NONLINEAR MARKOVIAN FILTRATION AS APPLIED TO LIDAR SOUNDING OF OZONE BY THE DIFFERENTIAL ABSORPTION METHOD

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Received March 29, 1992

The possibility of the application of the optimum Markovian filtration to lidar sounding by the differential absorption (DA) method is shown on the basis of the Markovian model of the altitude fluctuations of the gas concentration smoothed by the lidar pulse. The algorithms for optimum estimation of the fluctuating profile of the gas concentration and its variance are found. The efficiency is analyzed by the numerical simulation method for four hypothetic lidars with the acceptable power as applied to the ozone sounding in the Hartley and Higgins bands.

Introduction. Stochasticity of the fields of optical and physical atmospheric parameters and the short noise fluctuations in the optical detection channel of the lidar receiver essentially limit the sounding efficiency, i.e., the set of the accuracy characteristics and spatiotemporal resolutions. But the requirements of concrete applications, in particular, the determination of the ozone concentration profiles by the differential absorption (DA) method to the quality of the lidar data are continually raising. For this reason the idea of applying the nonlinear Markovian filtration of lidar signals proposed and developed in Refs. 1–3 is fruitful. Such a technique of processing improves the efficiency of sounding of the fluctuating profiles of optical and physical parameters due to the use of the *a priori* data on their statistical structure.

This elaboration was continued in Refs. 4–7 as applied to lidar sounding of thermodynamic atmospheric parameters, in Refs. 8 and 9 – to lidar sounding of aerosol, in Ref. 10 –– to measurement of the gas content along the paths, and in Refs. 11 and 12 – to the temporal filtration of lidar returns.

In this paper the nonlinear Markovian filtration is used for the optimum separation of the gas concentration profile as applied to ozone sounding by the DA method, and the calculations are made in the Hartley and Higgins bands.

Physical premises. Let us consider a ground–based monostatic lidar with normalized power function $f(\tau)$ operating at the wavelength λ and sounding the atmosphere in the altitude range $[h_0, h_{max}]$. The power $P_s(h)$ of the signal component at the detector input in the single scattering approximation at the distance h is determined by the lidar equation[13]

$$P_s(h) = \chi_1 E_0 S_a \int\limits_0^h \mathrm{d}h' f[2(h - h')/c]\, \tilde{\beta}(h')\, \tilde{Y}_a^2(0, h') \times$$

$$\times\, \tilde{Y}_R^2(0, h')\, \tilde{Y}_g^2(0, h')/(h')^2 , \tag{1}$$

where χ_1 is the total efficiency of the optical train; E_0 is the radiated pulse power; S_a is the efficient area of the receiving aperture; $\tilde{Y}_a$, $\tilde{Y}_R$, and $\tilde{Y}_g$ are the transmission functions associated with aerosol and molecular scattering and gas absorption, respectively; c is the speed of light; $\tilde{\beta}(h)$ is the profile of aerosol and molecular backscattering coefficient; $\tau = 2h/c$; and, tilde denotes the true profiles.

To describe the shape of the true pulses one can use the following time dependence:[13]

$$f(\tau;\, m) = \frac{(\tau/\tau_0)^{m-1}}{\tau_0\Gamma(m)} \exp\{-\tau/\tau_0\} , \tag{2}$$

where $m = 1, 2...$ and $\tau_0 > 0$ are the parameters and $\Gamma(m)$ is the gamma function.

Let us define the efficient sounding pulsewidth as $\tau_p = f^{-1}(\tau_{max};\, m)$.

Since $P_s(h)$ is caused by backscattering in the efficient altitude range $[h - L, h]$, where $L = c\tau_p/2$, we can neglect the variations in the factor $1/(h')^2$ and $\tilde{Y}_a^2$ and $\tilde{Y}_R^2$ under the integral sign in Eq. (1) given that $L \ll h$. Let us assume that the profiles of aerosol and molecular backscattering can be deterministic during one sounding run, but unknown altitude functions. In this case smoothing over the sliding interval $[h - L, h]$ significantly change only the profile $\tilde{Y}_g(0, h)$ and related with it the vertical profiles of concentration and gas absorption characteristics. As a result, we can write Eq. (1) in the following form:

$$P_s(h) = \chi_1 E_0 S_a\, h^{-2} \frac{c}{2}\, \beta(h)\, Y_a^2(0, h)\, Y_R^2(0, h)\, J(h) , \tag{3}$$

$$J(h) = \frac{2}{c} \int\limits_0^h \mathrm{d}h' f[2(h - h')/c]\, \tilde{Y}_a^2(0, h') . \tag{4}$$

Following the approach of Refs. 14 and 15, let us consider the models for the fluctuating parameters, which have such stochastic properties that their dependence on time or distance must be described by realization of a random process. Let us represent the random values of the concentration $\tilde{N}(h)$ in the form of $\tilde{N}(h) = \bar{N}(h) + \Delta\tilde{N}(h)$, where the bar denotes ensemble averaging.

Let us expand $\tilde{Y}_g^2(0, h)$ in the Taylor series in the profile $\Delta\tilde{N}(h)$ around the vertical profile smoothed by the sounding pulse

$$\Delta N(h;\ m) = \frac{2}{c}\int\limits_0^h \mathrm{d}h' f[2(h-h')/c;\ m]\,\Delta\tilde{N}(h')\ . \qquad (5)$$

As the profiles $\Delta\tilde{N}(h)$ and $\Delta N(h;\ m)$ are close in values, for functional (4) the following approximation:

$$J(h) \simeq \bar{Y}_g^2(0,\ h)\exp\left\{-2\int\limits_0^h \mathrm{d}h'\ \sigma_g(h')\,\Delta N(h';\ m)\right\}$$

is valid where $\sigma_g(h)$ is the gas absorptional cross section, $\bar{Y}_g(0,\ h)$ is the transmission associated with the absorption by the gas molecules with average concentration $N(h)$. Thus, the fluctuations $\Delta P_s(h) = P_s(h) - \bar{P}_s(h)$ and $\Delta N(h;\ m)$ are related nonlinearly, while $\Delta N(h;\ m)$ is the profile of natural fluctuations of the concentration $\Delta\tilde{N}(h)$ efficiently smoothed according to Eq. (5) over the sliding interval L.

Model of signals and noise. Let us study the statistical structure of the process $\Delta N(h,\ m)$. To this end, let us define $\Delta N(h;\ m)$ in terms of the variable $\eta_m(\tau)$ of the state in the form $\Delta N(h;\ m) = \sigma[\Delta N(h;\ m)]\ \eta_m(\tau)$, where $\sigma^2[\Delta N(h;\ m)]$ is the variance of the fluctuations of the concentration smoothed according to Eq. (5), and let us differentiate Eq. (5) m times. Combining the state variables η_j, where $1 \le j \le m$, to the state vector $\boldsymbol{\eta} = \{\eta_1,\ \eta_2,\ \ldots,\ \eta_m\}^{\mathrm{T}}$, we can write the stochastic differential equation (SDE) of the following form:

$$\boldsymbol{\eta}(\tau) = A\boldsymbol{\eta}(\tau) + \mathbf{w}(\tau)\ . \qquad (6)$$

It is convenient to represent the matrix of coefficients A as

$$A = \alpha_0 \begin{Vmatrix} -1 & 0 & 0 & \ldots & 0 & 0 \\ 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & . & . & \ldots & . & . \\ 0 & 0 & 0 & \ldots & 1 & -1 \end{Vmatrix},$$

where $\alpha_0 = 1/\tau_0$ and $\omega(\tau) = \{\omega_1(\tau),\ 0,\ \ldots,\ 0\}^{\mathrm{T}}$ is the m–dimensional vector. Its nonzero component $\omega_1(\tau)$ is the white Gaussian noise with the correlation function[15] $<\omega_1(\tau)\ \omega_1(\tau')> = W_1/2\delta(\tau-\tau')$, when $L_0 = c\tau_0/2 \gg h_{\mathrm{k}}^{\mathrm{n}}$, where $h_{\mathrm{k}}^{\mathrm{n}}$ is the spatial correlation length of the concentration fluctuations $\Delta\tilde{N}(h)$ without smoothing. By virtue of the central limit theorem, $\Delta N(h;\ m)$ and consequently $\eta_m(\tau)$ are approximately Gaussian functions regardless of the distribution $\Delta\tilde{N}(h)$ for $L \gg h_{\mathrm{k}}^{\mathrm{n}}$. It is natural that when the fluctuations $\Delta\tilde{N}(h)$ without smoothing are Gaussian, the distribution function of the fluctuations $\Delta N(h;\ m)$ linearly related to them is also Gaussian for any L.

Particulary, the result of processing of the radiosonde data cannot characterize the form of the concentration distribution $\Delta\tilde{N}(h)$ O_3 after significant temporal and spatial smoothing. However, the prevalence of normal and lognormal distributions in all measurements[16] enables us to expect that the distribution of fluctuations $\Delta N(h,\ m)$ of the concentration of O_3 is close to normal at small time intervals. Thus, for $m = 1$ $\eta_m(\tau) = \eta_1(\tau)$ is the Gaussian Markovian process while for $m \ne 1$ $\eta_m(\tau)$ is fitted by the component of the m–dimensional Markovian process.

For the monostatic lidar with the receiver operating in the sounded altitude range in the linear detection regime the condition of the weak coherent selection[14] is realized in most cases. This fact makes it possible to apply the approximate asymptotically exact distributions of the number of photoelectrons. Particulary, for a prescribed realization of the power profile $P_s(h)$ the distribution of the signal photoelectrons for the given time interval $\Delta\tau$ is the Poisson one. Resultant distribution of the external and internal noise photoelectrons caused by the background radiation and the dark current of the photodetector, is also the Poisson one.

Assuming the backscattering intensity to be the random function of τ, we will obtain the doubly stochastic Poisson process at the output from the photodetector whose conditional intensity averaged over the ensemble of short noise fluctuations is modulated by the vector process $\boldsymbol{\eta}(\tau)$.

As a result, the total intensity function is given in the form:

$$\nu_\Sigma(\tau;\ \eta) = \bar{\nu}_s(\tau)\exp\left\{-2\int\limits_0^h \mathrm{d}h'\ \bar{\gamma}(h')\,\mu(h')\,\eta_m(h')\right\} + \nu_N, \qquad (7)$$

where

$$\bar{\nu}_s(\tau) = \chi_{\mathrm{R}} E_0 S_a\, h^{-2}\,\beta(h)\,\frac{c}{2}\,\bar{Y}_\Sigma^2(0,\ h)/h\nu \qquad (8)$$

is the function of the intensity of signal photoelectrons with an account of the gas absorption for the average absorption coefficient $\bar{\gamma}(h)$; $\mu(h)$ is the coefficient of gas concentration variations at the altitude h; $\bar{Y}_\Sigma(0;\ h) = Y_a Y_R \exp\{-\int_0^h dh'\bar{\gamma}(h')\}$; $\chi_\Sigma = \chi_{ph}\chi_1$; χ_{ph} is the quantum efficiency of the photodetector; $\nu_N = [\chi_{\mathrm{ph}} P_{bg} + \nu_d]$ is the total density of the dark photoelectrons with the intensity ν_d and the background photoelectrons, where P_{bg} is the power of the background radiation incident on the receiving aperture.

According to the classification of the photodetection regimes introduced in Ref. 14, for the photon counting mode, the average total intensity $\bar{\nu}_\Sigma(\tau) = \bar{\nu}_s(\tau) + \nu_N$ must satisfy the condition

$$\bar{\nu}_\Sigma(2h/c) \le -\ln q/\tau_{sp}\ , \qquad (9)$$

where q is the threshold value of the dip probability, $\tau_{sp} = \sqrt{\tau_g + \tau_{bg}}$ is the efficient single–electron pulsewidth, τ_g is the time constant of the photodetector, $\tau_{bg} = 1/2\Pi$, Π is the bandwidth of the postdetector filter.

Filtration equation. Let $\mu(h) = \mu_0 = \mathrm{const}$ in the sounded altitude range. Let us introduce the state variable $\eta_{m+1}(\tau)$, for which the SDE has the form

$$\eta_{m+1} = c\bar{\gamma}(h)\ \eta_m(\tau)/2\ . \qquad (10)$$

In this case the fluctuations $\Delta\tau(0,\ h)$ of the optical depth can be written down in terms of $\eta_{m+1}(\tau)$ as

$\Delta\tau(0, h) = \mu_0\eta_{m+1}(\tau)$, and the intensity of signal photoelectrons can be determined in terms of the generalized state vector $\eta_0 = \{\eta_1, \eta_{m+1}\}^T$, using Eq. (7), we obtain

$$\nu_s(\tau; \eta) = \bar{\nu}_s(\tau) \exp\{-\mathbf{C}^T\eta_0\}, \tag{11}$$

where $\mathbf{C} = \{0, -2\mu_0\}^T$. In the linear state space the generalized vector $\eta_0(\tau)$ satisfies the SDE in the form of formula (6) with the matrix of coefficients

$$A_0 = \begin{pmatrix} A & 0 \\ c\bar{\gamma}(h)/2 & 0 \end{pmatrix}$$

and $(m + 1)$–dimensional Gaussian noise $\omega_0(\tau) = \{\omega(\tau), 0\}^T$ with the matrix $b_0 = \{b_{0ij}\}$ of the bilateral spectral power densities, where $b_{0ij} = 0$ for $(i, j) \neq (1, 1)$ and $b_{011} = 2\alpha_0$.

By virtue of the above–given relations, $\Delta N(h)$ and $\nu_s(\tau, \eta_0)$ are determined uniquely in terms of η_0. The problem is the optimum estimate of the realization $\eta_0(c\tau/2)$ from the input data. In the photon counting photodetection mode the sequence of the random numbers of photoelectrons

$$n_\Sigma(\tau; \Delta\tau) = n_s(\tau; \Delta\tau) + n_{bg}(\Delta\tau) \tag{12}$$

is the sampling data of the doubly stochastic Poisson process $N(\tau; \eta_0)$ with the intensity function $\nu_s(\tau, \eta_0)$. Let us find the processing algorithm for the population $n_\Sigma(\tau, \Delta\tau)$ which provides the optimum estimate η_0^* of the maximum of the *a posteriori* probability density.

The *a priori* probability density $W_0(\eta_0)$ related to the Markovian vector process $\eta_0(\tau)$ at the moment τ satisfies the equation in partial derivatives of the second order, which is well known as the Fokker–Planck–Kolmogorov equation (FPK)[17]

$$\frac{\partial W_0(\eta_0)}{\partial\tau} = L_{pr}\{W_0(\eta_0)\},$$

where L_{pr} is the *a priori* FPK operator, whose drift coefficient η_0 is the linear function of η_0 and the diffusion coefficient is independent of it. Following Refs. 18 and 19, we can describe the evolution or the *a posteriori* probability density (APD) of the Markovian process, which modulate the intensity function of the inhomogeneous Poisson process. According to Refs. 18 and 19, for the APD $W(\eta_0/N(\tau))$ of the state vector $\eta_0(\tau)$ we have the relation

$$dW(\eta_0/N(\tau)) = L_{pr}[W(\eta_0/N(\tau))]d\tau + W(\eta_0/N(\tau)) \times$$
$$\times [\nu(\tau; \eta_0) - \bar{\nu}]\, \bar{\nu}^{-1}(\tau; \eta_0)\, [dN(\tau) - \bar{\nu}(\tau; \eta_0)d\tau], \tag{13}$$

where $\bar{\nu}(\tau, \eta_0)$ is the conditional estimate of the intensity function and $dN(\tau)$ is the increment to the Poisson process $N(\tau)$. The Kolmogorov–Feller equation (13) is the main result solving the problem of filtration of η_0 for observation of the doubly stochastic Poisson process $N(\tau; \eta_0)$. The direct way of solving Eq. (13), as a rule, appears to be irrational.[18,19] Therefore, it is more expedient to apply Eq. (13) in various approximate algorithms capable of obtaining the estimate of the process $\eta_0(\tau)$ without direct solution of Eq. (13). The parametrization of the APD is one of the natural approaches to the development of the approximate algorithms. Particulary, for the Gaussian approximation of the APD the optimum estimate can be obtained by means of the solution of the system of SDE of the quasioptimum nonlinear filtration for the conditional average η_0^* and the correlation matrix $K = \langle(\eta_0 - \eta_0^*)(\eta_0 - \eta_0^*)^T\rangle$ (see Ref. 18). However, in view of the fact that the practical implementation of the estimate $\eta_0(\tau)$ in the general case is difficult, let us restrict ourselves only to the Calman–Bucy filtration, when we can linearize Eq. (11) in η_0. Equations for the quasioptimum linear filtration have the form[19]

$$d\eta_0^* = A_0(h)\eta_0 d\tau + \frac{K\mathbf{C}}{\bar{\nu}_\Sigma(\tau)}[dN(\tau) - \bar{\nu}_\Sigma(\tau)d\tau - \mathbf{C}^T\eta_0^* d\tau], \tag{14}$$

$$K = A_0(h)K + KA^T(h) + b_0 - K\mathbf{C}\mathbf{C}^T K/\bar{\nu}_\Sigma(\tau) \tag{15}$$

with the initial conditions $\eta^*(\tau_0) = 0; K_{mm}(\tau_0) = 1$; $K_{ij}(\tau_0) = 0$ for $(i, j) \neq (m, m)$.

The optimum processing includes a simultaneous solution of the system of Eqs. (14) and (15) as the input data $n_R(\tau, \Delta\tau)$ come given that the profiles $\bar{N}(h)$, L_0, $\bar{\nu}_\Sigma(\tau)$, etc. are determined *a priori* with the above–indicated initial conditions by a suitable finite–difference method. The recurrent finite–difference solution of this system of equations yields the optimum estimate η_{0m}^* thereby giving the estimate of the profile $N(h)$

$$N^*(h) = \bar{N}(h)\, [1 + \mu_0\eta_{0m}^*(\tau)], \tag{16}$$

and the estimate $K_{mm}(\tau)$ of the variance for the realization $\eta_{0m}^*(\tau)$ thereby giving the variance $D[N^*(h)] = \mu_0^2\bar{N}^2(h)K_{mm}(\tau)$ of the profile $N^*(h)$.

The necessity for determining the average profiles of $\bar{N}(h)$ and $\bar{n}_R(\tau)$ *a priori* is caused by the fact that the statistical structure of the profiles, which forms the basis for the optimization of data processing, is determined for the fluctuations rather than for the average profiles. The most natural way of estimating the average profiles is to include without optimization the parallel channel of the postdetection processing into the receiver. Resulting from spatiotemporal smoothing in this channel, the variations of the estimates caused by the gas concentration fluctuations are smoothed, and the estimate accuracy is quite sufficient for its application as an average profile.

Let us explain the application of the Calman–Bucy filtration algorithm (14) and (15) in different variants of the DA method as applied to sounding in the Hartley and Higgins absorption bands. As a rule, two wavelengths are used: the short one corresponds to the strong absorption while the long one – to the weak absorption. In order to estimate the efficiency of the Markovian filtration let us consider the practically realizable situation, when one wavelength lies in the Hartley and Higgins bands while the required data on the elastic scattering characteristics are estimated from the data of simultaneous sounding in the visible ($\lambda = 532$ nm) and near–UV ($\lambda = 351$ and 353 nm) wavelength ranges, in which the ozone absorption is negligible.

Then we can use only the linear part of the expansion $\nu_s(\tau; \eta_0)$ in a power series of η_{m+1}, if the condition

$$2\,\overline{\Delta\tau(h_0,\,h)}\,]^{1/2} \simeq 2\mu_0 \int_{h_0}^{h} dh'\overline{\gamma}(h') \ll 1 \tag{17}$$

is satisfied.

Analysis of the filtration efficiency. Let us consider the altitude dependence of the variance of the ozone concentration estimate as the figure of merit of the filtration. According to Eq. (16)

$$D[N^*(h)] = \mu_0^2\overline{N}^2(h)\, D\{\eta^*_{0m}(h)\}\,, \tag{18}$$

where $D\{\eta^*_{0m}(h)\} = K_{mm}(h)$ is the corresponding diagonal element of the matrix K satisfying the dispersive equation (15). In turn, we can obtain the relation for $K_{mm}(h)$ from Eq. (18) in the form

$$K_{mm}(h) = \frac{D[N^*(h)]}{D[N(h)]}\,,$$

since $D[N(h)] = \mu_0^2\overline{N}^2(h)$. Thus, $K_{mm}(h)$ is the ratio of the *a posteriori* variance of the estimate $N^*(h)$ to the *a priori* variance of the fluctuating profile $N(h)$ of the gas concentration.

It is convenient to elucidate the main features of the dynamics of the filtration efficiency with the use of one of realizable simple models of the concentration fluctuations $\Delta N(h)$ smoothed by the sounding pulse, particulary, for the exponential shape of the sounding pulse ($m = 1$) and two–dimensional vector $\eta_0 = \{\eta_1,\,\eta_2\}^{\mathrm{T}}$. According to Eq. (15), the elements of the correlation matrix K satisfy the following system of differential equations:

$$\begin{cases} \dfrac{dK_{11}(h)}{dh} = -\dfrac{2}{L_0}K_{11}(h) + \dfrac{2}{L_0} - \dfrac{4\overline{\nu}_s^2(h)\mu_0^2}{\overline{\nu}_\Sigma(h)}K_{12}^2(h); \\ \dfrac{dK_{12}(h)}{dh} = -\dfrac{2}{L_0}K_{12}(h) + \overline{\gamma}(h)K_{11}(h) - \dfrac{4\overline{\nu}_s^2(h)\mu_0^2}{\overline{\nu}_\Sigma(h)}K_{12}(h)K_{22}(h)\,; \\ \dfrac{dK_{22}(h)}{dh} = \overline{\gamma}(h)K_{12}(h) - \dfrac{4\overline{\nu}_s^2(h)\mu_0^2}{\overline{\nu}_\Sigma(h)}K_{22}(h)\,, \end{cases} \tag{19}$$

where the altitude $h = c\tau/2$ is an independent variable; therefore, the profiles of relative variances $K_{11}(h)$ and $K_{22}(h)$ characterize the filtration efficiency and its altitude dependence.

Taking into account the complicated dependence of $\overline{\nu}_s(h)$ and $\overline{\nu}_\Sigma(h)$ on h we cannot expect the exhaustive analytical study of the dependences $K_{11}(h)$, $K_{12}(h)$, and $K_{22}(h)$. To do this, it is necessary to model the profiles $\overline{\nu}_s(h)$ and $\overline{\nu}_N$ taking into account all factors accompanying the sounding of O_3 in the UV range and then to make the numerical integration of Eq. (19).

We can obtain the vertical dependence $K_{11}(h)$ by replacing $K_{12}(h)$ in the first equation of the system of equations (19) by its approximation

$$\tilde{K}_{12}(h) \simeq \overline{\gamma}(h)\, L_0\, K_{11}(h)$$

which is obtained for $L_0 \ll h - h_0$. In this case we can integrate the relation for $K_{11}(h)$ independently of all the other relations of the system of equations (19). As a result, we have

$$\frac{dK_{11}(h)}{dh} = -\frac{2}{L_0}K_{11}(h) + \frac{2}{L_0} - \frac{2}{L_0}Q(h;\,\lambda)\,K_{11}^2(h)\,, \tag{20}$$

where

$$Q(h;\,\lambda) = \frac{2\overline{\nu}_s^2(h)\,\mu_0^2L_0}{\overline{\nu}_\Sigma(h)}\,[\overline{\gamma}(h)L_0]^2\,. \tag{21}$$

Let us term the value $Q(h,\,\lambda)$ which, as it is mentioned below, determines to a large extent the filtration efficiency in the sounded altitude range the generalized signal–to–noise ratio, in analogy with Refs. 12, 14, and 15 as applied to the one– and two–channel filtration of lidar signals of the elastic scattering. It can be seen that $Q(h;\,\lambda)$ in the form of formula (21) differs from the ratio $Q(h)$ introduced in Refs. 2 and 15 by the factor

$$\overline{\tau}_{O_3}(0,\,L_0) = \overline{\gamma}(h)L_0$$

which takes into account the ozone absorption at the interval L_0 determining the spatial resolution of the lidar.

The qualitative character of the $K_{11}(h)$ behavior in different sounding variants is the same:[15] fast decrease from the initial value $K_{11}(h_0) = 1$ down to a fixed value $\overline{K}_{11}$ (the transient regime), then much slower increase with the asymptotical approach to $K_{11}(\infty) = 1$. The approximate analytical study of Eq. (20) yields the duration of the transient process

$$h - h_0 \simeq \{2Q(h_0;\,\lambda)\}^{-1}\,.$$

Since the spatial scale of variation of the profiles $\overline{\nu}_s(h)$ and $\overline{\gamma}(h;\,\lambda)$ is, as a rule, much greater than L_0, we can find the so–called quasistationary solution of Eq. (20). To this end, let us set $dK_{11}(h)/dh = 0$ and write down the solution $\overline{K}_{11}(h)$ of the quadratic equation in $\overline{K}_{11}(h)$ in the form

$$\overline{K}_{11}(h) = \frac{1}{2Q(h;\,\lambda)}\{\sqrt{1 + 4Q(h;\,\lambda)} - 1\}\,. \tag{22}$$

If $Q(h,\,\lambda) \gg 0.25$, then $\overline{K}_{11}(h) \simeq \{Q(h,\,\lambda)\}^{-0.5}$. It can be seen from Eq. (22) that $Q(h,\,\lambda)$ is the most important parameter determining the filtration efficiency. To analyze the vertical profile and spectral behavior of $Q(h,\,\lambda)$, we calculated the profiles $Q(h,\,\lambda)$ at the given wavelengths of a Kr–F excimer laser with cells filled with H_2 and D_2 and of the Xe–Cl and Xe–Br excimer lasers with real lidar parameters. We can conclude from the results of calculations of $Q(h,\,\lambda)$ that the multipulse sounding during one measurement run must be done in order to provide $Q(h;\,\lambda) \gg 1$.

It is evident that the optimum filtration makes sense only for those altitudes at which $K_{11}(h) \ll 1$, since the *a priori* determined profile $\bar{N}(h)$ can be considered as a trivial estimate of the realization $N(h)$ with the variance being equal to $D[N(h)]$, i.e., with $K_{11}(h) = 1$. It can be seen from Eq. (22) that $\bar{K}_{11}(h)$ is inversely proportional to $Q(h; \lambda)$; therefore, for optimum filtration the inequality $Q(h; \lambda) \gg 1$ must be satisfied.

Four hypothetic lidars with the parameters given in Tables I and $\Pi = 5{\cdot}10^7$ Hz and $\nu_\delta = 10^2$ s^{-1} operating at night, were chosen as an example for calculating the sounding efficiency.

The dependences $Q(h; \lambda)$ for $L_0 = 300$ m averaged over $M = 10^4$ sounding runs were calculated using the Elterman model of aerosol scattering[20] and taking into account the ozone absorption in the Hartley and Higgins bands at the laser wavelengths of the lidars A–D that corresponds, for example, to the sensing period $\Delta t_s = 200$ s given that the sounding pulse repetition frequency $f_r = 50$ Hz. The vertical profile $K_{11}(h)$ calculated from Eq. (22) in the case of sounding with the lidars A – D are shown in Fig.1.

TABLE I. Parameters of hypothetic lidars

Lidar	*A*	*B*	*C*	*D*
λ, vm	282	291.6	308	313
Laser	XeBr	Kr–F+D_2^2	XeCl	Kr–F+H_2^2
E_0, J	0.1	0.057	0.4	0.095
S_a, m^2	0.785	0.785	0.785	0.785
χ_{ph}	0.2	0.2	0.2	0.2
χ_1	$9.35{\cdot}10^{-2}$	$1.02{\cdot}10^{-1}$	$1.15{\cdot}10^{-1}$	$1.15{\cdot}10^{-1}$

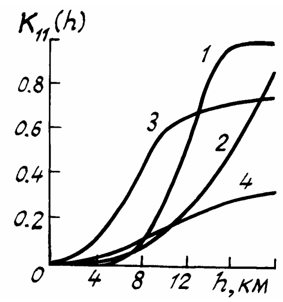

FIG. 1. Profiles of the efficiency of filtration of the ozone concentration for lidar sounding at the wavelengths λ = 282 nm (1), 291.6 nm (2), 313 nm (3), *and* 308 nm (4).

For the above–indicated lidar parameters, the spatial filtration is efficient up to the altitudes of 9, 12, 16, and 20 km, respectively, for the lidars A–D, while above these altitudes the sounding is inefficient from the standpoint of the sensitivity of the DA method to the absorption optical depth of the atmospheric column and the accuracy of the estimate of the fluctuating ozone concentration.

We can obtain the smaller values of $K_{11}(h)$ by means of deterioration of the spatiotemporal resolution, thereby increasing the accuracy of the estimate $N^*(h)$ of the ozone concentration $N(h)$ according to Eq. (18), or the sensing range with the acceptable accuracy.

Conclusion. Thus, the Markovian model of the gas concentration fluctuations smoothed in the course of sounding by the DA method is substantiated. The algorithms for the lidar signal processing in the photon–counting detection mode are found. It is shown that the efficiency of filtration of the fluctuating gas concentration profile depends on the generalized signal–to–noise ratio introduced in the paper. This approach is easily generalized for the case of sounding of other gases by the DA method both in the bands and lines of absorption.

Authors would like to thank A.I. Popkov for his help in performing calculations.

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