D_t$. Depending on the purpose and for convenient interpretation, they can be called the functions of source influence or danger, information value, information content of the monitoring system, etc.

The carriers of these functions can be interpreted as territory regions observed with a monitoring system located in the receptor region. In terms of the theory of

differential equations, they can also be related to definitions of domains of dependence and domains of influence for the state functions of the model (2) in the receptor zone.

The information meaning of the danger function for the functionals determining the atmospheric quality in the receptor zone can be described in the following way. Its value at the point $(\mathbf{x}, t) \in D_t$ is the relative contribution of pollutant emissions from a source located at that point during its operation period to the total amount of pollutants coming to the atmosphere of the receptor zone for the observation period.

From the ecological viewpoint, every source, even operating in the so-called standard mode, constitutes a threat to the environment. Therefore, equations and sensitivity functions contain some quantitative information for assessing the ecological risk for the receptor zone.

The sensitivity functions are calculated through solving the main and conjugate problems for the model (2)–(5) with unperturbed input data, and therefore they have a deterministic character. At the same time, variations of parameters, initial and boundary conditions, and sources may be both deterministic and random. As for sources, variations may arise due to some abnormal situations, which are usually random.

To quantitatively assess ecological risks, we introduce some thresholds for variations of the functionals (9) and denote them as $\Delta_k^s$, $k = \overline{1, K}$. Then the conditions, at which the inequalities

$$|\delta\Phi_k| \le \Delta_k^s, \qquad (12)$$

are fulfilled, can be conditionally thought ecologically safe, while the conditions, at which these inequalities break down, correspond to situations of ecological risk.

In the physical meaning, the state functions in the models (2)–(4) are nonnegative, and numerical schemes for discretization of these models can be so constructed that they are monotonic and transportable. This means that at a certain form of the functionals (9), i.e., when the conditions $(\partial F_k/\partial\varphi)\,\chi_k \ge 0$ are fulfilled, the sensitivity functions of these functionals to source variations are nonnegative as well.

It follows from the sensitivity equations (10) that at the given sensitivity functions and available quantitative information about parameter variations it is not difficult in principle to check the "ecological safety" inequalities (12). Actually, in the case of deterministic source and parameter variations, functional variations can be assessed as

$$|\delta\Phi_k| \le \sum_{i=1}^{N} |\Gamma_{ki}|\,|\delta Y_i|. \qquad (13)$$

Using these assessments along with the inequalities (12), one can categorize a situation as ecologically safe or risky.

If parameter and source variations are random, then it is somewhat more difficult to assess the functional variations, as compared to the deterministic case, since in this case one has to deal with multidimensional spaces of sensitivity functions and parameters. Consider one of approaches to obtaining the needed assessments in the deterministic-stochastic case based on the methods of the sensitivity theory[3] and the mathematical risk theory.[17]

Two methods are commonly used in practice to describe variations. The first one consists in the following. Based on *a priori* information on the character of a parameter or a source, some form of the distribution law for variations is postulated and, accordingly, some parameters needed to match the supposed distribution with the actual one are specified. The normal distribution is used most often, because most actual distributions are close to the normal one and the normal law is convenient for solution of different problems.

In the second case, the first two moments of the multidimensional distribution law (mathematical expectation and covariance matrix) are specified, but the law itself is not always specified. The both methods are similar, because the normal law is completely determined by the mathematical expectation and the covariance matrix. The second method is more general. The results it gives are valid for any, not necessarily normal, distribution of variations with the given characteristics.

Denote the mathematical expectation and the covariance matrix of the vector $\delta\mathbf{Y}$ as $E(\delta\mathbf{Y})$ and $D(\delta\mathbf{Y})$, respectively. The mathematical expectation is a vector of the following form

$$E(\delta\mathbf{Y}) = [E_i \equiv E(\delta Y_i),\ (i = \overline{1, N})], \qquad (14)$$

where $E(\delta Y_i)$ is the mathematical expectation of the vector component $\delta Y_i$; $N$ is the dimension of the parameter vector. The $N \times N$ covariance matrix $D(\delta\mathbf{Y})$ is nonnegative definite. It has a block structure matched with the structure of the vector $\delta\mathbf{Y}$. If some components of $\delta\mathbf{Y}$ are uncorrelated, the appropriate blocks of $D(\delta\mathbf{Y})$ are diagonal, while full blocks of $D(\delta\mathbf{Y})$ correspond to the correlated components of $\delta\mathbf{Y}$.

The vector $\mathbf{\Gamma}_k$ depends on the unperturbed values of parameters and the state vector. Therefore, for particular situations it can be thought non-random. Taking into account the properties of mathematical expectation and covariance matrices at linear transformations of random vectors, we obtain the following estimates for the mathematical expectation $E(\delta\Phi)$ of variations of the functional $\delta\Phi$:

$$E(\delta\Phi) = \sum_{i=1}^{N} \Gamma_i E(\delta Y_i) \qquad (15)$$

and for the variance $D(\delta\Phi)$

$$D(\delta\Phi) = [D(\delta\mathbf{Y})\ \boldsymbol{\Gamma},\ \boldsymbol{\Gamma}]. \quad (16)$$

Hereinafter the number index of the functional is omitted.

Thus, the variance is determined through a square form of the covariance matrix of the variation vector and the vector, whose components coincide, in view of Eq. (10), with the sensitivity functions of this functional.

It is obvious that $D(\delta\Phi)$ is nonnegative. The scalar product in Eq. (16) is introduced by analogy with Eq. (10), as in a finite-dimensional vector space.

In a particular case, when errors in the components of $\delta\mathbf{Y}$ are uncorrelated, the covariance matrix is diagonal:

$$D(\delta\mathbf{Y}) = \mathrm{diag}\ [D_i \equiv D(\delta Y_i),\ (i = \overline{1, N})], \quad (17)$$

where $D(\delta Y_i)$ is the variance of the $i$th component of $\delta\mathbf{Y}$, and Eq. (16) takes the simplest form

$$D(\delta\Phi) = \sum_{i=1}^{N} \Gamma_i^2\ D_i\ . \quad (18)$$

Note that in Eqs. (15)–(18) the sum includes only those terms, in which the sensitivity function is larger than some threshold.

According to Eq. (10), variation of $\delta\Phi$ is determined as a linear combination of random parameters. If $N$ is quite large and all components of $\delta\mathbf{Y}$ do not differ largely from the normal distribution, then, based on the central limit theorem of the probability theory,[18] we can assume that the distribution law for $\delta\Phi$ tends to the normal one.

The assumption of the normal distribution law considerably facilitates the problem, because to characterize this distribution completely, it is sufficient to know its mathematical expectation $E(\delta\Phi)$ and the variance $D(\delta\Phi)$ or the corresponding covariance matrix.

Using the values of $E(\delta\Phi)$ and $D(\delta\Phi)$ and the assumption of the normal distribution of $\delta\Phi$ as a random parameter, we can obtain some estimates of $\Phi(\varphi)$. The technique for constructing these estimates is rather well-developed now.[19] In its consideration we follow Ref. 3.

The probability that the value of $\delta\Phi$ falls in a given region $\Delta$ (i.e., $|\delta\Phi| \le \Delta$) is determined as

$$P(\delta\Phi \in \Delta) = \int_{\Delta} f(x)\mathrm{d}x, \quad x \equiv \delta\Phi\ , \quad (19)$$

where $f(x)$ is the density of the distribution of $x$:

$$f(x) = \frac{1}{\sqrt{2\pi\ D(x)}}\ \mathrm{e}^{-[x-E(x)]^2/[2D(x)]}, \quad (20)$$

and $E(\delta\Phi)$ and $D(\delta\Phi)$ are determined by Eqs. (15), (16), or (18). The integral is taken over the region $(-\Delta \le x \le \Delta)$.

Of interest are the probabilities of fulfillment of the inequalities (12) expressing the conditions for the situation under analysis to be categorized as ecologically safe:

$$R^{\mathrm{s}} = P(|\delta\Phi| \le \Delta^{\mathrm{s}}). \quad (21)$$

Assuming the normal distribution law for $\delta\Phi$ with the distribution density (20), we have for the sought probabilities

$$R^{\mathrm{s}} = \frac{1}{\sqrt{2\pi\ D(x)}} \int_0^{\Delta^{\mathrm{s}}} \mathrm{e}^{-[x-E(x)]^2/[2D(x)]}\ \mathrm{d}x =$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^{\lambda} \mathrm{e}^{-t^2/2}\ \mathrm{d}t = \Psi(\lambda), \quad (22)$$

where $\lambda = \lambda(R^{\mathrm{s}}) = [\Delta^{\mathrm{s}} - E(x)]/\sqrt{D(x)}$; $\Psi(\lambda)$ is the probability integral. The probability that the situation is categorized as ecologically hazardous $R^{\mathrm{r}}$ is estimated as

$$R^{\mathrm{r}} = 1 - R^{\mathrm{s}} \equiv P\{\delta\Phi > \Delta^{\mathrm{s}}\}. \quad (23)$$

On the other hand, if we specify a certain acceptable level of the safety probability $R^{\mathrm{s}}$, then, based on it, we can determine $\lambda$ from Eq. (22) and estimate the safety range:

$$|\Delta^{\mathrm{s}} - E(\delta\Phi)| = \lambda \sqrt{D(\delta\Phi)}.$$

In problems of ecological designing, in addition to assessment of a situation as a whole, it is necessary to consider the worst possible cases for the atmospheric quality in some receptor region. For this purpose, zones of local maxima of the sensitivity function and zones of location of potentially high-power sources (in respect to pollutant emission) are being sought. If these zones coincide, situations of high ecological risk/fragility can occur. In such cases, more thorough investigation is necessary, for example, initiation of direct modeling scenarios with a given set of sources and various versions of emissions.

## Some scenario estimates

As an example, we present the results for a typical scenario demonstrating assessment of ecological outlook for a region under the anthropogenic impact. This scenario describes diffusion of gas and aerosol pollutants from aggregated sources. The source function in the scenario parametrically specifies pollutant emissions by the scheme, which roughly describes actual emissions initiated by United States bombardment of Afghanistan. Since the relations between sources and receptors are beforehand unknown, it is worth beginning the analysis of the situation from direct modeling in order to estimate the characteristic scales of the processes. For realization of scenarios, we used a set of three-dimensional global circulation models for the atmosphere in the Northern Hemisphere. The atmospheric circulation was reconstructed from

REANALYSIS archive data[20] for the period from October 5 to November 5 of 1999 in the assimilation mode using the system[21] based on the hydrothermodynamic model with 30-min discreteness. For this purpose, we used the procedure of quick data assimilation in the 20-level (vertical) version of the model in hybrid coordinates following the Earth topography.

Sources of pollution were specified parametrically in the north of Afghanistan. The source operational mode was assumed discrete with the 24-hour periodicity. Pollutants were emitted into the atmosphere at night, similarly to that as it took place during the US campaign in Afghanistan. Powerful and extensive explosions produced aerosol clouds of soil particles. Under the effect of thermal convection, they rose, as estimated, up to the heights of 500 mbar. In calculations it was assumed that these particles had several size grades, and particles of different sizes were diffused with different modes of deposition onto the surface. Scenarios were realized in the direct modeling mode in parallel by the processes describing diffusion of particles of different sizes. Since we estimated the characteristic scales of polluted areas, here we present the results for particles of two grades, which are transported to long distances and can affect the state of the climatic system and the atmospheric quality in regions neighboring to Afghanistan under the conditions of trans-border transport.

Figures 1 and 2 show the field of total concentrations for the computational period in the surface atmospheric layer. It should be emphasized that the results of scenarios should be considered only as tentative, because the data on hydrodynamics are dated to 1999. They demonstrate the possibilities of estimating the characteristic scales of processes with the developed technique and the set of models.

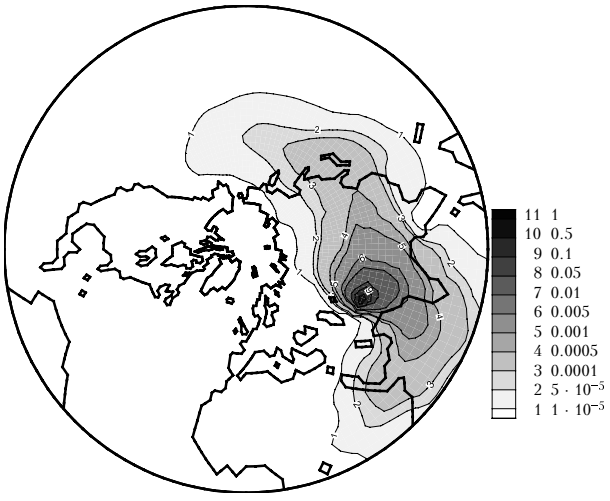


**Fig. 1.** Total concentration of gas pollutants and submicron aerosol particles in the surface atmospheric layer (rel. units.) from aggregated sources located at the territory of Afghanistan.

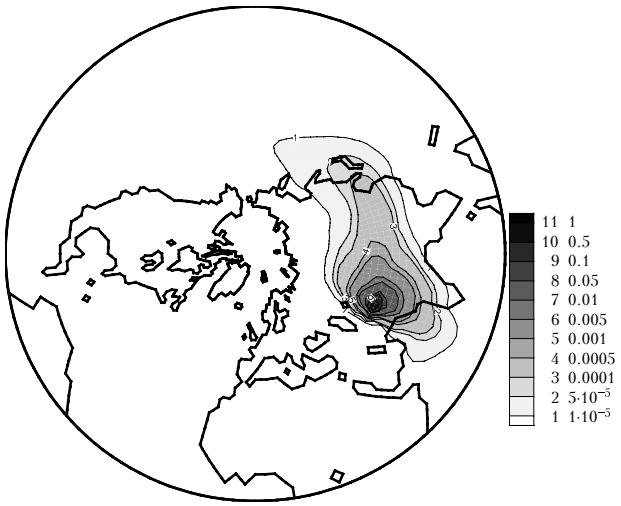


**Fig. 2.** The same as in Fig. 1, but for aerosol particles of about 50 μm in diameter.

The estimates show that specific pollutants can be transported through borders to long distances to Middle and Central Asia and even southern Siberia and China. Keeping in mind the character of the anthropogenic impact, we can say that the pollution mostly consists of fine dust. The dust interacts with the solar and Earth's radiation in a more complex way than other aerosols do. This is caused by the fact that mineral particles can absorb and scatter the radiation in the ultraviolet, visible, and infrared regions, that, in turn, may lead to both cooling and heating of the climatic system on the regional scale. Calculations show that the pollutant concentration fields in dynamics are very variable in space and time, and in zones of increased pollution they have rather large optical thickness. The lifetime of mineral aerosols in the atmosphere is relatively short – about several weeks. However, since their sources operate in the pulsed modes for rather long periods, i.e., new portions of pollutants are emitted into the atmosphere from time to time, the superposition effect takes place. It follows herefrom that the aerosol burden of the atmosphere has a complex spatiotemporal structure and not only changes the environment quality at vast territories, but also affects the thermodynamics of the regional climate. Regions with increased pollution can be considered as zones of potential ecological risk. As the receptor regions, they are further studied with the use of inverse modeling methods by the scheme described in Refs. 10, 13, and 15.

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