ON THE GENERATION OF SOUND IN AIR BY MODULATED 10.6 μм LASER RADIATION

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Simple formulas for the dependence of the amplitude of the pressure of sound, generated when laser radiation is absorbed in air, on the frequency of modulation of the laser beam, on the spatial spectrum of the intensity distribution in the beam, as well as on the parameters of kinetic cooling of air are derived.

The experimental investigation of the generation of monochromatic sound accompanying the absorption of modulated CO_2-laser radiation in air[1] showed that this sound can be reliably detected against the background of acoustic noise. Since the amplitude and the phase of the sound waves carry information about both the intensity distribution in the laser beam and the process of absorption of laser radiation by the atmospheric gases and aerosol, acoustic measurements can be employed to obtain this information.

In this paper we derive simple relations for the dependences of the amplitude and phase of the sound wave on the frequency of modulation of the laser radiation, on the spatial spectrum of the intensity distribution in the beam, and on the parameters determining the absorption kinetics. These relations could be useful both for calculating the intensity of the sound excited by laser beams with an arbitrary intensity distribution across the beams and for solving inverse problems of reconstructing the intensity distribution in the beams or the absorption parameters.

The change In the pressure P, the density ρ, the enthalpy h, the Internal energy E_v, and the velocity of the air $\vec{V}$ accompanying absorption of radiation with power density I by the air are described by the following equations:

$$\partial\rho/\partial t + \rho\, \mathrm{div}\vec{V} + \vec{V}\,\nabla\rho = 0; \tag{1}$$

$$\partial\vec{V}/\partial t + (\vec{V}\nabla)\vec{V} = -\nabla P/\rho; \tag{2}$$

$$\rho(\partial h/\partial t + \vec{V}\,\nabla h) - (\partial P/\partial t + \vec{V}\,\nabla P) = \alpha_t I; \tag{3}$$

$$\rho\left(\partial E_v/\partial t + \vec{V}\,\nabla E_v\right) + \rho E_v/\tau = 2.44\alpha_{CO_2} I; \tag{4}$$

$$h = \frac{\gamma}{\gamma - 1}\,\frac{P}{\rho} + E_v. \tag{5}$$

In these equations α_t is the total coefficient of absorption of laser radiation by water vapor, carbon dioxide, and other absorbing impurities in the atmosphere; α_{CO_2} is the absorption coefficient, of carbon dioxide gas (this coefficient is separated out because the energy absorbed by carbon dioxide is not transformed immediately into heat, but rather at first it resonantly excites the vibrations of nitrogen molecules at a frequency 2.44 times higher than the transition frequency of the CO_2 molecule corresponding to the wavelength 10.6 μm); τ is the thermalization time (the lifetime of nitrogen molecules in the excited state); and, γ is the ratio of the heat capacities C_p/C_v.

Linearizing Eqs. (1)–(5) with respect to the initial values of the quantities P_0, ρ_0, and h_0, setting $V_0 = 0$ and $E_{v,0} = 0$, we obtain the following equations for the change p produced in the pressure by the cylindrical laser beam:

$$\partial^2 p/\partial t^2 - u^2\Delta_{\perp} p = \alpha_t(\gamma - 1)(\partial I/\partial t - \partial^2 E/\partial t^2), \tag{6}$$

$$\partial E/\partial t + (1/\tau)E = CI, \tag{7}$$

where

$$u^2 = \gamma P_0/\rho_0, \quad E = \rho_0 E_v, \quad C = 2.44\alpha_{CO_2}/\alpha_t.$$

The solution of Eqs. (6)–(7) in a polar coordinate system (r, φ) centered at the point M at which the sound pressure is measured in the case of harmonic modulation of the intensity of the laser radiation $I(r, \varphi, t) = I_0 f(r, \varphi)\exp(i\omega t)$ will be

$$p(M, t) = \left[\alpha_t(\gamma - 1)/4u^2\right]\omega g(\omega\tau)\, e^{i\omega t} \times \int_{-\pi}^{\pi} d\varphi \int_0^{\infty} I_0 f(r, \varphi) H_0^{(2)}(\omega r/u)\, r dr, \tag{8}$$

where $g(\omega\tau) = [1 + i(1 - C)\omega\tau)/(1 + i\omega\tau)$ and $H_0^{(2)}$ is a Hankel function.

If the intensity in the beam is different from zero in a region whose dimensions are much smaller than the distance r_0 from the point of observation to the center of gravity of the beam and the modulation frequency ω is such that $\omega r_0 \gg u$, then the function $H_0^{(2)}$ can be replaced by its asymptotic approximation

$$H_0^{(2)}(\omega r/u) \approx \sqrt{2u/(\pi\omega r)}\ \exp(-\iota\omega r/u - \iota\pi/4). \quad (9)$$

$\sqrt{r}$ can be replaced by $\sqrt{r_0}$, and the lower limit of integration over r in Eq. (8) can be replaced by $-\infty$.

$$\int_{-\pi}^{\pi} d\varphi \int_0^{\infty} f(r, \varphi)\, H_0^{(2)}(\omega r/u)\, r dr \approx$$

$$\approx \sqrt{2r_0 u/\pi\omega} \int_{-\infty}^{\infty} f_1(r)\, \exp(-\iota\omega r/u - \iota\pi/4)\, dr, \quad (10)$$

where $f_1(r) = \int_{-\pi}^{\pi} f(r, \varphi) d\varphi$.

In a Cartesian coordinate system x, y, for large distances from the beam to the observation point, i.e., when $r_0 \gg a$, where a is an effective radius of the beam

$$\int_{-\pi}^{\pi} d\varphi \int_{-\infty}^{\infty} f(r, \varphi) e^{-\iota\omega r/u} dr \approx$$

$$\approx \frac{1}{r} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} f(x, y)\ e^{-\iota\omega x/u} dx.$$

Thus the approximation formula for calculating the pressure has the following form

$$p(M, t) \approx \frac{\alpha_t(\gamma - 1)}{4u^2}\, g(\omega\tau) \sqrt{\frac{2u\omega}{\pi r_0}} \times$$

$$\times e^{\iota(\omega t - \pi/4)} (2\pi)^2\, \Phi_I(\kappa_x = \omega/u,\ \kappa_y = 0), \quad (11)$$

where

$$\Phi_I(\kappa_x, \kappa_y) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} I(x, y)\, \exp\left(-\iota\kappa_x x - \iota\kappa_y y\right) dxdy$$

is the two-dimensional spatial spectrum of the intensity distribution across the laser beam.

To estimate the error in the formula (11), we calculated the sound pressure from beams with a uniform distribution and a Gaussian distribution of the intensity across the beam. For the uniform distribution,

$$I(x,y) = \begin{cases} I_0 & \text{within the circle } (x - r_0)^2 + y^2 = a^2 \\ 0 & \text{outside the circle,} \end{cases}$$

$$\int_{-\infty}^{\infty} I(x, y)\, dy = 2I_0\sqrt{a^2 - (x - r_0)^2}, \quad |x - r_0| < a,$$

$$\Phi(\kappa_x, 0) = I_0 \frac{a^2}{2\pi} \frac{J_1(a\kappa_x)}{a\kappa_x} \exp(-\iota\kappa r),$$

where J_1 is a Bessel function.

$$p(M, t) \approx \alpha_t(\gamma - 1) I_0 \frac{a}{u}\, g(\omega\tau) \sqrt{\frac{\pi a}{2r_0} \frac{a\omega}{u}} \times$$

$$\times \frac{J_1(a\omega/u)}{(a\omega/u)} \exp\left[\iota\left(\omega t - \omega r_0/u - \pi/4\right)\right]. \quad (12)$$

The exact solution of Eqs. (6) and (7), which is not difficult to find by Fourier transforming with respect to the transverse coordinates, will be

$$p(M, t) = \alpha_t(\gamma - 1) I_0 \frac{a}{u}\, g(\omega\tau) \frac{\pi\omega a}{2u} \times$$

$$\times \frac{J_1(a\omega/u)}{(a\omega/u)} H_0^{(2)}\left(r_0\omega/u\right) \exp(\iota\omega t). \quad (13)$$

for $r_0 > a$.

The accuracy of the formula (12) is therefore determined only by the accuracy of the asymptotic representation (9) of the Hankel function.

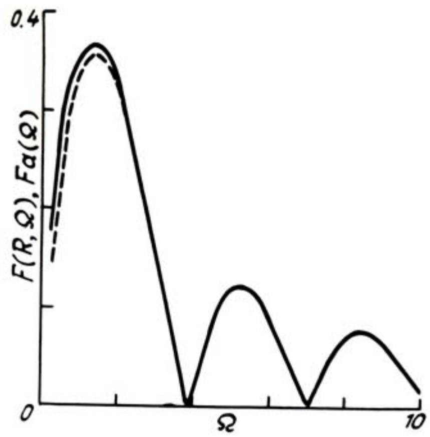

FIG. 1. The amplitude of the sound pressure versus the dimensionless frequency of modulation of laser radiation $\Omega = \omega a/u$ for a uniform distribution of the intensity across the laser beam. The solid line was calculated using the approximate formula (12) and the dashed line was calculated using the exact formula (13). The dimensionless distance $R = r_0/a = 1$.

Figures 1 and 2 show the spectra of the pressure amplitude:

the exact spectrum $F(R, \Omega) = \sqrt{R}\,|J_1(\Omega)|\,|H_0(R\Omega)|$, where $R = r_0/a$, and $\Omega = \omega a/u$ and the approximate spectrum $F_a(\Omega) = \sqrt{2/\pi\Omega}\,|J_1(\Omega)|$.

The relative error of the approximate formula in the region of the first maximum ($\Omega \approx 1.4$) is less than 2% for $R = 1$ and less than 0.5% for $R \geq 3$. At low frequencies $\Omega \leq 0.5$ the relative error is higher.

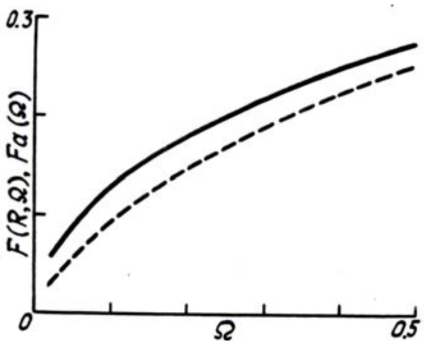

FIG. 2. Same as Fig. 1 but for $\Omega < 0.5$.

In the case of a beam with a Gaussian Intensity distribution

$$I(x, y) = I_0 \exp\left[-y^2/a^2 - (x - r_0)^2/a^2\right]$$

the approximate formula for the pressure has the form

$$p(M, t) \approx \alpha_t(\gamma - 1)\, I_0 \frac{a}{2u} g(\omega\tau) \sqrt{\frac{\pi a}{2r_0}\frac{\omega a}{u}} \times$$
$$\times \exp\left[-\frac{\omega^2 a^2}{4u^2} + i\left[\omega t - \frac{\omega r_0}{u} - \frac{\pi}{4}\right]\right]. \qquad (14)$$

The accuracy of this formula was checked by integrating numerically the expression

$$p(M, t) = \alpha_t(\gamma - 1) I_0 a^2 g(\omega\tau)\; i\omega \times$$
$$\times \int_0^\infty \frac{J_0(\kappa r_0)\kappa\, d\kappa}{2(\kappa^2 u^2 - \omega^2)} \exp\left[-\frac{\kappa^2 a^2}{4} + i\omega t\right], \qquad (15)$$

which is the solution obtained for Eqs. (6) and (7) by Fourier transforming with respect to the transverse coordinates. The accuracy of the approximate formula (14) was approximately the same as in the case of a beam with a uniform distribution of the intensity.

We note the formula (14) gives a frequency dependence of the pressure amplitude that differs from the dependence presented in Fig. 2 of Ref. 1, which was also calculated for a Gaussian beam, by the fact that. In particular, according to Ref. 1, the frequency ω_0 at which the amplitude is maximum is equal to $\omega_0 = 2\pi f_0 = 2.8 \cdot u/a$. According to the formula (14), however, $\omega_0 \approx u/a$. The calculations performed in Ref. 1 contain some errors. In Ref. 1 the frequency dependence of the pressure for $r_0 = 0$ is obtained from Eq. (15), if the imaginary part is neglected in calculating the integral. In addition, it was incorrectly assumed that the frequency dependence $p(r_0, \omega)$ is proportional to $p(0, \omega)$. Insofar as the dependence $p(r_0, \omega)$ observed in the experiment of Ref. 1 agrees satisfactorily with the incorrectly computed dependence, a possible explanation of this fact could be that the intensity distribution in the region of the path where the sound was measured, was not Gaussian. For the geometry of the experiment of Ref. 1, this could have been caused, for example, by the reflection of the laser beam from the mirror.

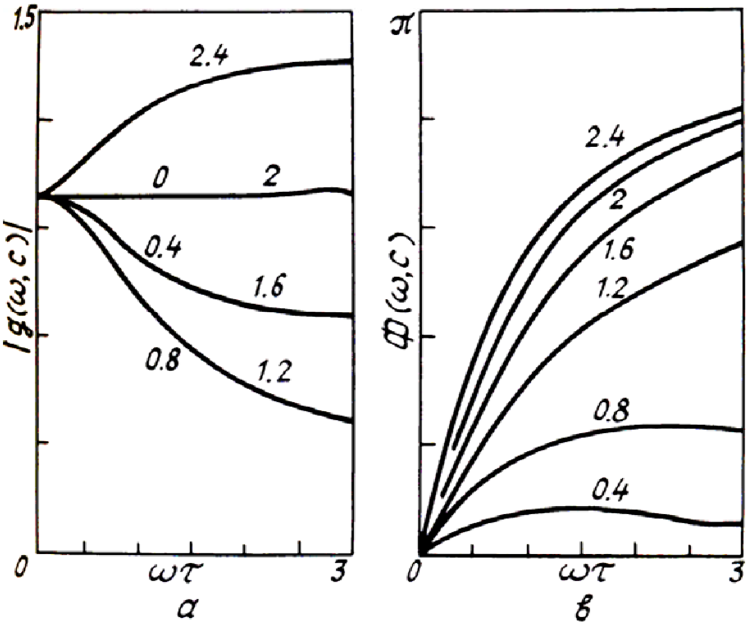

FIG. 3. The modulus (a) and argument (b) of the function $g(\omega\tau)$ for the values of the parameter C indicated on the curves.

Kinetic cooling could also have an appreciable effect on the frequency dependence. The effect of kinetic cooling on the shape of the sound pulse excited in the air was discussed earlier in Ref. 3. The dependences of the amplitude and phase of the function $g(\omega\tau)$, which describes the effect of kinetic cooling, are presented in Figs. 3*a* and *b* for different values of the parameter C (C can vary from 0 to 2.44).

For $C < 2$ kinetic cooling causes the amplitude of the sound pressure to decrease and the maximum of $p(\omega)$ to shift into the region of lower frequencies than in the case of the position of the maximum when there is no cooling effect. For $C = 1$ the amplitude of the sound pressure approaches zero as $\omega\tau - \infty$. In this case the heating owing to absorption of 10.6 μm radiation by water vapor and aerosol is compensated by cooling owing to absorption by carbon dioxide gas. For $C > 2$ the function $|g(\omega)|$ increases monotonically, so that the maximum of $p(\omega)$ can shift into the region of high frequencies.

If acoustic measurements are used to determine the parameters C and τ, then for this purpose it is best to use phase measurements, since the phase shift is not affected by the form of the intensity distribution across the laser beam.

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