CALCULATION OF THE ACOUSTIC RESPONSE GENERATED BY LASER BEAM PROPAGATING TROUGH THE ATMOSPHERE

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Received October 2, 1990

The amplitudes and shapes of sound pulses generated by laser beams propagating through the atmosphere are numerically investigated. Families of curves are plotted for three different types of the beam time modulation, namely, a triggering pulse, a short laser pulse, and a harmonically modulated radiation. These curves describe acoustic response at an arbitrary distance from the beam axis at any time. Approximate relations for quantitative estimates of the amplitude of sound pulses at various distances from the beam axis are given. They describe the changes in sound pressure with distance to an accuracy of 5%. The frequency dependence of the amplitude of a periodic component of the acoustic signal on the beam axis is also studied.

Propagation of laser radiation through an absorbing medium is accompanied by generation of sound in the propagation channel. Since, in the atmosphere electrostriction may but weakly contribute to changes in the pressure and density in the region of the beam,[1] the basic physical mechanism of sound generation when the medium retains its state of aggregation is the thermal one. Therefore, the excited acoustic waves may be easily monitored and their spatial characteristics and temporal behavior may be controlled.[2] Thus, changes in the spatial configuration of the acoustic field may be obtained by adjusting the radiation regime and its parameters, because the acoustic response, generated by thermal expansion of the medium, is associated with the changes in the beam intensity.

The conditions needed to increase the efficiency of recording of sound pulses or to obtain sound fields of a given configuration may be chosen on the basis of the theoretical models of such fields corresponding to a certain type of modulation of a laser beam. In this connection, it is of interest to consider several regimes of radiation, which are most often encountered in practice, and to calculate corresponding acoustic fields.

The phenomenon of laser excitation of sound in the atmosphere was theoretically studied in Refs. 3, 4, and 5. The analytical formulas for sound pressure, given in Refs. 4 and 5, were written in integral form, and therefore were not plotted. However, in certain particular cases, e.g., on the beam axis and at long distances from the axis, the solutions of the equation for pressure perturbations were obtained in terms of special functions, and the corresponding sound pulses were shown in the figures. However, such a description of the sound field excited by laser radiation modulated in a certain way is insufficient for practice.

The aim of this paper is to reconstruct numerically the amplitude and shape of the sound pulses generated during propagation of laser beams through the atmosphere. The calculations were carried out for three different types of modulation of laser radiation, i.e., a triggering pulse (a stepwise function), a short laser pulse, and a harmonically modulated radiation. In the first case, the results of numerical integration representing the solution obtained in Ref. 4 and describing pressure perturbations are compared with the solution given in Ref. 5, and the limits of applicability of the last are found. In the case of the harmonic modulation of radiation, a frequency dependence of the acoustic amplitude is examined. A technique is proposed for numerical calculation of the radiation power and of the air absorption coefficient.

The pressure perturbation p at a distance r from the beam axis is described by the well-known equation

$$\frac{\partial^2 p}{\partial t^2} - u^2 \Delta_r p = \alpha(\gamma - 1)\frac{\partial I}{\partial t}, \tag{1}$$

where u is the sound velocity, α is the coefficient of light absorption in air, and γ is the gas constant.

If the beam is Gaussian and is amplitude modulated by the function $f(t)$

$$I = \frac{W}{\pi a^2}\exp\left(-\frac{r^2}{a^2}\right)f(t),$$

the solution of Eq. (1) can be represented in the form[4]

$$p(\bar{r}, \bar{t}) = \int\limits_0^\infty \hat{p}(\mathrm{k}, \bar{t}) J_0(\kappa \bar{t})\, \kappa \mathrm{d}\kappa, \tag{2}$$

where W is the beam power, I is the radiation power density, a is the characteristic beam size, $\bar{r} = \dfrac{r}{a}$, and $\bar{t} = \dfrac{tu}{a}$ are the dimensionless parameters which describe the distance and time, and $J_0(\kappa\bar{r})$ is the zero–order Bessel function. The relation for $\hat{p}(\kappa, \bar{t})$ is determined by the form of the dependence $I(r, t)$, in particular, by the function $f(t)$.

TRIGGERING PULSE

Solution (2) acquires the following form for the stepwise function $f_1(t) = H(t)$:

$$p_1(\bar{r}, \bar{t}) = \alpha(\gamma - 1)\frac{W}{2\pi a u}\int\limits_0^\infty \exp\left(-\frac{\kappa^2}{4}\right)\sin(\kappa\bar{t})\, J_0(\kappa\bar{t})\, \mathrm{d}\kappa. \tag{3}$$

Integral solution (3) is the rigorous solution of the equation describing the change in the pressure at the distance r from the beam axis.

According to Ref. 5, we have for the considered case

$$p_1(r,\ t) = \frac{\alpha(\gamma - 1)W}{2\pi\sqrt{\pi a}\ u\sqrt{r + ut}}\, f\left(\frac{r - ut}{a}\right), \qquad (4)$$

where

$$f(x) = \int\limits_x^\infty \frac{\exp(\xi^{-2})}{\sqrt{\xi - x}}\,d\xi = \sqrt{\pi}\ 2^{-1/4}\exp\left(-\frac{x^2}{4}\right) D_{-1/2}(\sqrt{2x})$$

and $D_{-1/2}(\sqrt{2x})$ is the parabolic cylinder function.

Solution (4) gives an approximate formula for sound pressure, since it is obtained on the assumption that the observation point is far from the beam axis ($r \gg a$).

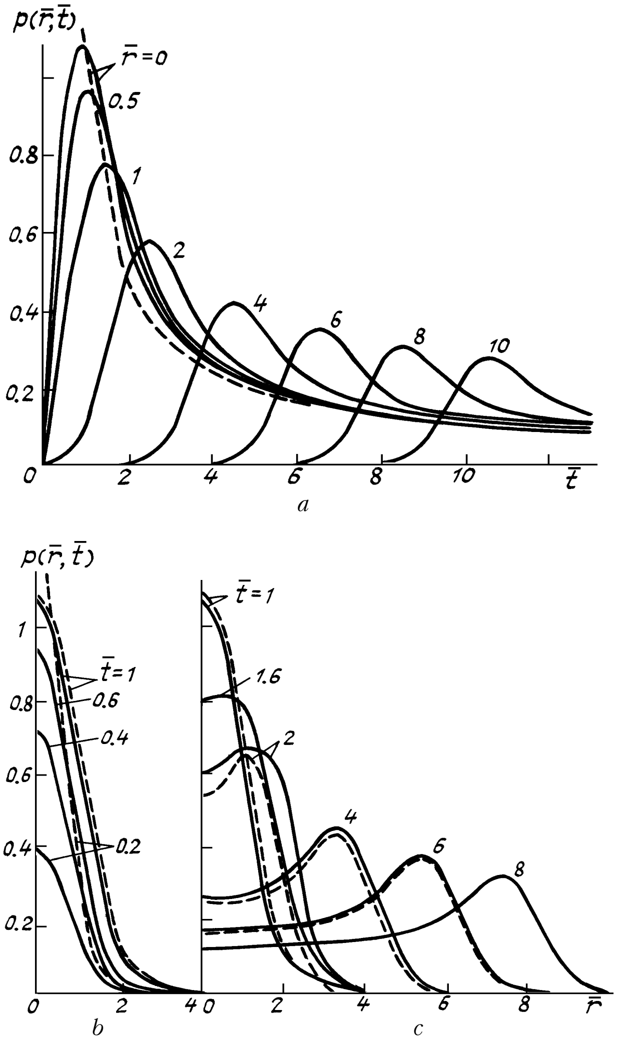

FIG. 1. Sound pulses generated by the stepwise leading front of laser signal at various distances from the beam axis $\bar{r}$ indicated on the curves (a) and sound fields generated by the same front at different times $\bar{t}$ indicated on the curves (b and c). Solid lines denote the rigorous solutions and dashed lines – the approximate solution.

We compared solutions (3) and (4) with the help of numerical integration. The plots describing the shapes of sound pulses and sound fields are shown in Figs. 1*a*, *b*, and *c*. Solid lines show the rigorous solution and dashed lines – the approximate solution. As can be seen from these figures, the assumption $r \gg a$ used to derive solution (4) makes it virtually inapplicable for short times, particularly within the region of the beam. However, the approximate solution can be applied for calculations of the sound field with good accuracy, even within the region of the beam, if time $t \geq \frac{6a}{u}$. Solution (3) is preferable for numerical calculations, since it allows one to estimate the values of the sound pressure at arbitrary distances from the beam axis at any time, which is quite important for solving various practical problems.

It should be noted that the dimensional coefficients in formulas describing $p(\bar{r},\ \bar{t})$ were neglected for the considered types of modulation, because they determine the amplitude of sound pulses rather than their shape. The dimensionless values of $p(\bar{r},\ \bar{t})$ were calculated following the trapezoid rule and applying the Romberg extrapolation to the discrete sets of $\bar{r}$, and $\bar{t}$. The upper limit of integration was bounded by the domain of existence of the integrand. Based on the results of calculations, we plotted the families of curves which described the changes in the sound pressure as functions of time for fixed $\bar{r}$ and the distance for fixed $\bar{t}$.

It can be seen from Fig. 1*a* that the acoustic response generated by a stepwise leading front of a laser pulse decreases rather slowly with increase of the distance from the beam axis and at $r = 2a$ it is about 53% of the maximum value obtained at the beam center. The general trend of changes in $p_{\max}(\bar{r})$ may be approximated by the function $\frac{1}{\sqrt{1 + r^2}} + 0.08$ for distances varying from 0 to $2a$ and by $\frac{1}{\sqrt[4]{1 + r^2}} - 0.06$ for distances $r > 2a$. The accuracy of approximation in this case is about 5%. The same accuracy is obtained in other cases considered below.

SHORT LASER PULSE

In the case of a short laser pulsewidth $\tau \ll \frac{a}{u}$, we have $f_2(t) = \frac{E}{W}\,\delta(t)$, where E is the energy, and solution (2) describing pressure perturbation takes the form

$$p_2(r,\ \bar{t}) = \alpha(\gamma - 1)\,\frac{E}{2\pi a^2}\ \times$$

$$\times \int\limits_0^\infty \exp\left(-\frac{\kappa^2}{4}\right)\cos(\kappa,\ \bar{t})\ J_0(\kappa\bar{t})\ \kappa d\kappa. \qquad (5)$$

Sound pulses on the beam axis and at distances divisible by a from the axis are shown in Fig. 2*a*. The temporal behavior of the sound field can be seen in Figs. 2*b* and *c*.

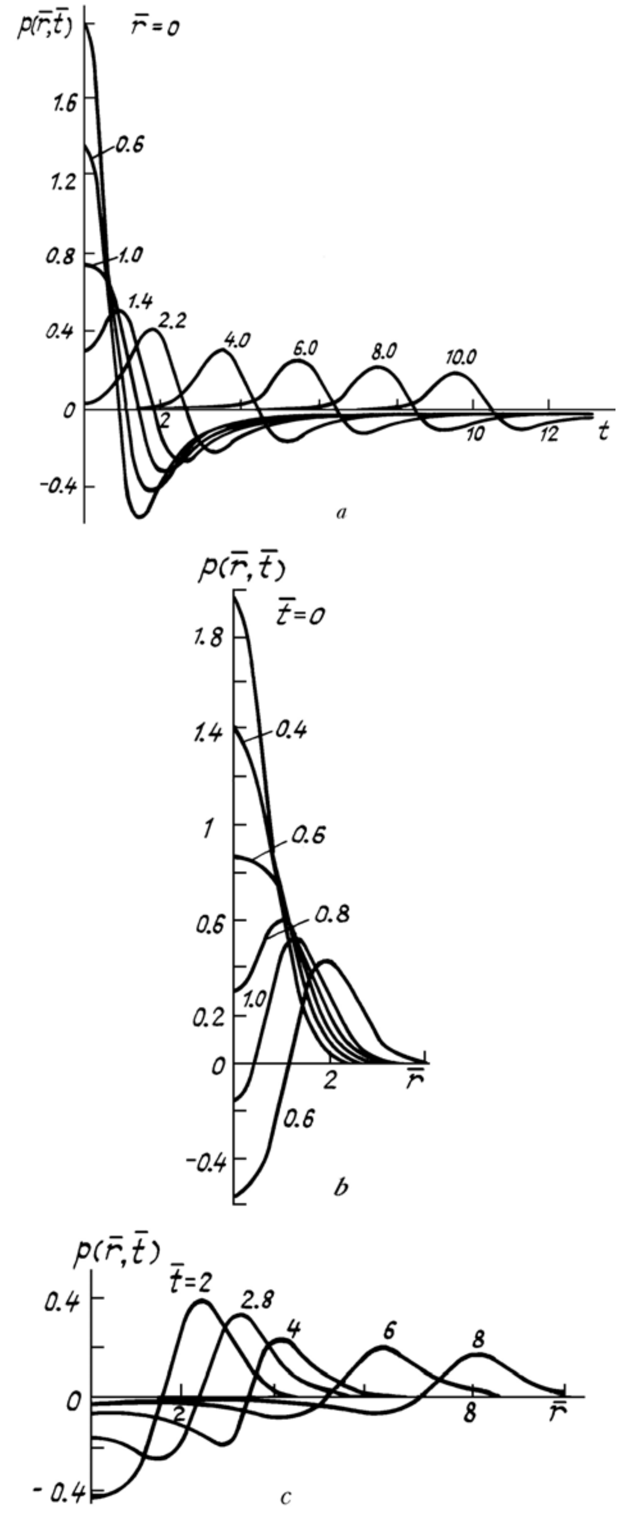

FIG. 2. Sound pulses generated by the short laser pulse at various distances from the beam axis $\bar{r}$ indicated on the curves (a) and sound fields generated by the same pulse at different times $\bar{t}$ indicated on the curves (b and c).

The acoustic response on the beam axis is almost two times greater than the corresponding response excited by the stepwise leading front of the signal. However, at $r = 2a$ the maximum sound pressure is about 21% of the corresponding value on the beam axis (see Fig. 2*a*). The changes in $p_{\max}(r)$ may be approximated by the functions $2\exp(-r^2)$ within the region of the beam and by $\dfrac{1}{\sqrt[3]{(1+r)^2}}$ for $r > a$. At distances [1.4; 5.0] the latter function should be corrected for (–0.04).

HARMONIC MODULATION

If the beam intensity is modulated by the function $f_3(t) = H(t)(1 + \sin(\omega t))$ we have the following solution of Eq. (1) taken from Ref. 4:

$$p_3(\bar{r},\ \bar{t}) = p_1(\bar{r}\ \bar{t}) + \alpha(\gamma - 1)\frac{W}{2\pi a u}\bar{\omega}\times$$

$$\times\int\limits_0^\infty \exp\left(-\frac{\kappa^2}{4}\right)\frac{\cos(\kappa\bar{t}) - \cos(\bar{\omega}\,\bar{t})}{\bar{\omega}^2 - \kappa^2} J_0(\kappa\bar{r})\,\kappa\mathrm{d}\kappa\,, \qquad (6)$$

where $\bar{\omega} = \dfrac{\omega a}{u}$.

Sound pulses and fields, calculated according to this formula for a fixed frequency **Ошибка!** = 1.6, are shown in Figs. 3*a*, *b*, and *c*.

It should be noted that the repetition frequency of the sound pulses at arbitrary distance from the beam axis coincides with the beam modulation frequency. The amplitude of the first acoustic response is greater than the amplitude of the subsequent periods of sound oscillations. This effect may be attributed to superimposing of the triggering pulse on the harmonically modulated laser radiation. The change in the maximum sound pressure with increase of the distance from the beam axis (see Fig. 3*a*) is slower than in the foregoing case and may be described by a function $\dfrac{2}{\sqrt[3]{1 + 1.7r^2}}$ for the first sound pulse at distances $0 \le r \le 4a$ and by $\dfrac{2}{\sqrt[3]{1 + r^2}}$ at $r > 4a$. The amplitudes of the subsequent sound pulses at the same distances are described by the functions $1/(1 + r^2) + 0.54$ and $\dfrac{1}{\sqrt[4]{1 + r^2}}$, respectively. At distances [4.0; 10.0] the latter function should be corrected for (+0.05).

The dependence of the periodic component of the acoustic signal on the beam axis on the modulation frequency obtained from Eq. (6) is given by the formula

$$\tilde{p}_3(\bar{r} = 0,\ \bar{t},\ \bar{\omega}) =$$

$$= \alpha(\gamma - 1)\frac{W}{2\pi a u}\bar{\omega}\cos(\bar{\omega}\,\bar{t})\int\limits_0^\infty \exp\left(-\frac{\kappa^2}{4}\right)\frac{\kappa}{\bar{\omega}^2 - \kappa^2}\,\mathrm{d}\kappa =$$

$$= \alpha(\gamma - 1)\frac{W}{2\pi a u}\bar{\omega}\cos(\bar{\omega}\,\bar{t})\exp(-\bar{\omega}^2/4)\,E_i\left(\bar{\omega}^2/4\right)\,, \qquad (7)$$

where $E_i\left(\bar{\omega}^2/4\right)$ is the integral exponent.

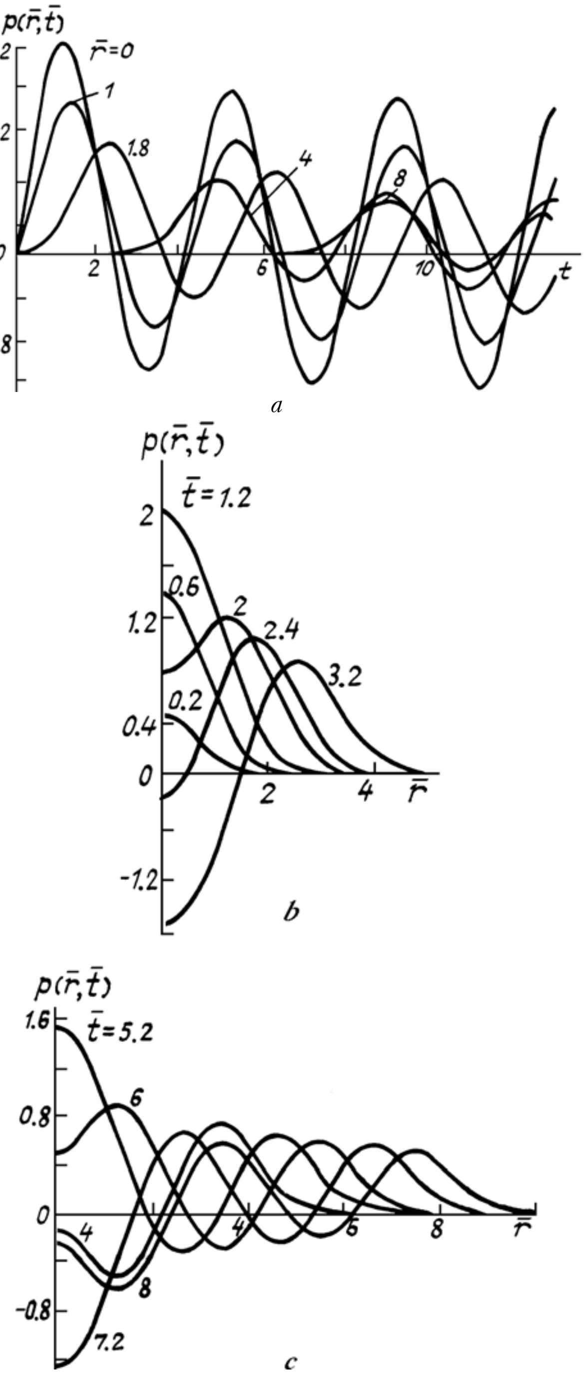

FIG. 3. Temporal behavior of the sound pressure at various distances $\bar r$ from the beam axis for harmonic modulation of the beam intensity with the frequency $\bar\omega = 1.6$ (*a*). *Distances from the beam axis are indicated on the curves. Changes in the sound pressure with distance at different times $\bar t$ indicated on the curves* (*b and c*).

Frequency dependence of the amplitude of the periodic component calculated from Eq. (7) is shown in Fig. 4a.

The results of calculations confirm the existence of two extrema at frequencies $f_1 = 0.07u/a$ and $f_2 = 0.45u/a$ that were discussed in Ref. 4. Their amplitudes are now refined

$$p_3(\bar r = 0, \bar\omega_1) = -0.252\alpha(\gamma - 1)\frac{W}{\pi a u};$$

$$\tilde p_3(\bar r = 0, \bar\omega_2) = 0.474\alpha(\gamma - 1)\frac{W}{\pi a u}.$$

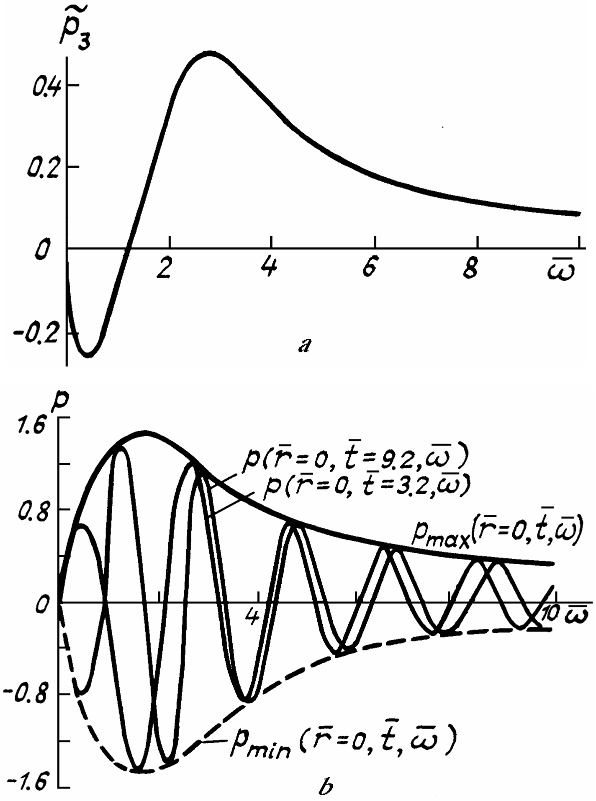

FIG. 4. Frequency dependence of the amplitude of the periodic component of the acoustic response on the beam axis (*a*) *and the dependence of the maximum* (*solid lines*) *and minimum* (*dashed lines*) *sound pressures on the beam axis on the modulation frequency* (*b*).

It follows from Fig. 4a that the amplitude of the periodic component of sound attains its maximum at $f_2 = 0.45u/a$. In addition, there exists a frequency $f_0 = 0.194u/a$, at which the periodic component in the acoustic signal vanishes.

For comparison we calculated the frequency dependences of the maxima and minima in the sound pulse on the beam axis in the case of the time modulation of the beam of the form $f_4(t) = H(t)\cdot\sin(\omega t)$. These results are shown in Fig. 4b. They represent the envelops of the curves describing the changes in the sound pressure vs modulation frequency at different times $\bar t$. It can be seen from this figure that the nonharmonic modulation results not only in the deviation of the frequency at which the acoustic signal is at maximum ($f = 0.27u/a$) but also in the variation of the character of frequency dependence.

Thus, our calculated results give full description of the acoustic fields generated during the propagation of modulated laser beams through the atmosphere.

Combining an experiment with such numerical calculation will permit one to determine either the radiation power or the absorption coefficient of air for a given type of modulation of the beam, since the calculated dimensionless sound pressure differs from the experimental one in the coefficient, which is constant in Eqs. (3), (5), and (6) for every type of modulation and contains the sought-after parameters.

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