PROPAGATION OF THE AMPLITUDE–MODULATED LASER BEAM THROUGH A GAS OF TWO–LEVEL ATOMS

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Propagation of the amplitude–modulated electromagnetic wave through a gas of resonant atoms is considered. It is shown that the nonlinear energy transfer from a strong field to weak fields takes place under certain conditions. Numerical calculations of the field intensities are performed.

The problem on the propagation of amplitude–modulated radiation through a medium of two–level quantum systems is usually discussed in the approximation of a strong field.[1–5,6] The authors of Ref. 6 claim that the "coupling reaction", i.e. , the diminishment of the intensity of a nonlinear field, whose energy is "transferred" with two-level systems to "side" components, cannot be neglected: moreover this is the coupling reaction which should classify the type of the problem. In our paper we develop a consistent semi classical electrostatics of this process, which uses the small parameters natural in this case.

Let us represent the electromagnetic field incident on the medium in the following form:

$$E_{\text{inc}} = E^{(0)}(1 + \mu\cos\Omega\, t)\cos\omega t\ , \qquad (1)$$

where $E^{(0)}$ is the amplitude of a linearly polarized field, μ and Ω are the depth and frequency of modulation, ω is the carrier frequency, and $\Omega \ll \omega$. The field in the medium can be represented in the form

$$E = E_0(z)\exp(-i\omega t) + E_1(z)\exp(+i\omega_1 t) + E_2(z)\exp(-i\omega_2 t) + \text{c. c.}\ , \qquad (2)$$

where $\omega_1 = \omega + \Omega$, $\omega_2 = \omega - \Omega$, and z is the coordinate counted off from the point of incidence of the beam in the direction of its propagation inside the medium.

Let us assume that the medium consists of two–level atoms and the transition frequency $\omega_{21} = \omega$.

Let the system of equations for a density matrix be written as follows:

$$i\hbar\left(\frac{\partial u}{\partial t} + \frac{u-1}{T_1}\right) + 2d(E_0(z)\exp(-i\omega t) + \text{c. c.}) \times (\rho_{21} + \text{c. c.}) = -2d(E_1(z)\exp(-i\omega_1 t) + E_2(z)\exp(-i\omega_2 t) + \text{c. c.})(\rho_{21} + \text{c. c.})\ ,$$

$$i\hbar\left(\frac{\partial r_{21}}{\partial t} + \rho_{21}\left(\frac{1}{T_2} + i\omega\right)\right) + d(E_0(z)\exp(-i\omega t) + \text{c. c.})\,u = -d(E_1(z)\exp(-i\omega_1 t) + E_2(z)\exp(-i\omega_2 t) + \text{c. c.})\,u\ , \qquad (3)$$

where $u = \rho_{11} - \rho_{22}$, d is the dipole moment of the transition, T_1 and T_2 are the longitudinal and transverse relaxation times. Here and below the matrix coefficients $\rho_{ij}(i, j = 1, 2)$ are considered to be dependent of z and t.

Solving the system of Eqs. (3), we may determine the polarization of the medium

$$P = \text{N}tr\rho d = N\,(d\rho_{21} + \text{c. c.})\ , \qquad (4)$$

where N is the density of atoms.

THE PERTURBATION THEORY

We shall consider now the regime of weak modulation when m $\ll$ 1, i.e., the fields $E_1(z)$ and $E_2(z)$ are assumed to be weak. Let us construct the perturbation theory for these fields using the system of Eqs. (3). In zero approximation, this system can be solved by equaling its right side to zero. It is a well-known approximation which describes the saturation effect

$$u^{(0)} = \left(1 + 4T_1T_2a^2(z)\right)^{-1},\ \rho_{21}^{(0)} = \chi^{(0)}(z)\,E_0(z)\exp(-i\omega t)\ ,$$

$$\chi^{(0)}(z) = i\,\frac{d\cdot T_2}{\hbar}\,u^0(z)\ ,\ a(z) \equiv \frac{|E_0(z)|\cdot d}{\hbar}\ . \qquad (5)$$

In the first approximation, we substitute $u^{(0)}$ and $\rho_{21}^{(0)}$ to the right side of system (3) and $u = u^{(0)} + u^{(1)}$, $\rho_{21} = \rho_{21}^{(0)} + \rho_{21}^{(1)}$ to the left side. The resulting system of equation is similar to Eq. (3) in which ρ_{21} and u replaces $\rho_{21}^{(0)}$ and $u^{(0)}$ in its right side and ρ_{21} and u replaces $\rho_{21}^{(1)}$ and $u^{(1)}$ in its left side. Then it becomes evident that $\rho_{21}^{(1)} \sim \exp(-i\omega_1 t)$ and $\exp(-i\omega_2 t)$ and $u^{(1)} \sim \exp(\pm i\Omega\, t)$.

Thus, we obtain in the first approximation

$$\rho_{21}^{(1)} = \chi_1(z)\,E_1(z)\exp(-i\omega_1 t) + \chi_2(z)\,E_2(z)\exp(-i\omega_2 t)\ , \qquad (6)$$

in addition, $\chi_1 = -\chi_2^*$.

$$\text{Re}\chi_1 = \frac{\Omega\cdot d}{\hbar\gamma}\,u^0\left[4a^2(z)\left(2 + \frac{T_2}{T_1}\right) - \Omega^2 - \frac{1}{T_1^2}\right],$$

$$\text{Im}\chi_1 = \frac{T_2\cdot d}{\hbar\gamma}\,u^0\left[\left(4a^2(z) + \frac{1}{T_1T_2}\right)\left(\frac{1}{T_1T_2} - 4a^2(z)\right) + \Omega^2\left(4a^2 + \frac{1}{T_2^2}\right)\right],$$

$$\gamma \equiv \left(4a^2(z) + \frac{1}{T_1T_2} - \Omega^2\right)^2 + \Omega^2\left(\frac{1}{T_1} + \frac{1}{T_2}\right)^2 . \qquad (7)$$

Substituting Eqs. (6) and (7) into Eq. (4), we can see that in the first approximation polarization arises at frequencies of weak fields. Since $a(z) \sim |E_0(z)|$, it is possible to note that in this approximation weak fields depend on the strong field $E_0(z)$.

The second order of the perturbation theory can be obtained in analogy with the first order of this theory: i.e., for u and ρ_{21} in the right side of Eqs. (3) we substitute $u^{(0)} + u^{(1)}$ and $\rho_{21}^{(0)} + \rho_{21}^{(1)}$ and in the left side we substitute $u^{(0)} + u^{(1)} + u^{(2)}$ and $\rho_{21}^{(0)} + \rho_{21}^{(1)} + \rho_{21}^{(2)}$. In this case $\rho_{21}^{(2)} \sim \exp(-i\omega t)$ and $\exp(-i(\omega\pm 2\Omega)t)$. It can easily be shown, that for the fields $E' \frac{\partial |E'|^2}{\partial z} \sim |E_1|^2\, |E'|$ at frequencies $(\omega\pm 2\Omega)$, and in the second order of the perturbation theory these fields can be neglected. Thus, in the second order we have

$$\rho_{21}^{(2)} = \chi^{(2)}(z)\, E_0(z) \exp(-i\omega t) ,$$

$$\chi^{(2)}(z) = -i\, \frac{8|E_1(z)|^2 \cdot d^3T_2}{\hbar^3\gamma}(u^{(0)})^2\left[\left(4a^2(z) + \frac{1}{T_1T_2}\right)\times \right.$$

$$\left. \times \left(T_1T_2(\Omega^2 - 4a^2(z)) + 3\right) + \frac{\Omega^2(T_2 - T_1)^2}{T_1T_2}\right], \qquad (8)$$

$$u^{(2)} = -\, 4T_1 \frac{d}{\hbar}\left[\, |E_0(z)|^2\, \mathrm{Im}\chi^{(2)} + 2|E_1(z)|^2\, \mathrm{Im}\chi_1(z)\, \right] . \qquad (9)$$

The second order of the perturbation theory reveals a coupling reaction of the waves; the expression for $\chi^{(2)}$ shows how variations of the fields at frequencies $\omega\pm\Omega$ affect the propagation of the strong field ($\chi^{(2)} \sim |E_1|^2$). Thus, eliminating a strong field in the zero approximation, it is possible to construct the perturbation theory which correctly describes the behavior of three waves (2) in a resonance medium.

The motion of atoms can be taken into account going over to a reference system affixed to the atom, and the problem can be reduced to the Doppler frequency shift: $\omega' = \omega - k\nu$, $\omega_1' \approx \omega_1 - k\nu$, and $\omega_2' \approx \omega_2 - k\nu$, where ν is the projection of the atomic velocity onto the direction of wave propagation, k is the wave vector. Corresponding formulas, which take into account the Doppler effect, are given in Appendix.

EQUATION FOR FIELD INTENSITIES

Let us write an equation for the field

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2} , \qquad (10)$$

where the field E and the polarization P have, in our case, the form

$$E = |E_0(z)|\, \exp(i(\varphi_0 - \omega t)) + |E_1(z)|\, \exp(i(\varphi_1 - \omega_1 t)) +$$
$$+\, |E_2(z)|\, \exp(i(\varphi_2 - \omega_2 t)) + \text{c. c.}$$

$$P = Nd\Big[(\chi^{(0)}(z) + \chi^{(2)}(z))|E_0(z)|\, \exp(i(\varphi_0 - \omega t)) +$$
$$+\, \chi_1(z)\, |E_1(z)|\, \exp(i(\varphi_1 - \omega_1 t)) +$$
$$+\, \chi_2(z)\, |E_2(z)|\, \exp(i(\varphi_2 - \omega_2 t))\Big] + \text{c. c.} \qquad (11)$$

Here $\varphi_\alpha = n_\alpha \frac{\omega_\alpha}{c} z$, n_α is the index of refraction at the frequency ω_α, and $\alpha = 0, 1, 2$. By substituting Eq. (11) into Eq. (10), we derive the following system of equations:

$$1 - n_\alpha^2 = -\, 4\pi Nd \cdot \mathrm{Re}\chi_\alpha , \qquad (12)$$

$$\frac{\partial |E_\alpha|}{\partial z} = -\frac{2\pi\omega}{c} Nd\, |E_\alpha(z)|\, \mathrm{Im}\chi_\alpha . \qquad (13)$$

Here $\chi_0 \equiv \chi^{(0)} + \chi^{(2)}$. From Eq. (12) it follows that $2\varphi_0 = \varphi_1 + \varphi_2$. From Eq. (13) as well as from the fact that $\chi_1 = -\chi_2^*$ it follows that the boundary amplitudes are identical $|E_1| = |E_2|$. Let us rewrite Eq. (13) for the intensities

$$\frac{\partial I_0}{\partial z} = -\frac{4\pi\omega}{c} Nd\, I_0(z)\left[\mathrm{Im}\chi^{(0)} + I_1(z)\eta\right] ,$$

$$\frac{\partial I_1}{\partial z} = -\frac{4\pi\omega}{c} Nd\, I_1(z)\, \mathrm{Im}\chi_1(z) , \qquad (14)$$

$$I_\alpha(z) \equiv |E_\alpha(z)|^2 ,\ \ \eta(z) = \mathrm{Im}\chi^{(2)}/I_1(z) . \qquad (15)$$

It should be noted that χ_1, $\chi^{(0)}$, and η depend on $I_0(z)$, since $a \equiv \frac{|E_0(z)|\ d}{\hbar}$. It is clear from Eq. (14) that the behavior of weak fields is determined by $\mathrm{Im}\chi_1$ which is a function of the strong ield amplitude in the medium, and when $\mathrm{Im}\chi_1 < 0$ weak fields become stronger. Using Eq. (7), we may write this condition in the form

$$4a^2(z) > \frac{\Omega^2}{2} + \left[\left(\frac{\Omega^2}{2} + \frac{1}{T_2^2}\right)^2 + \frac{1}{T_2^2}\left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right)\right]^{1/2} . \qquad (16)$$

From here it is possible to find the amplitude of the strong field $E_{0s}(z)$ at which weak fields can be amplified:

$$|E_{0s}(z)|^2 = \frac{\hbar^2}{4d}\left\{\frac{\Omega^2}{2} + \left[\left(\frac{\Omega^2}{2} + \frac{1}{T_2^2}\right)^2 + \frac{1}{T_2^2}\left(\frac{1}{T_1^2} - \frac{1}{T_2^2}\right)\right]^{1/2}\right\} \qquad (17)$$

Shown in Fig. 1 is the function $f \equiv -\frac{\hbar}{T^2 d}\,\mathrm{Im}\chi_1 \cdot 10^4$ vs $\xi \equiv \lg \frac{|E_0(z)|\, T_2 d}{\hbar \cdot 0.8}$, i.e., the dependence of the function $\mathrm{Im}\chi_1$ on the amplitude of the strong field $E_0(z)$ taking into account the Doppler effect. In the calculations the following values of the parameters were used: $T_1 = T_2 = 1.6\cdot 10^{-8}$ s, $\Omega = 2\cdot 10^8\ \text{s}^{-1}$, $\omega_0 = 3.2\cdot 10^{15}\ \text{s}^{-1}$, and m indicated the atomic mass of Na. An averaging over velocity was based on a Maxwell velocity distribution at a temperature of 300 K.

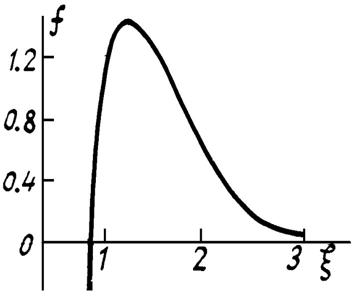

FIG. 1. A plot of the function f vs $|E_0(z)|$. *Here* $f \equiv -\frac{\hbar}{dT_2}\,\mathrm{Im}\chi_1 \cdot 10^4$, $\xi \equiv \lg\left(\frac{|E_0(z)|\,dT_2}{0.8\,\hbar}\right)$.

It becomes clear from the foregoing discussions that the approximation of the strong field appears to be inadequate and the second order of the perturbation theory describing the coupling reaction of waves became important. An amplification of the weak fields arising from the strong field will take place until the strong field diminishes up to the values determined by Eq. (17). It is evident from the system of Eq. (14) that the attenuation of the strong field depends on the weak fields. Thus, Fig. 2 shows the z–behavior of the intensity $I_1(z)$. Here z_s is the distance at which the strong field intensity I_0 diminishes up to the values given by Eq. (17). At sufficiently large z, i.e., when $z \gg z_s$, the coupling of the waves E_1 and E_0 fails and the ordinary dissipative attenuation of the fields occurs.

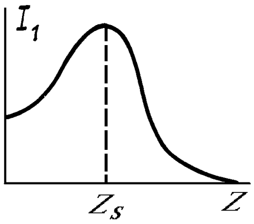

FIG. 2. The intensity I_1 *as a function of distance z inside the medium.*

Let us now estimate the distance z_s in the zero approximation. In doing this, we write the first equation of the system (14) in the form

$$\frac{\partial I_0}{\partial z} = -\frac{4\pi\omega N d^2 T_2 I_0(z)}{c\hbar\left(1 + \frac{T_1 T_2 4 I_0(z) d^2}{\hbar^2}\right)},$$

from which one can easily obtain

$$z_s \approx \frac{c\hbar}{4\pi\omega N d^2 T_2}\left[\frac{4T_1T_2d^2}{\hbar^2}\Big(I_0(0) - I_0(z_s)\Big) - \ln\frac{I_0(z_s)}{I_0(0)}\right], \quad (18)$$

where $I_0(z_s) = |E_{0s}|^2$ and $I_0(0) = |E_0(0)|^2 = \frac{E^{(0)2}}{4}/E^{(0)}$ is given by formula (1).

Let us now examine the dependence of the function η given by Eq. (15) on $|E_0(z)|$ which describes the coupling reaction of the waves. For simplicity, we assume that $T_1 = T_2 = T$. It is then clear from Eq. (6) that $\eta < 0$ when $4a^2 < \Omega^2 + \frac{3}{T^2}$ and $\mathrm{Im}\chi_1 < 0$ when $4a^2(z) > \Omega^2 + \frac{1}{T^2}$. Thus, for such amplitudes of the strong field that $\frac{1}{T^2} + \Omega^2 < 4\left(\frac{|E_0(z)|\,d}{\hbar}\right)^2 < \frac{3}{T^2} + \Omega^2$ weak fields become stronger and the strong field attenuates slower than it does in the absence of the fields E_1 and E_2. This effect is associated with decrease of the population of the upper level $\rho_{22} = (1 - u)/2$ due to the interaction between the waves and, as a result, with decrease of the relaxation losses of the field energy $E_0(z)$, which is proportional to the population of this level, in comparison with the zero approximation. The relaxation loss of the wave energy per unit time is $Q = \frac{\hbar\omega}{2T}(1 - u)$. It follows from Eq. (9) that when $\eta < 0$ and $\mathrm{Im}\chi_1 < 0$, the value of $u^{(2)}$ is always positive, i.e., Q actually decreases in comparison with the zero approximation. For the values of $a^2(z)$ being beyond the interval $\left(\frac{1}{T^2} + \Omega^2, \frac{3}{T^2} + \Omega^2\right)$, the values η and $\mathrm{Im}\chi_1$ are opposite in sign, i.e., amplification of weak fields results in corresponding attenuation of the strong field.

CONCLUSION

The paper describes the propagation of the amplitude–modulated wave with small depth of modulation through the resonance medium. The latter circumstance made it possible to develop the perturbation theory which is based on the small value of the modulation depth or, in other words, on the smaller amplitudes of the two "side" waves compared to the strong "central" field. It has been shown that under certain conditions there occurs amplification of weak waves affecting the strong wave propagation. In particular, we have found the possibility of amplifying weak waves and simultaneously decreasing the strong wave attenuation in comparison with the propagation of a single wave. This effect has been interpreted. The distance was determined, at which weak fields are being amplified, as a function of the strong–field amplitude at the medium boundary. The estimates confirm an essential role of the coupling reaction of the during their waves propagation.[6]

APPENDIX

Let us introduce the notations $b \equiv dT_2/(1 + T_2T_1 4a^2 + (T_2\,k\mathrm{v}^2)\,\hbar)$. After averaging of the velocity over a Maxwell velocity distribution, we obtain the following expression for χ, χ_1 and χ_2:

$$\langle \mathrm{Im}\chi^{(0)} \rangle = \langle b \rangle, \ \langle \mathrm{Re}\chi^{(0)} \rangle = 0, \quad (A.1)$$

$$\langle \mathrm{Imc}_1 \rangle = \left\langle b\,\frac{c_1 s - c_2 r}{r^2 + s^2} \right\rangle, \ \langle \mathrm{Re}\chi_1 \rangle = \left\langle b\,\frac{c_1 r + c_2 s}{r^2 + s^2} \right\rangle, \quad (A.2)$$

where

$$r = 4\big(aT_2\big)^2 \cdot \frac{1 + T_2^2\,\Omega(\Omega + k\mathrm{v})}{1 + T_2^2\,\Omega(\Omega + k\mathrm{v})^2} + \frac{T_2}{T_1} - \Omega\,T_2^2\,(\Omega - k\mathrm{v}),$$

$$s = \Omega\,T_2 + \frac{(\Omega - k\mathrm{v})\,T_2^2}{T_1} - \frac{4aT_2^3\,k\mathrm{v}}{1 + T_2^2\,\Omega\,(\Omega + k\mathrm{v})^2},$$

$$c_1 = \Omega\, T_2\left(1 + (T_2 k\nu)^2\right),\ c_2 = \frac{T_2}{T_1}\left(4a^2 T_1 T_2 - T_2^2\, k^2\nu^2 - 1\right),$$

$$\langle\chi_2\rangle = -\langle\chi_2^*\rangle\ , \tag{A.3}$$

$$\langle \mathrm{Im}\chi^{(2)}\rangle = 2\left|E_1\right| \left\langle b\left(\mathrm{Re}\alpha - \frac{2T_1\left|E_1\right| d}{\hbar}\,\mathrm{Im}\left(\chi_1 + \chi_2\right)\right)\right\rangle \tag{A.4}$$

$$\alpha \equiv -\frac{\left|E_1\right|}{\left|E_0(z)\right|\ a}\left[\frac{b}{T_2}\left(1 + (T_2 k\nu)^2\right) + \chi_1\left(\frac{i}{T_2} + \Omega - k\nu\right)\right].$$

REFERENCES

1. E.J. Robinson, Phys. Rev. A **31**, 3986 (1985).
2. D.T. Pegg, J. Phys. B: At. Mol. Phys. **18**, 415 (1985).
3. D.T. Pegg, Opt. Comm. **57**, 185 (1986).
4. F.T. Hioe, Phys. Rev. A **30**, 2100 (1984).
5. M.V. Danileiko, A.M. Negriiko, V.I. Romanenko, et al., Zh. Teor. Fiz. **58**, No. 6, 1174 (1988).
6. I.M. Al'tshyl', V.P. Lopasov, and S.D. Tvorogov, "*Propagation of waves parametrically connected by Rabi-resonance through a two-level medium*", Preprint No. 67, Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk (1988).