CALIBRATION OF MULTIFREQUENCY LIDAR ACCORDING TO THE SIGNALS OF MOLECULAR SCATTERING

V.E. Zuev, V.V. Zuev, and B.S. Kostin

Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk

Received March 24, 1992

A technique of calibration of a multifrequency lidar according to the signals of molecular scattering is considered. It is shown that an a priory *assignment of the thermodynamic parameters and molecular scattering coefficients are not required for the calibration of the lidar having three and more working frequencies.*

One of the most important problems of lidar application for sounding of the aerosols of the upper atmosphere is its calibration. The technique of lidar calibration according to the signals of molecular scattering[1] is widely used for solving the given problem. This technique is based on the assumption that at some altitudes (above 30 km, as a rule) aerosol scattering is negligible and the lidar signals are associated only with molecular scattering. The technique makes it possible to determine the calibration parameters to within the constant being equal to the square atmospheric aerosol transmissivity. However, various alternatives of the technique assume an *a priori* assignment of the molecular scattering coefficients or of the thermodynamic atmospheric parameters, which is equivalent. We propose the technique which is capable of solving the problem of calibration of multifrequency lidar in sounding of the aerosols of the upper atmosphere without additional information.

Let the lidar with n working wavelengths be used for sounding. Starting from the altitude h_0 the lidar signals are assumed to be associated with molecular scattering solely. The lidar equations then can be given in the form

$$N(\lambda_i, h_l) = B(\lambda_i)\, h_l^{-2}\, \beta^{\rm m}_\pi(\lambda_i, h_l)\, \exp\left\{-2\int\limits_0^{h_l} \beta^{\rm m}_{\rm sc}(\lambda_i, z){\rm d}z\right\},$$
$$i = 1, 2, \dots, n \qquad (1)$$

where

$$h_l = h_0 + (l-1)\Delta h\,, \quad l = 1, 2, \dots, \quad B(\lambda_i) = b\,(\lambda_i)\, T^2_{\rm a}(\lambda_i, h_0)\,,$$

$$T^2_{\rm a}(\lambda_i, h_0) = \exp\left\{-2\int\limits_0^{h_0} \beta^{\rm a}_{\rm ex}(\lambda_i, z){\rm d}z\right\}. \qquad (2)$$

Here $N(\lambda_i, h_l)$ is the number of photopulses recorded from the altitude h_l, λ_i is the wavelength, $\beta^{\rm m}_{\rm p}(\lambda_i, h_l)$ is the molecular backscattering coefficient, $b(\lambda_i)$ is the calibration constant, $T^2_{\rm a}(\lambda_i, h_0)$ is the square of the atmospheric aerosol transmissivity, $\beta^{\rm a}_{\rm ex}(1_i, z)$ and $\beta^{\rm m}_{\rm sc}(\lambda_i, z)$ are the coefficients of the aerosol extinction and molecular scattering, respectively.

For each value of h_l Eqs. (1) represent the system of n equations in 3 n unknowns. We can reduce the number of the unknowns in this system if we make use of the analytical expression for the spectral dependence of the molecular scattering coefficients. Each coefficient in Eqs. (1) can be represented as a product

$$\beta^{\rm m}(\lambda_i, h_l) = p(\lambda_i)\, \beta^{\rm m}(h_l)\,, \qquad (3)$$

where $\beta^{\rm m}(h_l)$ is the corresponding coefficient for the shortest wavelength λ_1 and $p(\lambda_i)$ is the rational–fractional expression of the well–known form being determined in the theory of molecular scattering. If we neglect the dispersive properties of air then $p(\lambda_i)=(\lambda_1/\lambda_i)^4$. The substitution of Eq. (3) into Eq. (1) gives

$$N(\lambda_i, h_l) = B(\lambda_i)\, p(\lambda_i)\, h_l^{-2}\beta^{\rm m}_\pi(h_l)\exp\left\{-2p(\lambda_i){\rm Q}(h_l)\right\},$$
$$i = 1, 2, \dots, n, \qquad (4)$$

where

$${\rm Q}(h_l) = \int\limits_0^{h_l} \beta^{\rm m}_{\rm sc}(z){\rm d}z\,.$$

Equations (4) contain $n + 2$ unknowns now; moreover, the molecular optical characteristics are the functions of the altitude alone. The altitude range for which the starting assumption of the technique of calibration according to the molecular scattering signals is valid is assumed to be sufficiently wide and to contain q counts (strobes). It is obvious that the problem of lidar calibration can be completely defined by means of the proper choice of the number q. Starting from the necessary condition for solving the system of equations by the least–squares technique, let us specify the relation between the number of the operating wavelengths and the necessary number of strobes in the form

$$nq \ge n + 2q. \qquad (5)$$

This inequality shows that 3–4 strobes are needed for calibration of the trifrequency lidar.

The system of transcendental equations (4) written down for q altitudes can be reduced, by taking the logarithm, to the system of linear algebraic equations of the form

$$f_{il} = x_i + y_l - 2{\rm p}_i{\rm Q}_l\,, \quad i = 1, 2, \dots, n, \quad l = 1, 2, \dots, q\,, \qquad (6)$$

where

$$f_{il} = \ln\frac{N(1_i, h_l)\, h_l^2}{p(1_i)},\ x_i = \ln B(\lambda_i)\,,\ y_l = \ln \beta^{\rm m}_\pi(h_l)\,,\ {\rm Q}_l = {\rm Q}(h_l)\,.$$

0235-6880/92/06 415-02 $02.00 © 1992 Institute of Atmospheric Optics

The system of equations (6) consists of nq equations and contains $n + 2q$ unknowns. Taking into account the specific features of equations of this system and using the standard procedure of the least–squares technique, we obtain

$$\sum_{i=1}^{n} f_{il} - \sum_{i=1}^{n} x_i - n y_l + 2Q_l \sum_{i=1}^{n} p_i = 0 ,$$

$$\sum_{i=1}^{n} p_i f_{il} - \sum_{i=1}^{n} p_i x_i - y_l \sum_{i=1}^{n} p_i + 2Q_l \sum_{i=1}^{n} p_i^2 = 0 , \quad (7)$$

$$\sum_{l=1}^{q} f_{il} - q\, x_i - \sum_{l=1}^{q} y_l + 2p_i \sum_{l=1}^{q} Q_l = 0 ,$$

The system of equations (7) has dimensionality $n + 2q$. The solution of this system yields

$$y_l = \frac{1}{D} \sum_{i=1}^{n} \sum_{j=1}^{n} p_j (p_i - p_j)(f_{il} - x_i) ,$$

$$Q_l = \frac{2}{D} \sum_{i=1}^{n} \sum_{j=1}^{n} (p_i - p_j)(f_{il} - x_i) , \quad (8)$$

$$x_i + \frac{1}{D} \sum_{j=1}^{n} \sum_{k=1}^{n} (p_i - p_j)(p_k - p_j) x_k =$$

$$= \frac{1}{q} \sum_{l=1}^{n} \left[f_{il} + \frac{1}{D} \sum_{j=1}^{n} \sum_{k=1}^{n} (p_i - p_j)(p_k - p_j) f_{kl} \right] , \quad (9)$$

where

$$D = \sum_{i=1}^{n} \sum_{j=1}^{n} p_i (p_i - p_j) .$$

The desired quantities are found by a numerical solution of the system of equations (9) of the dimensionality n (see Ref. 3).

The solution of the system of equations (9) determines unambiguously the product $b(\lambda_i) T_{\rm a}^2(\lambda_i, h_0)$. However, the accuracy of determining of this quantity depends on the altitude h_0 at which the lidar is calibrated. The required altitude h_0^* can be determined by means of variation of h_0 within the range from 30 km to the maximum sensing altitude and of a subsequent analysis of x_i behavior. If x_i remains unchanged as h_0 varies, then the initial value of h_0 is chosen as h_0^*. Otherwise, that value of h_0 is chosen at which x_i have their minimum. If the effect of aerosol is pronounced, then the value of x_i will be overestimated when solving the system of equations (9).

In conclusion we note that this variant of the technique does not require additional information about the molecular component or thermodynamic parameters of the atmosphere. On the contrary, the characteristics of molecular scattering can be calculated based on formulas (8) and can be compared with the corresponding model if any.

REFERENCES

1. V.E. Zuev, G.M. Krekov, and M.M. Krekova, in: *Remote Sensing of the Atmosphere* (Nauka, Novosibirsk, 1978), pp. 3–46.

2. Yu.V. Linnik, *The Least–Squares Technique and the Principles of the Theory of Data Processing* (State Physical and Mathematical Press, Moscow, 1962), 349 pp.

3. V.L. Zaguskin, *Reference Book on Numerical Techniques for Solving the Equations* (State Physical and Mathematical Press, Moscow, 1960), 216 pp.