A theoretical research into mechanism of formation of coherent structures in distribution of admixture in the lower troposphere under convective conditions

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A hydrodynamic-statistical model is proposed that explains qualitatively the presence of quasi-regular structures in the distribution of an admixture in the atmospheric convective boundary layer.

Introduction

Observations show that in summer fine dust particles suspended in the atmosphere, as well as temperature and velocity fields sometimes form coherent convective structures with the period from one to several kilometers over arid areas.[1] This paper is devoted to theoretical explanation of this phenomenon. The interaction between convective cells containing an admixture is studied, and simplified Boussinesq equations supplemented with the equation for a passive admixture are used. It is shown that the cells can merge and break down at interaction. This result allows us to put forward a hypothesis on the ways of formation of the statistic structure of an ensemble of convective cells and to obtain the relatively simple equations for their spectra.[2] Under simplifying assumptions it is shown that to parametrically take the total effect of small convective cells into account when modeling an ensemble of convective cells of the next hierarchical level, the coefficient of molecular and turbulent viscosity, heat conductivity, and diffusion should be multiplied by a constant factor. Thus, the convective cells of the height H_k coinciding with the thickness of the convective layer have the maximum spatiotemporal scale in the proposed theory, and the concentration fields of the admixture have the horizontal scale approximately six times larger than H_k, what coincides with measurements from Ref. 1 at $H_k = 1$ km.

Simplified hydrodynamic model

Let us introduce the following simplifying assumptions.

1. Convective cells arise under the effect of two axisymmetric thermal pulses arranged above one another.

2. Only interactions between cells situated along the buoyancy vector (i.e., lying along the same vertical line) are taken into account. The first two assumptions allow us to consider the process as axisymmetric.

3. It is assumed that the vertical dimension of the cells is larger than the horizontal one, because viscosity and Archimedes' force affect propagation of convective perturbation along the vertical. This assumption allows us to simplify the Boussinesq equations due to the theory of the vertical boundary layer.[2]

4. Temperature and density of liquid are related by the following linear law: $\rho = \rho_0(1 - k\theta)$, where θ is the deviation of temperature from its value $\Theta = \Theta_0 - \alpha z$ in unperturbed liquid (α = const); ρ and ρ_0 are the liquid density and its mean value in the unperturbed liquid; k = const is the coefficient of linear expansion of liquid or gas.

5. The convective layer is stratified unstably: $\alpha > 0$.

6. The background concentration of an admixture decreases with height according to the linear law: $S = S_0 - \beta z$ at $z \le S_0/\beta$, $S = 0$ at $z \ge S_0/\beta$.

7. The coefficients of viscosity ν, thermal conductivity, and diffusion are constant and equal to each other.

With allowance for the above simplifications, the Boussinesq equations supplemented with the equation for s (s is the deviation of admixture concentration from its background value S) take the following form[2]:

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \lambda\theta + \frac{\nu}{r}\frac{\partial}{\partial r} r\frac{\partial w}{\partial r} + \nu\frac{\partial^2 w}{\partial z^2}, \quad (1)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial r} + w\frac{\partial \theta}{\partial z} = \alpha w + \frac{\nu}{r}\frac{\partial}{\partial r} r\frac{\partial \theta}{\partial r} + \nu\frac{\partial^2 \theta}{\partial z^2}, \quad (2)$$

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial r} + w\frac{\partial s}{\partial z} = \beta w + \frac{\nu}{r}\frac{\partial}{\partial r} r\frac{\partial s}{\partial r} + \nu\frac{\partial^2 s}{\partial z^2}, \quad (3)$$

$$\frac{\partial ur}{\partial r} + \frac{\partial wr}{\partial z} = 0, \quad (4)$$

where t is time, r and z are the cylindrical radial and vertical coordinates (the axis z is directed upward); u and w are, respectively, the radial and vertical components of velocity, $\lambda = kg$; g is the acceleration of gravity.

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Let us set several vertically elongated axisymmetric thermal pulses as initial conditions for Eqs. (1)–(4): at $t = 0$

$$\theta = \frac{4\nu^2}{\lambda r_0^2} R_0\, f_0(z); \quad s = 0; \quad w = 0, \qquad (5)$$

where $R_0 = \exp[-r^2/(2r_0^2)]$; $f_0(z)$ is nonzero at several non-adjoining segments.

Then the 2D Cauchy problem (1)–(5) is reduced to the 1D one[2]:

$$w = 4\nu^2\alpha\varphi R f;\; u = \frac{4\nu^2\varphi(R-1)}{r}\frac{\partial f}{\partial z};\; \theta = \frac{\psi w}{\lambda\varphi};\; s = \frac{\beta w}{n}; \qquad (6)$$

$$\varphi = \frac{\sinh(nt)}{n};\; \psi = \cosh(nt);\; a = \frac{1}{2\nu t + r_0^2};\; R = \exp(-ar^2); \qquad (7)$$

$$\frac{\partial f}{\partial t} + 4\nu^2\alpha\varphi f\frac{\partial f}{\partial z} = \nu\frac{\partial^2 f}{\partial z^2}; \quad \text{at } t = 0 \quad f = f_0(z); \qquad (8)$$

$$2\pi\int\limits_0^\infty\int\limits_{-\infty}^\infty vr\mathrm{d}r\mathrm{d}z = Q;\; \int\limits_{-\infty}^\infty f\mathrm{d}z = Q_e;\; Q_e = \frac{\lambda q_e}{8\pi c_p\rho\nu^2}, \qquad (9)$$

here $Q = Q_e\,\psi$ is buoyancy of the cell (the value proportional to the heat accumulated in the cell); $n = \sqrt{\alpha\lambda}$.

Let us derive the criterion of cell instability to the effect of perturbations of finite amplitude. To do this, let us divide t into quite small discrete intervals $t = i \times \Delta t$, $i = 0,1,\ldots,n$, in each of which the coefficient $b = 2\nu\alpha\varphi$ of the non-linear term in Eq. (8) can be considered as constant: $b = 2\nu\alpha\varphi = b_i = \text{const}$ at $\Delta t{\cdot}i \le t \le \Delta t(i+1)$. Let us affect the unstable system with a thermal pulse at the initial time $t = 0$ and assume that a thermic of the buoyancy Q is formed by the time $t = t_i = \Delta t{\cdot}i$. Besides, let us very weakly affect the atmosphere with $Q_2 = \varepsilon \ll 1$ at the time $t = t_i$. In this case, at $\Delta t{\cdot}i \le t \le \Delta t(i+1)$ we have

$$\frac{\partial f}{\partial t} + 2\nu b_i f\frac{\partial f}{\partial z} = \nu\frac{\partial^2 f}{\partial z^2}, \qquad (10)$$

instead of Eq. (8) at $t = t_i$, $f = f_1(z) + f_2(z)$. Here f_1 is the solution of Eq. (10) at $t = \Delta t{\cdot}i$; f_2 is the weak effect at $t = t_i$.

Using the fact that Eq. (10) at $\Delta t{\cdot}i < t < \Delta t(i+1)$ is the Burgers equation, let us solve the following problem instead of Eq. (10):

$$f = \frac{\exp Q(F_1 + \varepsilon F_2)}{b_i\left[1 + \exp Q\left(\int\limits_z^\infty (F_1 + \varepsilon F_2)\,\mathrm{d}z\right)\right]}, \qquad (11)$$

where F_1 and F_2 satisfy the following conditions:

$$\int\limits_{-\infty}^\infty F_1\mathrm{d}z = \int\limits_{-\infty}^\infty F_2\mathrm{d}z = 1,\; \frac{\partial F}{\partial t} = \nu\frac{\partial^2 F}{\partial z^2},\; F = F_1 + F_2.$$

The denominator in Eq. (11) reduces to zero, and the solution loses the sense at

$$\varepsilon = -(\exp(Q_1) + \int\limits_z^\infty F_1\mathrm{d}z)/\int\limits_z^\infty F_2\mathrm{d}z. \qquad (12)$$

Thus, the values of ε depend on t and z, but, as fluctuations can arise in the temperature field at any moment of time and at any point of the convective layer, we should take the minimum absolute value of ε from Eq. (12). This value corresponds to the minimum thermal fluctuation that destroys the thermic. It is achieved at $\int\limits_{-\infty}^\infty F_1\,\mathrm{d}z = 0$ and $\int\limits_{-\infty}^\infty F_2\,\mathrm{d}z = 1$. Substituting these values in Eq. (12), we finally have

$$\varepsilon = \varepsilon_{\text{cr}} = -\exp(-Q_1). \qquad (13)$$

Weaker fluctuations do not destroy the thermic. Just the same equation for ε in the solution for two simultaneous point-like thermal pulses set for $\alpha = 0$, $r_0 = 0$ at $t = 0$ is given in Ref. 2. Let us note that the obtained solutions do not describe the process of the cell destruction (because the solution is absent in this case). However, the numerical solution of an analogous problem under the same conditions but by the Boussinesq equations without simplifications of the theory of the vertical boundary layer has shown that the process of "wave reversal" initiated by collision of upward and downward going cells, accompanied by entrainment of the surrounding air into the cell, and leading to its quick dissipation corresponds to cell destruction in the more complete model. Such dissipation is assumed instant and called cell destruction in the simplified theory developed here.

Comparison of the theory with calculations by the model without simplification of the vertical boundary layer has shown that the life cycle of every cell consists of two stages: laminar (for microscale pulsations) or quasi-laminar (for thermics and convective clouds) and turbulent ones. At the first stage, the cell grows spontaneously, and at the second stage it breaks down in the simplified model and dissipates gradually in the more complete model. This is caused by instability of convective cells and, finally, determines the probabilistic properties of the proposed simplified model.

Statistic model of an ensemble of convective cells

According to Eq. (13) (for details see Ref. 2), we assume that the following relation is fulfilled for the probability density of the cell distribution $P(Q)$:

$$P(Q) = \exp(Q_n - Q),\quad \int\limits_{Q_n}^{Q} P(x)\mathrm{d}x = 1. \qquad (14)$$

However, Eq. (14) is unsuitable for testing. Let us try to express Q in terms of the parameters that can be approximately estimated based on the measured data. To do this, let us use the solution of Eq. (8) obtained in Ref. 2 without vertical viscosity at $t \gg 1/n$:

$$w = nzR,\ \theta = \alpha zR,\ s = \beta zR \quad (15)$$

at $0 \le z \le h$ and

$$w = 0,\ \theta = 0,\ s = 0 \quad (16)$$

at $z < 0$ and $z > h = 2(Q\nu/n)^{1/2}$.

It is easily seen that

$$w_m = 2(Q\nu n)^{1/2};\ \theta_m = 2\alpha(Q\nu/n)^{1/2};$$
$$s_m = 2s_0(Q\nu/n)^{1/2}; \quad (17)$$

$$H = w_m\theta_m = 4\nu\alpha Q;\ E = w_m w_m = 4\nu nQ;$$
$$M = w_m s_m = 4\nu\beta Q; \quad (18)$$

$$Q^2 = \alpha\lambda h^4/(16\nu^2) = gk\Delta\Theta h^3/(16\nu^2) = Ra_q/16, \quad (19)$$

where w_m, θ_m, and s_m are equal to w, θ, and s, respectively, at $z = h$ and $r = 0$; $\Delta\Theta = \Theta_{z=h} - \Theta_{z=0} = \alpha h$; and Ra_q is the Rayleigh number of the cell with buoyancy Q.

Thus, the simple relation of Q with h, w_m, θ_m, and H is obtained. This relation allows derivation of the following equations for the density of distribution of the convective cells:

$$P(Y_i)\mathrm{d}Y_i = 2(Y_i - X_i)\exp\{-[(Y_i - X_i)^2/D_i]^2\}/D_i^2\,\mathrm{d}Y_i,$$
$$P(H)\mathrm{d}H = \exp[-(H - H_m)/H_0]/H_0\mathrm{d}H,$$
$$P(E)\mathrm{d}E = \exp[-(E - E_m)/E_0]/E_0\mathrm{d}E,$$
$$P(M)\mathrm{d}M = \exp[-(M - M_m)/M_0]/M_0\mathrm{d}M,$$

where $i = 1, 2, 3, 4$; $Y_1 = h$, $Y_2 = w_m$, $Y_3 = \theta_m$; $Y_4 = s_m$; D_i are the variances of Y_i: $D_1 = h_0 = 2(\nu/n)^{1/2}$, $D_2 = w_0 = 2(\nu n)^{1/2}$, $D_3 = \theta_0 = 2\alpha h_0$; $D_4 = s_0 = 2\beta h_0$; $X_1 = h_n = Q_n^{1/2}h_0$, $X_2 = w_n = Q_n^{1/2}w_0$, $X_3 = \theta_n = Q_n^{1/2}\theta_0$; $X_4 = s_n = Q_n^{1/2}s_0$; $H_0 = 4\nu\alpha$, $H_n = Q_nH_0$; $E_0 = 4\nu(\alpha\lambda)^{1/2}$, $E_n = Q_nE_0$; $M_0 = 4\nu\beta$, $M_n = Q_nS_0$.

Averaging over the ensemble

$$\bar{x} = \int_{X_i}^{\infty} Y_i P(Y_i)\mathrm{d}Y_i,\ \bar{F}_j = \int_{F_n}^{\infty} F_j P(F_j)\mathrm{d}F_j,\ j = 1, 2, 3, \quad (20)$$

we obtain $\bar{f}_j = h_{0i}(Q_n^{1/2} + \pi^{1/2}/2)$, $\bar{F}_j = F_{0j}(Q_n + 1)$, where $f_1 = \bar{h}$, $f_2 = \bar{w}_m$, $f_3 = \bar{\theta}_m$, $f_4 = \bar{s}_m$, $f_{01} = \bar{h}_0$, $f_{02} = \bar{w}_0$, $f_{03} = \bar{\theta}_0$, $f_{04} = \bar{s}_0$; $F_1 = \bar{H}$, $F_2 = \bar{E}$, $F_3 = \bar{M}$, $F_{01} = \bar{H}_0$, $F_{02} = \bar{E}_0$, $F_{03} = \bar{M}_0$.

It is seen therefrom that Q_m can be determined in terms of the mean values and their variances:

$$Q_m + 1 = \bar{F}_j/F_{0j};\ Q_m + \pi^{1/2}/2 = (Y_i/D_i)^2. \quad (21)$$

It is obvious that $\bar{F}_j$ is proportional to the convective fluxes of heat and momentum through the upper boundary of the convective cell into the surrounding atmosphere. Upon accepting any hypothesis on the horizontal distribution of cells and averaging over the horizontal, we obtain

$$\hat{H} = c\alpha(Q_m + 1),\ \hat{E} = cn(Q_m + 1),\ \hat{M} = c\beta(Q_m + 1), \quad (22)$$

where $c = 4\nu C$, C is inversely proportional to the mean distance between cells ($C \approx 0.7$ at horizontal distance between cells $l = 2\pi h$).

In the case that convective perturbation arises not in the rest atmosphere but inside a larger convective cell, one should take $\alpha_2 = \partial\Theta_2/\partial z$ instead of $\alpha = \mathrm{d}\Theta/\mathrm{d}z$, where Θ_2 is temperature inside this larger cell. (We use the subscript 2 for the distribution of the convective ensemble of the second hierarchical level).

Let us study now the behavior of several interacting cells of the second hierarchical level situated above each other. The fluxes of heat and momentum due to micro-scale convection in this ensemble are parameterized as follows:

$$\hat{H} = c_1\partial\Theta_2/\partial z;\ \hat{E} = c_1\partial w_2/\partial z;\ \hat{M} = c_1\partial s_2/\partial z; \quad (23)$$

$$\hat{vu} = c_1\partial\Theta_2/\partial x;\ \hat{wu} = c_1\partial w_2/\partial x;\ \hat{su} = c_1\partial s_2/\partial x, \quad (24)$$

where $c_1 = 4C\nu(Q_m + 1)$.

Equations (23) are obtained without involving any additional hypotheses. Since this model ignores the side interaction between the cells, the correct values of horizontal heat and momentum fluxes can hardly be calculated with it. That is why we accept Eq. (24) as a hypothesis complementing those we accepted when stating the problem and developing the statistical model. The form of Eq. (24) is similar to that of Eq. (23). In this case, the ensemble of the interacting cells of the second hierarchical level situated above each other is described by equations similar to Eqs. (1)–(4) but with u, w, θ, s, ν, and α replaced with u_2, w_2, θ_2, s_2, $\nu_2 = 4C\nu(Q_m + 1) + \nu$, and α_2, where α_2 determines the temperature stratification with allowance made for the mean effect of convection of the first hierarchical level. The equations of the type (23) and (24) and the conclusions following from them are valid for all hierarchical levels. Thus,

$$\frac{\bar{h}_{i+1}}{\bar{h}_i} = LN_i^{-1/4};\ \frac{\bar{w}_{i+1}}{\bar{w}_i} = LN_i^{1/4};$$

$$\frac{\bar{u}_{i+1}}{\bar{u}_i} = L;\ \frac{\bar{\theta}_{i+1}}{\bar{\theta}_i} = LN_i^{3/4}, \quad (25)$$

where $N_i = \alpha_{i+1}/\alpha_i$; $L = 2[C(Q_m + 1)]^{1/2}$.

Let us now determine Q_m. Unfortunately, convective atmospheric turbulence of the first hierarchical level is indistinguishable from dynamic turbulence. So let us consider the experimental data[3] on convection of water in a wide pan heated from below by water heated up to 100°C. Heating was performed in such a way that the heat inflow from below was equal to its outflow through top and sides. When the effect of bottom and sides on convection was minimum, the mean speed of very small suspended particles was equal to 1.8 cm·s^{-1}, and the variance was 20% of this speed. Assuming $D_2/\bar{w} = 0.2$ and substituting this value into Eq. (21), we obtain $\theta_m \approx 25$. Assuming $\nu = 10^{-2}$ cm$^2 \cdot$ s^{-1}, $\alpha = 1$°C $\cdot$ s^{-1}, and $\lambda = gk = 3$ cm $\cdot$ s^{-1} (°C)$^{-1}$, we have $\bar{w} \approx (2Q_m\nu)^{1/2}(\alpha\lambda)^{1/4} \approx 1.8$ cm $\cdot$ s^{-1}, what coincides with measurements from Ref. 3. Finally, assuming that the height of the cells of the last hierarchical level coincides with the height of the mixing layer H_k, we obtain $l = 2\pi H_k$ of 6 km at $H_k = 1$ km, what is confirmed by measurements from Ref. 1. It is also easy to show that the mean lifetime of a cell increases as the hierarchical level of the convective ensemble increases.

Conclusion

The theoretical results show that a convective ensemble consists of cells having significantly different size. According to Eq. (21), the linear dimensions of convective cells of the first hierarchical level are approximately 10 cm. They form cells of the second hierarchical level, whose linear dimensions, according to Eq. (25), are larger by approximately one order of magnitude. This rule proves to be valid for cells of the next hierarchical levels.

According to the theory as well as the data of observations, atmospheric convection evolves as follows. First, a convective ensemble consisting of centimeter and decimeter cells is formed over the surface heated by the Sun. This ensemble is often seen over ploughed fields or asphalt as a haze. As the lower layer is heated, cells of the next hierarchical level arise. The measured results from Ref. 1 have shown that under convective conditions the fields of submicron aerosol particles over arid areas follows, to some degree, the structure of the ensemble of larger and regular convective cells, what is confirmed by this model.

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