LIDAR DETERMINATION OF ORIENTED PARTICLES IN CRYSTAL CLOUDS

B.V. Kaul’

Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk
Received December 19, 1994

Two possible ways of determining the preferred orientation of ice crystal cloud particles, without resorting to direct measurements of total backscattering phase matrix, are theoretically justified. One way is to rotate the whole lidar, and another is to rotate the plane of linear polarization of laser transmitter of the lidar.

Recent lidar experimental studies of the backscattering phase matrix (BPM)[1,2] have shown that particles of crystal clouds generally have preferred orientation of their axes in a certain direction in the horizontal plane. This is manifested through the presence of nonzero off–diagonal elements in the BPM. For model of an ensemble of crystals in the form of axisymmetric plates and columns, it is possible to find the direction of preferred orientation and its degree from measurements of the BPM elements.[2,3]

Preferable orientation profoundly influences the angular distribution of scattered radiation and must be considered in radiative transfer problems. Because of this, it is desirable to have procedure simpler than BPM measurements to identify clouds consisting of oriented crystal particles. Below we describe two such procedures of measurements by a lidar with linearly polarized laser radiation, which records two orthogonal components of the scattered radiation intensity, i.e., the first two Stokes parameters. The former employs the lidar rotatable about the sounding direction, while the latter requires that the lidar be supplemented with a $\lambda/2$ phase plate located on the optical axis of a laser transmitter.

For the ease of presentation, we reproduce basic formulae of polarization laser sensing of the atmosphere.

The equation of laser sensing, in the single scattering approximation generalized for the Stokes vector, is of the form[4]

$$I(h)\,\mathbf{s}(h) = (1/2)\,c\,\Delta t\,\kappa\,P_0\,T^2(h)\,\hat{\beta}(h)\,\mathbf{s}_0\,, \tag{1}$$

where $I(h)$ is the intensity of scattered light coming from volume scattering element of extent $c\Delta t$ at the altitude h; c is the velocity of light; Δt is the laser pulse duration; κ is the efficiency of lidar optical elements; P_0 is the power of laser radiation pulse; $T^2(h)$ is the atmospheric transmission on the path from the lidar to the volume scattering element and backward; $\hat{\beta}(h)$ is the backscattering phase matrix of the volume element; $\mathbf{s}(h)$ and $\mathbf{s}_0$ are the Stokes vectors of scattered radiation and laser radiation, respectively, normalized by their intensities.

The normalized Stokes vectors $\mathbf{s}(h)$ and $\mathbf{s}_0$ are the column vectors of the form

$$\mathbf{s}(h) = \begin{pmatrix} 1 \\ q(h) \\ u(h) \\ v(h) \end{pmatrix}, \qquad \mathbf{s}_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \tag{2}$$

where the $\mathbf{s}_0$ form is specialized for the case of lidar with linearly polarized laser radiation and with the receiver whose polarization basis is chosen to have one axis (x–axis) parallel to the oscillation plane of electric field of laser radiation.

The polarization basis of the lidar receiver is normally formed by prisms splitting the incident ray into two rays with orthogonal linear polarization states. In the case in which a pair of prism sides is parallel to the plane of linear polarization of laser radiation, the Stokes vectors of laser beams exiting through the prism are given by the matrix equations

$$\mathbf{s}_{\parallel} = L(0)\,\mathbf{s},$$

$$\mathbf{s}_{\perp} = L(\pi/2)\,\mathbf{s}, \tag{3}$$

where $\mathbf{s}$ is the column vector of the form given by Eq. (2), $L(\theta)$ are the matrix operators

$$L(0) = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\; L\left(\frac{\pi}{2}\right) = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4}$$

Detectors placed across the two beams record only their intensities; so, of the vector equations (3) of interest are only the scalar products of the form

$$I_{\parallel} = \frac{1}{2}(1\ 1\ 0\ 0)\begin{pmatrix} 1 \\ q \\ u \\ v \end{pmatrix} = \frac{1}{2}(1 + q),$$

$$I_{\perp} = \frac{1}{2}(1\ -1\ 0\ 0)\begin{pmatrix} 1 \\ q \\ u \\ v \end{pmatrix} = \frac{1}{2}(1 - q). \tag{5}$$

0235-6880/95/06 439-02 $02.00

We denote the row vectors by $\mathbf{D}_{\|} = (1/2)(1\ 1\ 0\ 0)$ and $\mathbf{D}_{\perp} = (1/2)(1\ {-1}\ 0\ 0)$. By taking the scalar product by multiplying both sides of Eq. (1) first by $\mathbf{D}_{\|}$ and then by $\mathbf{D}_{\perp}$, we obtain two equations for the intensities, namely,

$$I_{\|}(h) = (1/2)\, I(h)\, [1 + q(h)] = A\, T^2(h)\, \mathbf{D}_{\|}\, \hat{\beta}(h)\, \mathbf{s}_0 , \quad (6)$$

$$I_{\perp}(h) = (1/2)\, I(h)\, [1 - q(h)] = A\, T^2(h)\, \mathbf{D}_{\perp}\, \hat{\beta}(h)\, \mathbf{s}_0 , \quad (7)$$

where A is the instrumental constant.

By adding and subtracting Eqs. (6) and (7), we have

$$q = \frac{I_{\|} - I_{\perp}}{I_{\|} + I_{\perp}} = \frac{(\mathbf{D}_{\|} - \mathbf{D}_{\perp})\, \hat{\beta}\, \mathbf{s}_0}{(\mathbf{D}_{\|} + \mathbf{D}_{\perp})\, \hat{\beta}\, \mathbf{s}_0} . \quad (8)$$

Here $\mathbf{D}_{\|}$ and $\mathbf{D}_{\perp}$ are added and subtracted by the rules of vector summation. For simplicity, the h dependence is not indicated, but is implied.

We note that the normalized Stokes parameter measurements eliminate the problem of the variability of the transmission $T(h)$.

The subsequent analysis is based on the form of the backscattering phase matrix for axisymmetric polydispersions[3]

$$\hat{\beta} = \begin{pmatrix} a & k_1\, b\cos2\alpha & -k_1\, b\sin2\alpha & 0 \\ k_1\, b\cos2\alpha & \frac{a-c}{2} + k_2\frac{a+c}{2}\cos4\alpha & -k_2\frac{a+c}{2}\sin4\alpha & k_1\, d\sin2\alpha \\ k_1\, b\sin2\alpha & k_2\frac{a+c}{2}\sin4\alpha & \frac{c-a}{2} + k_2\frac{a+c}{2}\cos4\alpha & -k_1\, d\cos2\alpha \\ 0 & k_1\, d\sin2\alpha & k_1\, d\cos2\alpha & c \end{pmatrix}, \quad (9)$$

where a, b, c, and d have the meaning of the quadratic forms averaged over an ensemble of polydispersions and formed by the elements of the scattering amplitude matrix. The coefficients k_1 and k_2 vary between 0 and 1 and characterize the degree of orientation of particle axes in the direction lying in the plane perpendicular to the direction of radiation propagation.

The angle α is counted off from a plane of reference chosen *a priori* and containing the direction of the wave vector of laser radiation and the x axis of the receiver polarization basis (the direction $\|$, in our case).

Let us consider the first of the above possibilities of determining the particle orientation in ice crystal clouds. Upon rotating the lidar through the angle φ counterclockwise about the sounding direction, when looking in the direction of radiation, the BPM given by Eq. (9) is transformed by the law

$$\hat{\beta}(\varphi) = R(\varphi)\, \hat{\beta}(0)\, R(\varphi), \quad (10)$$

where the matrix operator of rotation $R(\varphi)$ has the form

$$R(\varphi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos2\varphi & \sin2\varphi & 0 \\ 0 & -\sin2\varphi & \cos2\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \quad (11)$$

Applying transformation (10) to Eq. (8), we derive

$$q(\varphi) = \frac{I_{\|}(\varphi) - I_{\perp}(\varphi)}{I_{\|}(\varphi) + I_{\perp}(\varphi)} = \frac{(\mathbf{D}_{\|} - \mathbf{D}_{\perp})\, R(\varphi)\, \hat{\beta}(0)\, R(\varphi)\, \mathbf{s}_0}{(\mathbf{D}_{\|} + \mathbf{D}_{\perp})\, R(\varphi)\, \hat{\beta}(0)\, R(\varphi)\, \mathbf{s}_0} . \quad (12)$$

Substitution of matrix (9) as well as the proper operations on the right side of Eq. (12) give

$$q(\varphi) = \frac{\frac{a-c}{2\,a} + k_1\frac{b}{a}\cos2(\varphi-\alpha) + k_2\frac{a+c}{2}\cos4(\varphi-\alpha)}{1 + k_1\frac{b}{a}\cos2(\varphi-\alpha)} . \quad (13)$$

For randomly oriented particles, $k_1 = k_2 = 0$ and the second Stokes parameter is independent of the lidar position. For preferred particle orientation, $k_1 = k_2 = 0$.

The right–hand side of the equation contains the modulation terms. The normalized Stokes parameter appears to be modulated with a period of π. The phase difference between the measured function $q(\varphi)$ and a certain function (13) calculated at $\alpha = 0$ can be found to give the direction of preferred orientation.

Technically, more feasible is the idea of rotating one of the polarization elements rather than the whole lidar. Such an element may be a half–wave plate inserted into the transmitted laser beam so the polarization is transformed in accordance with the equation

$$\mathbf{s}_0(\varphi) = K(\varphi)\, \mathbf{s}_0, \quad (14)$$

where $K(\varphi)$ is the operator of the $\lambda/2$ phase plate whose rapid axis is turned about the direction of radiation propagation through the angle φ counted off from the x axis of polarization basis of the lidar.

For the above–determined correspondence between the direction of linear polarization of laser radiation and the polarization basis of a receiving antenna, formula (14) is explicitly written as

$$\mathbf{s}_0(\varphi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos4\varphi & \sin4\varphi & 0 \\ 0 & \sin4\varphi & -\cos4\varphi & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ \cos4\varphi \\ \sin4\varphi \\ 0 \end{pmatrix} . \quad (15)$$

Substitution of matrix (9) as well as $\mathbf{s}_0(\varphi)$ for $\mathbf{s}_0$ into Eq. (8) yields

$$\frac{I_{\|} - I_{\perp}}{I_{\|} + I_{\perp}} = \frac{k_1\frac{b}{a}\cos2\alpha + \frac{a-c}{2\,a}\cos4\varphi + k_2\frac{a+c}{2\,a}\cos4(\varphi+\alpha)}{1 + k_1\frac{b}{a}\cos(4\,\varphi + 2\,\alpha)} . \quad (16)$$

In the case of randomly oriented particles, $k_1 = k_2 = 0$, and Eq. (16) contains a single nonzero term modulated with a period of $\pi/2$ and having zero mean. For water droplet clouds, $a = -c$ and the normalized Stokes parameter q varies between -1 and 1.

In the case of preferred particle orientation, a constant component appears in Eq. (16). It is positive when $|\alpha| < \pi/4$ and negative when $|\alpha|$ is within the limits from

$\pi/4$ to $\pi/2$. Also appearing are the modulation terms that produce the phase shift relative to $\cos 4\varphi$. This shift can be used to determine the direction of preferred orientation.

The above–described schemes of determining the preferred particle orientation can be realized with the help of simpler apparatus and for shorter period than total backscattering phase matrix measurements.

ACKNOWLEDGMENT

The work was supported in part by Russian Foundation of Fundamental Researches Grant No. 93–05–9376.

REFERENCES

1. B.V. Kaul', A.L. Kuznetsov, E.R. Polovtseva, and I.V. Samokhvalov, Atmos. Oceanic Opt. **6**, No. 4, 257–261 (1993).
2. B.V. Kaul', A.L. Kuznetsov, and I.V. Samokhvalov, Atmos. Oceanic Opt. **7**, No. 1, 4–7 (1994).
3. D.N. Romashov and R.F. Rakhimov, Atmos. Oceanic Opt. **6**, No. 8, 515–518 (1993).
4. V.E. Zuev, B.V. Kaul', I.V. Samokhvalov, et al., *Laser Sensing of Industrial Aerosol* (Nauka, Novosibirsk, 1984), 185 pp.