ESTIMATING THE COORDINATES OF A PULSED ISOTROPIC RADIATION SOURCE BY THE DIFFERENCE RANGE FINDING METHOD FROM THE DATA OF OBSERVATION THROUGH CLOUDS

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A model of the path of signal propagation through a cloudy layer of large optical thickness and a procedure for determining the coordinates of a pulsed isotropic radiation source from the data of observations through clouds obtained with a satellite system have been considered. An algorithm and a computer program have been given.

1. The method for determining an object coordinates from the measured delays of signal propagation from the object to some spaced reference points is well known in radar detection and ranging. This is the difference range finding method realized by means of a satellite system.[1] In this method, signals are transmitted from onboard spacecrafts and are recorded by a detector placed on the object. After choice of the constellation optimal for the formulated problem, the coordinates of the detector are calculated based on the time lags of signal arrival and the known satellite coordinates. A revised version of this method was proposed in Ref. 2 for estimating the coordinates of a pulsed isotropic source of optical radiation from the data obtained by means of few spacecrafts. Different spacecrafts record weakly disturbed, delayed, and possibly scaled up or down copies of the same signal, which usually has complex shape not known in advance. The initial data for calculating the source coordinates are the time lags of the copies.

This is the problem of statistical estimating. It is formulated with the help of a system of equations:

$$f_{i,j} \equiv f(x, y, z; x_i, y_i, z_i; x_j, y_j, z_j; p) = \Delta_{i,j} + \varepsilon_{i,j}, \quad i \neq j; \quad i, j = 1, 2, \dots, N, \tag{1}$$

where $f_{i,j}$ is the known function of the source coordinates (x, y, z) to be estimated and of the given coordinates of two spacecrafts (x_i, y_i, z_i) and (x_j, y_j, z_j), $i \neq j$. When a signal propagates in the free space, the function $f_{i,j}$ is represented by the expression

$$f_{i,j} = r_i - r_j, \quad r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}. \tag{2}$$

In these equations, r_i is the distance from the source to the ith spacecraft; $\Delta_{i,j} = c(t_i - t_j)$ is the time lag between the instants of signal arrival to the ith and jth spacecrafts determined from the measurement results (time delay of copies), multiplied by the light speed c; $\varepsilon_{i,j}$ is the error in estimating $\Delta_{i,j}$, considered as a random variable (usually it is the white Gaussian noise with zero mean); p are the additional parameters characterizing the conditions of the experiment. The error $\varepsilon_{i,j}$ also includes the errors connected with small disturbances in the signal shape during signal propagation along the path. However, one may consider these disturbances as small and disregard them in the explicit form only for a cloudless atmosphere. One should take them into account for the conditions of strong turbidity by adding the correction terms to Eq. (1). The form of these terms is determined below for the simplest case of a homogeneous cloudy layer of large optical thickness ($\tau \geq 20$), upon exiting of which radiation has only diffuse component.[3]

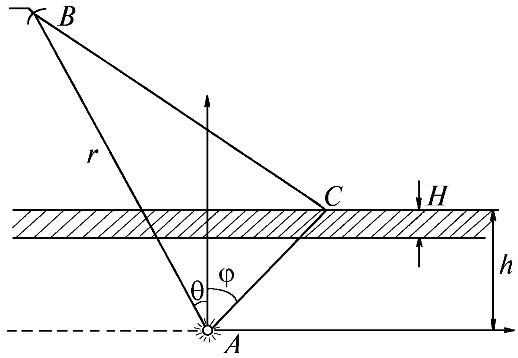

FIG. 1. Optical–geometric scheme of the path of signal propagation through a cloudy layer of large optical thickness.

Two effects are connected with the cloudy layer for the model under consideration: (a) radiation scattering that allows the rays deviating from the direct propagation path AB (Fig. 1) to enter the aperture of a receiving system of a spacecraft and (b) decrease of the effective velocity of signal propagation on the section of the path passing through the cloudy layer. The final result of the effect of the cloudy layer is the increase of the effective distance between a radiation source and a spacecraft by the value Δr depending on the layer thickness H and the angle θ at which the source is observed by instrument from onboard the spacecraft. Then expressions (2) in Eq. (1) are replaced by

$$f_{i,j} = (r_i + \Delta r_i) - (r_j + \Delta r_j). \tag{3}$$

2. In order to calculate the correction terms Δr in Eq. (3), we take advantage of the pattern of propagation of the diffuse component of pulsed radiation Ψ_d in optically thick layer proposed in Ref. 4. This pattern is analytically described by the equation

$$\left[\nabla^2 - \frac{3}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{D}\frac{\partial}{\partial t} + \gamma_a \gamma_{tr}\right]\Psi_d(t) = 0 \tag{4}$$

and is illustrated by Fig. 2. In Eq. (4), $D = c / 3(\gamma_a + \gamma_{tr})$ is the diffusion coefficient; $\gamma_{tr} = \gamma_s(1 - g) + \gamma_a$ is the transport coefficient; γ_s and γ_a are the scattering and absorption coefficients, in m^{-1}; and g is the average cosine of the scattering angle. The value of g is close to unity for cloud droplets, the transport coefficient γ_{tr} is much less than the total extinction coefficient $\gamma_{\mathrm{t}} = \gamma_{\mathrm{s}} + \gamma_{\mathrm{a}}$, and the transport mean free path $l_{\mathrm{tr}} = \gamma_{\mathrm{tr}}^{-1}$ is much greater than the total mean free path $l_{\mathrm{tr}} = \gamma_{\mathrm{t}}^{-1}$. The total intensity of radiation upon exiting a layer of optical thickness $\tau = \int_0^s \gamma_{\mathrm{t}}\, \mathrm{d}s$ is equal to

$$\Psi_{\mathrm{t}} = \Psi_{\mathrm{c}} + \Psi_{\mathrm{d}} , \qquad \Psi_{\mathrm{c}} = F_0 \exp(-\tau) , \tag{5}$$

where F_0 ($\mathrm{W/m^2}$) is the intensity of the plane–parallel radiation flux incident on the layer. The coherent component of the intensity Ψ_{c} at large thicknesses of cloudy layer ($\tau \geq 15$) is small in comparison with the diffuse component Ψ_{d}: $\Psi_{\mathrm{c}} \ll \Psi_{\mathrm{d}}$ and $\Psi_{\mathrm{t}} \approx \Psi_{\mathrm{d}}$.

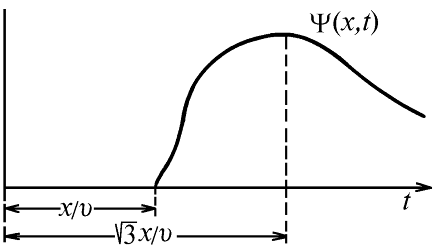

FIG. 2. Pulse shape upon exiting a layer of large optical thickness.

As is seen from Eq. (4) and Fig. 2, the wave front of a diffuse pulse propagates in the layer with the velocity $t = c/\sqrt{3}$. This means that the effective (assuming that $l/\upsilon = l_{\mathrm{eff}}/c$) length l_{eff} of any section of the wave path in the cloudy layer, not too close to its origin, exceeds the geometric length l of this section by the value

$$\Delta l = l_{\mathrm{eff}} - l = (\sqrt{3} - 1)\, l \approx 0.73\, l . \tag{6}$$

On the basis of assumption (6), we represent the effective length of the light ray path ACB for arbitrary angle φ by expression

$$L = r + h \left(\frac{1}{\cos(\varphi)} + \tan(\varphi)\sin(\theta) - \cos(\theta) \right) + \frac{k}{\cos(\varphi)}\, h , \quad k = 0.73\, H/h , \tag{7}$$

where r is the distance between the source A and the receiver B along the straight line AB. In the derivation of Eq. (7), we assumed r to be so long in comparison with h that the straight lines AB, CB, and OB can be assumed parallel, forming the same angle θ with the vertical direction.

Setting the derivative $\partial L/\partial\varphi$ equal to zero, we find the length $L_{\min} = r + \Delta r$ from Eq. (7), where

$$\Delta r = h \left(\sqrt{(1 + k)^2 - \sin^2(\theta)} - \cos(\theta) \right) , \tag{8}$$

and the corresponding angle $\varphi = \varphi_{\min}$ with

$$\sin(\varphi_{\min}) = -\sin(\theta) / (1 + k) . \tag{9}$$

The correction term Δr vanishes for $k = 0$. At $\theta = 0$ and $\theta = \pi/2$, it is equal to hk and $h\sqrt{2k + k^2}$, respectively.

The linearized problem of estimating the parameters[5] corresponding to Eqs. (1), (3), and (8) is written in the form

$$\nu_j = \mathbf{A}_j^{\mathrm{T}}\, \tilde{\Theta} + \xi_j , \qquad j = 2, 3, \ldots , N . \tag{10}$$

Here $\tilde{\Theta} = [\tilde{x}, \tilde{y}, \tilde{z}, \tilde{h}]^{\mathrm{T}}$ is the vector of deviation of the parameters x, y, z, and h from their rated (*a priori*) values x_0, y_0, z_0, and h_0:

$$\tilde{x} = x - x_0 , \quad \tilde{y} = y - y_0 , \quad \tilde{z} = z - z_0 , \quad \tilde{h} = h - h_0 ; \tag{11a}$$

$\nu_j = c(t_j - t_1) + f_j^0 = -c(t_j - t_1) + (r_{j0} - r_{10} - a_{j4} h_0)$ are the measurement data counted off from their rated value f_j^0, where

$$f_j^0 \equiv f_{j1}(x_0, y_0, z_0; x_j, y_j, z_j; x_1, y_1, z_1; h_0) = r_{j0} - r_{10} - a_{j4} h_0 ; \tag{11b}$$

$\mathrm{A}_j^{\mathrm{T}} = [a_{j1}, a_{j2}, a_{j3}, a_{j4}]$ is the row–vector with the elements

$$a_{j1} = \frac{x_j - x_0}{r_{j0}} - \frac{x_1 - x_0}{r_{10}} , \quad a_{j2} = \frac{y_j - y_0}{r_{j0}} - \frac{y_1 - y_0}{r_{10}} ,$$

$$a_{j3} = \frac{z_j - z_0}{r_{j0}} - \frac{z_1 - z_0}{r_{10}} ,$$

$$a_{j4} = -\left(\sqrt{(1 + k)^2 - \sin(\theta_{j0})} - \cos(\theta_{j0}) \right) + \left(\sqrt{(1 + k)^2 - \sin(\theta_{10})} - \cos(\theta_{10}) \right); \tag{11c}$$

θ_{j0} is the estimate of the angle θ_j between the vertical direction and the direction from the source to the ith spacecraft determined by the formula

$$\cos(\theta_{i0}) = \frac{x_0(x_i - x_0) + y_0(y_i - y_0) + z_0(z_i - z_0)}{r_0\, r_{i0}} ; \tag{11d}$$

r_{i0} is the estimate of the distance from the source to the ith spacecraft

$$r_{i0} = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} ; \tag{11e}$$

ρ_0 is the estimate of the distance from the origin of the coordinate system (the Earth's centre) to the source:

$$\rho_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} ; \tag{11f}$$

and, $\xi_j = \varepsilon_{j1}$. The choice of spacecraft number 1, relative to which the delays are determined, is arbitrary. In general, the delays can be determined between any pairs of spacecrafts. The parameter k in the above–presented equations is assumed to be known; in practice, it should be determined by the trial–and–error method.

3. The text of the program for determination of the source coordinates from Eqs. (10) and (11) written in PASCAL is given below. The first block of the program imitates a pulsed radiation source. The real source

coordinates x, y, z, the height of the cloudy layer upper boundary h, the time of the pulse arrival, and the limiting zenith angle of the source detection are taken as input parameters. The number and the coordinates of all the spacecrafts that have recorded the source are calculated for the instant of recording, as well as the instant of recording of the source by these spacecrafts, taking into account the decrease of the pulse velocity in the cloudy layer. If N, the number of satellites that have recorded the source, is greater than four, the second block of the program solves an inverse problem. The matrix $\mathbf{A}(4\times N)$ and the vector $\mathbf{V}(N)$ are then calculated. If $N > 5$, the left and right sides of Eq. (10) are multiplied by the matrix $\mathbf{A}^{\mathrm{T}}$ from the left. Then Eq. (10) is solved by inverting the matrix $(\mathbf{A}^{\mathrm{T}}\mathbf{A})$ (or the matrix $\mathbf{A}$ for $N = 5$). The so–determined values of the corrections to the radiation source coordinates and to the height of the upper boundary of the cloudy layer are used for calculating new **A** and **V**. Calculations were done for 2 880 variants of spacecraft constellations for each set of the pulsed radiation source coordinates (its latitude varied from 90° S to 90° N, and its height above the Earth's surface varied up to 100 km) and the height of the cloudy layer upper boundary (up to 10 km). The initial source coordinates were fixed on the subsatellite point of the first spacecraft which has recorded the source. The initial value of the upper cloudy layer boundary height was set equal to zero. The iteration process converged after the third step. The root–mean–square error in calculating x, y, z, and h was of the order of a few meters.

APPENDIX

The program for calculating the pulsed radiation source coordinates and the height of the cloudy layer upper boundary. The coordinates of the nth spacecraft are calculated in the subroutine $XYZ(n, t)$ for the time instant t. The number of spacecrafts is 24. Calculated results are put in the array orb. The function Det calculates the determinant of the matrix of the fourth order.

```
Label 1, 2, 3;
Const Rz: Extended = 6371000; (the Earth's radius, m)
Cs: Extended = 3.0e8; (light velocity in vacuum)
im = 15; (data sampling interval, min)
id = 30; (number of days)
ii = id*1440 Div im; (the number of satellite configurations under consideration)
Bet: Extended = 55; (geographical latitude of the source)
L: Extended = 38; (geographical longitude of the source)
Hz: Extended = 500; (source height above the Earth's surface, m)
H: Extended = 3000; (height of the cloudy layer upper boundary, m)
Upr: Extended = 75; (limiting angle of detecting the source)
NAm = 10; (maximum number of spacecrafts detecting the source)
dr: Extended = Pi / 180;
Nit = 4; (number of iterations)
KK = 0.35; (relative difference between the pulse velocities in a cloud and in vacuum)
KK1: Extended = KK*KK + 2*KK; Type Ar4 = Array [1 .. 4, 1 .. 4] of Extended;
Var nk, N, k, k1, i, i1, i2, j, j1, j2: Integer; orb: Array [1 .. 24, 2 .. 4] of Extended;
    tt, cb, au, aco, s, s1, mx, my, mz: Extended;
    X, E: Array [1 ..4] of Extended; A: Array [1 .. 4, 1 .. NAm] of Extended;
    cte, ti, Fk: Array [1 .. 24] of Extended; D, B: Ar4;
  Begin aco: = Cos(Upr*dr); cb: = cos(Bet*dr); j2: = 0;
 (Cartesian coordinates of the source) mx: = (Rz + Hz)*cb*Cos(L*dr);
my: = (Rz + Hz)*cb*Sin(L*dr); mz: = (Rz + Hz)*sin(Bet*dr);
For i: = 0 To ii Do Begin {ii} tt: = im*60*i; {cycle over the satellite configurations}
  For j1: = 1 To 24 Do Begin XYZ(j1, tt);
     (calculation of cosines of the zenith angles of the directions from the source to the spacecraft)
cte [j1] : = (mx*(orb[j1, 2] – mx) + my*(orb[j1, 3] – my) +
    mz*(orb[j1, 4] – mz)) / ((Rz + Hz)*Sqrt(Sqr(orb[j1, 2] – mx) +
     Sqr(orb[j1, 3] – my) + Sqr(orb[j1, 4] – mz))) End;
     (ordering the spacecrafts with respect to the zenith angle) For j1: = 1 to NAm Do Begin
  For k: = 23 down to n Do Begin k1: = k + 1; If cte[k] < cte[k1] then Begin
au: = cte[k1]; cte[k1]: = cte[k]; cte[k]: = au; au: = orb[k1, 2]; orb[k1, 2]: = orb[k, 2];
orb[k, 2]: = au; au: = orb[k1, 3]; orb[k1, 3]: = orb[k, 3]; orb[k, 3]: = au;
au: = orb[k1, 4]; orb[k1, 4]: = orb[k, 4]; orb[k, 4]: = au End End End;
      For j1: = 1 to NAm Do Begin
S: = Sqrt(Sqr(orb[j1, 2] – mx) + Sqr(orb[j1, 3] – my) + Sqr(orb[j1, 4] – mz));
    S1: = H / Cs*(Sqrt(KK*KK + 2*KK + cte[j1]*cte[j1]) – cte[j1]);
{calculation of detection time} ti[j1]: = tt + S / Cs + S1 End; {j1}
{if necessary, one can refine the spacecraft coordinates at the instant ti[n]}
{initial values of the source coordinates and the height H}
k: = 0; For j1: = 1 to 3 Do X[j1]: = orb[1, j1 + 1]*Rz / Sqrt(Sqr(orb[1, 2]) + Sqr(orb[1, 3]) + Sqr(orb[1, 4]));
X[4]: = 0; 1: N: = 0;
S: = Sqrt(Sqr(orb[1, 2] – X[1]) + Sqr(orb[1, 3] – X[2]) + Sqr(orb[1, 4] – X[3]));
cte[1]: =(X[1]*(orb[1, 2]–X[1])+X[2]*(orb[1, 3]–X[2])+X[3]*(orb[1, 4] – X[3])) / (Sqrt(Sqr(X[1]) + Sqr(X[2]) + Sqr(X3))*S;
For j1: = 2 to NAm do Begin
S1: = Sqrt(Sqr(orb[j1, 2] – X[1]) + Sqr(orb[j1, 3] – X[2]) + Sqr(orb[j1, 4] – X[3]));
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cte[n]: = (X[1]*(orb[j1, 2] – X[1]) + X[2]*(orb[j1, 3] – X[2]) + X[3]*(orb[j1, 4] – X[3]))/(Sqrt(Sqr(X[1])+ Sqr(X[2]) + Sqr(X3))*S1);
If cte[j1] > = aco Then Begin N: = N + 1;
A[1, N]: = (orb[j1, 2] – X[1]) / S1 – (orb[1, 2] – X[1]) / S;
A[2, N]: = (orb[j1, 3] – X[2]) / S1 – (orb[1, 3] – X[2]) / S;
A[3, N]: = (orb[j1, 4] – X[3]) / S1 – (orb[1, 4] – X[3]) / S;
A[4, N]: = Sqrt(KK1 + cte[1]*cte[1]) – cte[1] – Sqrt(KK1 + cte[n]*cte[n]) + cte[n];
Fk[N]: = S1 – S – X[4]*A[4, N] – Cs*(ti[j1]) End End;
If N < 4 Then GoTo 2; If N = 4 Then Begin For j: = 1 to 4 Do Begin E[j]: = Fk[j];
For i1: = 1 to 4 Do D[j, i1]: = A[j, i1] End; GoTo 3 End;
For j: = 1 to 4 Do Begin E[j]: = 0; For i2: = 1 to j1 Do E[j]: = E[j] + A[j, i2]*Fk[i2];
For i1: = 1 to 4 Do Begin D[j, i1]: = 0; For i2: = 1 to j1 Do
D[j, i1]: = D[j, i1] + A[j, i2]*A[i1, i2] End End;
3: S: = Det(D); For j: = 1 to 4 Do Begin For i1: = 1 to 4 Do For i2: = 1 to 4 Do If i1 = j Then B[i1, i2]: = E[i2] Else B[i1, i2]: = D[i1,i2];
S1: = Det(B);
X[j]: = X[j] + S1 / S End; If k = Nit Then Begin j2: = j2 + 1;
WriteLn(i:5', number of spacecrafts that detected the source', N + 1:2);
WriteLn('X = ', X[1], 'Y = ', X[2], 'Z = ', X[3], 'H = ', X[4]) End;
k: = k + 1; If k < = Nit Then GoTo 1; 2: End End.
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