VISION OF VOLUME OBJECTS THROUGH THE EARTH'S ATMOSPHERE

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In this paper we discuss the problem on observation of volume objects through the Earth's atmosphere. The results of our study are equations that describe the field of radiation scattered by a volume object as it is recorded with an imaging system through the atmosphere. It is shown that under certain conditions in the atmosphere the volume objects can be invisible.

In order to describe theoretically conditions of vision of various objects one has to know the fields of radiation which they produce in the atmosphere. Then one may find, using these fields, the distribution of irradiation over the image plane of an optical system used (the eye, in case of visual observation). And finally, by employing techniques of the theory of linear systems, one can estimate the conditions of these objects vision.

As to various plane emitters, the techniques to retrieve the fields they produce are quite well developed.[1–4] Therefore, they can be useful in the theory of vision when analyzing observations of plane objects on the Earth's surface. However, these techniques are insufficient for many other tasks.

In this paper we consider techniques of reconstructing of radiation fields from volume objects. Assume also that the object to be observed may be either on the Earth's surface or at an arbitrary height in the atmosphere. Assume, also, that an object of a certain shape is placed into the Earth's atmosphere at a height h. One needs to find the radiation field produced by the object at an arbitrary point $\mathbf{r}$ in the atmosphere. We search for an operator solution of this problem. We introduce operators which describe interaction between the radiation and the atmosphere free of any object L_a, interaction between an object and radiation L_o, interaction of radiation with the Earth's surface L_s. The equation for radiation brightness I in the atmosphere may then be written as follows:

$$L_a I = (L_o + L_s) I. \quad (1)$$

Let us introduce two auxiliary parameters s_o and s_s, using which equation (1) may be written in the form:

$$(L_a - s_o L_o - s_s L_s) I = 0. \quad (2)$$

We seek a solution of Eq. (2) in the form

$$I = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} I_{k\,l} (s_o)^k (s_s)^l. \quad (3)$$

The physical meaning of the function I_{kl} is that it decribes the radiation reflected from an object and the Earth's surface k times and l times, respectively. By substituting Eq. (3) into Eq. (2) and equating the terms with equal powers of s_o and s_s, we obtain:

$$L_a I_{k\,l} - L_o I_{k-1;\,l} - L_s I_{k;\,l-1} = 0. \quad (4)$$

As result for radiation reflected solely from the Earth's atmosphere (described by the function I_{00}), for radiation only once reflected from an object (function I_{10}), for radiation only once reflected from the Earth's surface (function I_{01}), and for radiation reflected once from the Earth's surface and once from the object (function I_{11}) equations (4) yield

$$\begin{aligned} &L_a I_{00} = 0; \\ &L_a I_{10} = L_0 I_{00}; \\ &L_a I_{01} = L_s I_{00}; \\ &L_a I_{11} = L_o I_{01} + L_s I_{10}. \end{aligned} \quad (5)$$

The first of these equations describes the atmospheric haze, the third – radiation reflected from the Earth's surface, second and fourth equations – the two components of radiation produced by a volume object placed into the atmosphere. The total radiation reflected by the observed object is, apparently, equal to

$$I_o = I_{10} + I_{11}. \quad (6)$$

By summing the second and the fourth equations of the system (5) we obtain:

$$L_a I_o = L_o(I_{00} + I_{01}) + L_s I_{10}. \quad (7)$$

Based on the equation (7) we may write an operator expression for the radiation brightness produced by the object observed

$$I_o(\mathbf{r};\Omega) = L_a^{-1} \{L_o(I_{00} + I_{01}) + L_s I_{10}\}, \quad (8)$$

where L_a^{-1} is the operator inverse to the operator L_a.

Expression (8) makes it possible to reconstruct the field of radiation produced by objects due to emission and reflection of light from their surfaces. However, it does not allow any description of one more factor affecting the conditions of objects vision, that is, the screening of radiation propagating through the atmosphere by objects. As a result of such a screening, "black" objects can be observed, while being neither reflector nor the emitter of radiation. According to Eq. (8), we have for such objects $L_o = 0$ and $I_o = 0$.

Screening may be accounted for if one describes the object by the operator of absorption $T(\rho; \Omega)$. If a beam,

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propagated along $\mathbf{\Omega}$ direction is incident into the point $\boldsymbol{\rho}$ of the object and is absorbed there, then $T = 1$, otherwise $T = 0$. The introduction of the operator T makes possible to relate brightness I_{ap} incident on the rear surface of the object to the radiation brightness at the front side $I^{+}_{a\,p}$

$$I^{+}_{a\,p} = T\, I_{a\,p}. \tag{9}$$

Taking this into account we may write an expression for the brightness of a volume object, which would account for screening, as follows:

$$I_{os}(\mathbf{r};\, \mathbf{\Omega}) = L^{-1}_{a}\, \{L_{o}(\, I_{00} + I_{01}) + L_{s}\, I_{10} - TI_{a\,p}\}. \tag{10}$$

The term $L^{-1}_{a}L_{s}I_{10}$ in expression (10) describes radiation coming from the object and to the plane of inlet of the optical system after reflection from the Earth's surface. For the majority of realistic situations, the influence of this factor can be neglected.

As is seen from expression (10), the function $I_{os}(\mathbf{r};\, \mathbf{\Omega})$ may be both positive and negative. The positive sign of the $I_{os}(\mathbf{r};\, \mathbf{\Omega})$ function indicates that the object is observed as a bright spot against the sky background, and the negative sign corresponds to the same object being observed as a dark spot. A situation is also possible when $I_{os}(\mathbf{r};\, \mathbf{\Omega}) = 0$. In this case the object will remain invisible against the background from sky and the Earth's surface.

To perform computations one needs to have explicit expressions for operators entering into Eq. (10). It is clear that the operator L_{a} is the operator of the equation of radiation transfer:

$$L_{a}\, I = \mathbf{\Omega}\cdot\nabla I - \varepsilon\, I - \int\limits_{4\pi} \sigma(\gamma)\, I(\Omega')\, d\Omega',$$

where $\varepsilon = \varepsilon(\mathbf{r}')$ and $\sigma(\gamma) = \sigma(\mathbf{r}';\, \gamma)$ are the coefficients of extinction and differential scattering of radiation by the atmosphere at the point $\mathbf{r}'$.

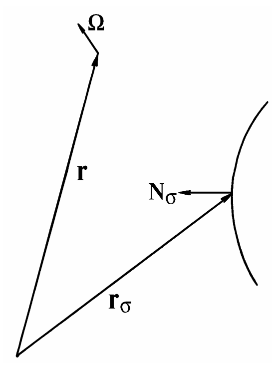

FIG. 1. On computing the reflected radiation brightness

Assume that the Earth's surface is described by equation $z = F(\boldsymbol{\rho})$, where $\boldsymbol{\rho}$ is the projection of radius-vector $\mathbf{r}$ on the horizontal surface (the X0Y plane). In this case the operator L_{s} may be written in the form:

$$L_{s}\, I = (1/\pi) \int\limits_{2\pi} d\Omega' \,|\, \Omega'\cdot\mathbf{N}(\mathbf{r}) \,|\, \beta(\Omega;\Omega';\mathbf{r})\, I(\Omega')\, d\Omega'\, \delta(\, z - F(\rho)),$$

where $\beta(\mathbf{\Omega};\, \mathbf{\Omega}';\, \mathbf{r})$ is the surface brightness coefficient at the point $\mathbf{r}$; $N(\mathbf{r})$ is the normal to the surface at that point; $\delta(z)$ is delta-function.

Let us now derive an explicit expression for the function $I_{os}(\mathbf{r};\, \mathbf{\Omega})$. Figure 1 shows an element of the surface of the object $d\sigma$ centered at $\mathbf{r}_{\sigma}$. This element has its normal $\mathbf{N}_{\sigma}$. According to the definition of radiation brightness the energy emitted by this element into the elementary solid angle $d\mathbf{p}$ is $d\Phi = B(\mathbf{r}_{\sigma};\, \mathbf{p}) \,|\, (N_{\sigma}\cdot\boldsymbol{\pi}) \,|\, d\sigma d\mathbf{p}$, where $B(\mathbf{r}_{\sigma};\, \mathbf{p})$ is the surface brightness. Energy transfered through the elementary surface $d\sigma_{01}$ normal to radius-vector $\mathbf{r}$, is

$$d\Phi = B(\mathbf{r}_{\sigma};\, \mathbf{p}) \,|\, \mathbf{N}_{\sigma}\cdot \mathbf{p}) \,|\, G(\mathbf{r}_{\sigma};\, \mathbf{r};\, \mathbf{\Omega};\, \mathbf{p})\, d\sigma\, d\mathbf{p},$$

where $G(\cdot)$ is the Greene's function for the equation of radiation transfer from mono-directional emitters. Energy transfered through the surface area $d\sigma$, normal to $\mathbf{\Omega}$, is $d\Psi = d\Phi\,|\mathbf{p}\cdot\mathbf{\Omega}|$. As follows from the definition of brightness the value $d\Psi$ is exactly equal to radiation brightness $dI(\mathbf{r},\, \Omega)$ produced by the element $d\sigma$. Brightness of radiation emitted by the object as a whole is equal to

$$I_{o}(\mathbf{r};\Omega) = \int d\mathbf{p} \int d\sigma B(\mathbf{r}_{\sigma};\, \mathbf{p}) \,|\, \mathbf{N}_{\sigma}\cdot\mathbf{p} \,|\,|\, \mathbf{p}\cdot\mathbf{\Omega} \,|\, G(\mathbf{r}_{\sigma};\, \mathbf{r};\Omega;\, \mathbf{p}). \tag{11}$$

Expression (11) does not account for the fact that the object surface may only be illuminated from outside, when $N_{\sigma}\cdot\mathbf{p} > 0$. This feature may be accounted for by introducing the following function:

$$H(\, \mathbf{p}\cdot\mathbf{N}_{\sigma}) = \begin{cases} 1 & \text{if } \mathbf{p}\cdot\mathbf{N}_{\sigma} > 0, \\ 0 & \text{if } \mathbf{p}\cdot\mathbf{N}_{\sigma} < 0. \end{cases} \tag{12}$$

The element $d\sigma$ may also be shadowed by the adjacent elements of the surface. The effect of such a shadowing may be easily accounted for by a function $F(\mathbf{p},\, \rho)$, its definition being illustrated in Fig. 2.

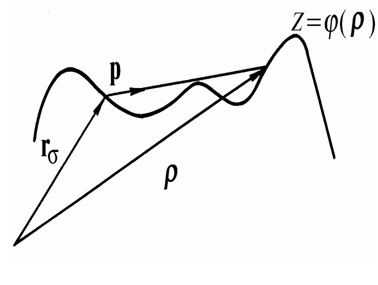

FIG. 2. On the definition of the shadowing function

Let the equation of the object surface be written in the form $z = \varphi(\rho)$. Equation for the ray coming to the point ρ_{σ} in the direction $\mathbf{p}$ has the form:

$$\rho = \rho_{\sigma} - [z - z_{\sigma}](\, \mathbf{p}_{\perp}/\mathbf{p}_{z}\,).$$

where $\mathbf{p}_{1}$ is the projection of $\mathbf{p}$ onto the X0Y plane, $\mathbf{p}_{z}$ is the direction cosine of the vector $\mathbf{p}$ with respect to the axis z; $\mathbf{r} = \{\rho,\, z\}$, $\mathbf{r}_{\sigma} = \{\rho_{\sigma},\, z\}$. Aparently, if this ray is shadowed by some element of the surface, there are solutions of the equation

$$\rho = \rho_\sigma - [\varphi(\rho) - z_\sigma](\mathbf{p}_\perp/\mathbf{p}_z). \tag{13}$$

Thus, the function of shadowing $F(\mathbf{p};\rho)$ is equal to unit for $\mathbf{p}$ and ρ such that no solution of Eq. (13) exists, and it is equal to zero for $\mathbf{p}$ and ρ, which satisfy Eq. (13).

With the account for shadowing, equality (11) can be written as follows:

$$I_o(\mathbf{r};\Omega)=\int d\mathbf{p}\int d\sigma B(\mathbf{r}_\sigma;\mathbf{p})\,|\mathbf{N}_\sigma\cdot\mathbf{p}|\,|\mathbf{p}\cdot\Omega|\times$$

$$\times G(\mathbf{r}_\sigma;\mathbf{r};\Omega;\mathbf{p})\,H(\mathbf{p}\cdot\mathbf{N}_\sigma)\,F(\mathbf{p};\rho_\sigma), \tag{14}$$

while the account for screening results in the following expression:

$$I_{os}(\mathbf{r};\Omega) = \int d\mathbf{p}\int d\sigma\, B(\mathbf{r}_\sigma;\mathbf{p})\,|\,\mathbf{N}_\sigma\cdot\mathbf{p}\,|\,|\,\mathbf{p}\cdot\Omega\,|\times$$

$$\times G(\mathbf{r}_\sigma;\mathbf{r};\Omega;\mathbf{p})\,H(\mathbf{p}\cdot\mathbf{N}_\sigma)\,F(\mathbf{p};\rho_\sigma) - \int d\mathbf{p}\int d\sigma I_{a\,p}(\mathbf{r}_\sigma;\mathbf{p})\times$$

$$\times T(\mathbf{r}_\sigma;\mathbf{p})\,G(\mathbf{r}_\sigma;\mathbf{r};\Omega;\mathbf{p}). \tag{15}$$

Within the small angle approximation[1–3] formula (15) may be significantly simplified, since within this approximation $\mathbf{p}\approx\Omega$. As a result we have

$$I_{os}(\mathbf{r};\Omega) = \int d\sigma\, B(\mathbf{r}_\sigma;\mathbf{p})\,|\,\mathbf{N}_\sigma\cdot\mathbf{p}\,|\,G_d(\mathbf{r}_\sigma;\mathbf{r};\Omega)\,H(\Omega\cdot\mathbf{N}_\sigma)\times$$

$$\times F(\Omega;\rho_\sigma) - \int d\sigma\, I_{ap}(\mathbf{r}_\sigma;\Omega)\,T(\mathbf{r}_\sigma;\Omega)\,G_d(\mathbf{r}_\sigma;\mathbf{r};\Omega), \tag{16}$$

where $G_d(\cdot)$ is the Green's function for the case of diffuse emitters.

As a rule, brightness $B(\mathbf{r}_\sigma;\Omega)$ entering into Eq. (15) consists of two components: brightness of the reflected solar radiation and brightness of thermal radiation. Brightness of the reflected solar radiation $B(\mathbf{r}_\sigma;\Omega)$ may be written as follows:

$$B(\rho_\sigma;\Omega) = (1/\pi)\int_{2\pi}\beta(\Omega;\Omega';\rho_\sigma)\,I_{as}(\rho_\sigma;\Omega')\,|\,\Omega'\cdot\mathbf{N}_\sigma\,|\times$$

$$\times H(-\Omega'\cdot\mathbf{N}_\sigma\,|\,F(\Omega;\rho_\sigma)\,d\Omega', \tag{17}$$

where the brightness of the point ρ_σ of the object surface due to radiation scattered by the atmospheric haze and by radiation reflected from the Earth's surface is $I_{as}(\rho_\sigma;\Omega) = I_{00}(\rho_\sigma;\Omega) + I_{01}(\rho_\sigma;\Omega)$.

Brightness of thermal radiation of the object may be written in the form

$$B_t(\rho_\sigma;\Omega) = (1/\pi)\,\varepsilon^0(\rho_\sigma;\Omega)\,\Phi^0[T^0(\rho_\sigma)]\,H(\Omega\cdot\mathbf{N}_\sigma)\,F(\Omega;\rho_\sigma),$$

where $\varepsilon^0(.)$ and $\Phi^0(.)$ are the coefficient of directional emission and the Planck function, respectively, and $T^0(.)$ is temperature.

Formula (17) permits further simplification for small-size objects, such that their size is much smaller than the width of the Green's function. In this case

$$I_0(\mathbf{r};\Omega)\simeq Q^*\,G_d(\mathbf{r}_0;\mathbf{r};\Omega), \tag{18}$$

where $\mathbf{r}_0$ is the coordinate of the object center,

$$Q^* = \int_r d\sigma\, B(\mathbf{r}_\sigma\cdot\Omega)\,|\,\mathbf{N}_\sigma\cdot\Omega\,|\,H(\Omega\cdot\mathbf{N}_\sigma)\,F(\Omega;\rho_\sigma) -$$

$$-\int d\sigma\, I_{a\,p}(\mathbf{r}_\sigma;\Omega)\,T(\mathbf{r}_\sigma;\Omega). \tag{19}$$

Below we shall call the value Q^* the "equivalent brightness of the object". It may be seen from Eq. (19) that this value may be either positive or negative.

As a rule, observations of objects use some optical and electronic instrumentation. As shown in Ref. 2, the "atmosphere – opto-electronic instrument" imaging system may often be considered isoplanar. Then, to describe optical image, one should use the convolution of the Green's function from the radiation transfer equation for an atmospheric layer with that for the opto-electronic system (formulas (16) and (18), with corresponding scaling). In this case, formulas (16) and (18) will describe irradiation in the image plane when the optical system is aimed at an object along the direction of the vector Ω.

The above expressions make it possible to consider both the statistical and deterministic versions of the theory of vision. Within the framework of statistical theory expressions for light fields will describe stochastic realizations of the field for a 3D-inhomogeneous stochastic atmosphere and a 2D-inhomogeneous stochastic Earth's surface[2]. Statistical characteristics of the detectability conditions in this case could be determined using methods of statistical estimating of the radiation fields in the Earth's atmosphere.[2] In this paper we only consider the case of the volume objects vision within the simplest deterministic models of the atmosphere and the Earth's surface.

Consider, by the way of example, the case of a spherical diffusely reflecting object. The equation of the object surface in this case has the form $x^2 + y^2 + z^2 - R^2 = 0$, where R is the radius of the revelant sphere, and the equation of the normal is $\mathbf{N} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})/R$, where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors along the x, y, and z axes of the Cartesian system of coordinates. We denote the polar and the azimuthal angles of the vectors of irradiation Ω'', observation Ω, and normal $\mathbf{N}$, as (θ'', φ''), (θ,φ), and (θ_n, φ_n), respectively. Then, taking into account only reflected solar radiation we have

$$Q^* = R^2\int_0^\pi \sin\theta_n\,d\theta_n\int_0^{2\pi} d\varphi_n\,B_p(\theta_n;\varphi_n)\,|\,\mathbf{N}_\sigma\cdot\Omega\,|\times$$

$$\times H(\Omega\cdot\mathbf{N}_\sigma) - \pi\, I_{a\,p}(\Omega)\,R^2, \tag{20}$$

where

$$B_p(\theta_n;\varphi_n) = \frac{b}{p}\int_0^\pi \sin\theta''\,d\theta''\int_0^{2\pi} d\varphi''\,I_{00}(\Omega'')\,|\,\Omega''\cdot\mathbf{N}_\sigma|\times$$

$$\times H(-\Omega''\mathbf{N}_\sigma); \tag{21}$$

$$\mathbf{N}_\sigma = \sin\theta_n \cos\varphi_n\, \mathbf{i} + \sin\theta_n \sin\varphi_n\, \mathbf{j} + \cos\theta_n\, \mathbf{k};$$

$$\mathbf{N}_\sigma\cdot\mathbf{\Omega} = \sin\theta \sin\theta_n \cos(\theta - \theta_n) + \cos\theta\cos\theta_n;$$

$$\mathbf{N}_\sigma\cdot\mathbf{\Omega}'' = \sin\theta' \sin\theta_n \cos(\varphi'' - \varphi_n) + \cos\theta''\cos\theta_n,$$

and β is the coefficient of brightness of the spherical surface.

The dependence of Q^* on the brightness coefficient of the sphere β is shown in Fig. 3 for several directions of sighting, typical cloudless atmospheric conditions, and for $R = 1$ m. Computations were conducted according to formulas (20) and (21) for an absolutely absorbing Earth's surface with its zero brightness coefficient. Atmospheric haze brightness was determined using the quasi-single-scattering small angle approximation, described in Ref. 2.

As seen from the data presented, the sphere will be seen differently depending on the observational conditions. At $\theta = 180°$ a black sphere on the black Earth's surface will naturally be invisible. The sphere will apparently seem lighter for larger β. At the same time, such a sphere at 6 km height in the atmosphere will be seen as a dark spot for $\beta = 0$. This is quite apparent physically, since in the latter case the dark sphere will be observed against the background of atmospheric haze, and at $\beta \approx 0.02$ it will become invisible. Depending on the value of β, conditions for observations along a horizontal direction will also vary quite significantly. For example, when observing the shadowed side of the sphere in the direction 1 (see the insert in Fig. 3), the sphere will seem dark, and when observing its illuminated side in the direction 3 it will seem dark for $\beta < 0.37$, and light for $\beta > 0.37$. At $\beta = 0.37$ the sphere will become invisible.

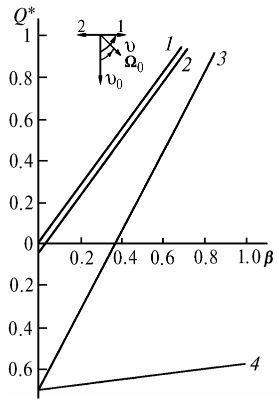

FIG. 3. Equivalent brightness of the sphere vs. its surface reflection coefficient for $\theta_0 = 60°$, $\theta = 180°$, $h = 0$ (*1*), $\theta = 180°$, $h = 6$ km (*2*), $\theta = 90°$, $h = 0$ (*3*).

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