One-dimensional radiation model with the explicit temperature dependence

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A simple radiation model from the class of one-dimensional radiation models is proposed. The model is described by ordinary differential equations for temperature of the layers. Temperature dependence is determined by the Planck's function, and this dependence is explicit. The model allows one to study the effect of variations in the concentration of minor atmospheric constituents on the evolution of temperature profile, the stability characteristics of the steady-state profile, and on the relaxation times. It is shown that variations of the concentration by a factor of 2 to 3 do not change the character of the steady state that remains a stable node, but affect the degree of stability by changing the relaxation times that increase with height. It is also shown that introduction of the albedo-temperature relation can lead to the appearance of an additional stable temperature profile with the temperature lower than that taking place nowadays.

Introduction

It is generally accepted now that the climatic system is, to a high degree, nonlinear and therefore it, theoretically, can demonstrate peculiarities of a catastrophic kind. Large models are poorly suited to seeking such peculiarities. At the same time, there are many questions concerning the behavior of the climatic system that can be answered "Yes" or "No" at least at the first stage. For example, how strong should be the variations in the concentration of minor atmospheric constituents, in particular, ozone to lead to catastrophic temperature jumps, and so on.

The corresponding processes are global and therefore they must be described, in addition to complete climatic models, by more simple models accounting just for these processes. Simple models can be a guide for more complicated models showing the domains of the parameters' values, at which the search for manifestation of qualitative peculiarities is likely to be successful. Qualitative peculiarities of a simple model can remain, in principle, at transition to more complicated models.

The behavior of simple nonlinear models can be comprehensively studied based on the theory of dynamic systems. In particular, for them we can find all possible steady states, the character of their stability, and change of their characteristics at variation of the system parameters. The domains of the parameter values, in which qualitative changes may manifest themselves, should be then tested within the framework of complete 3D models. Simple models of the global character[1,2] that have been developed in the 1970's showed that, in spite of a wide variety of factors affecting the climate, they adequately describe the main features of climate evolution.

The simplest climate model accounts for the radiant energy coming to the Earth and heat loss due to longwave emission. It is well known that with the allowance made for the albedo-temperature relation in the simplest zero-dimensional climate model describing the energy balance of the Earth as a whole, there exist three steady states, two of which are stable.[3,4] A slight change of the solar constant being a parameter in the equation of this model leads to the transition from the current steady state to the state with the much lower temperature.

The next step – from zero-dimensional to one-dimensional models – can be done in two directions, namely, by extending the models over latitude or altitude. Historically, the latitude models were first proposed by Budyko[1] and Sellers,[5] and interesting physical problem concerning the effect of variations of solar radiation on the ice sheet margin is connected with these models. The structure of steady states of these models is studied thoroughly in Refs. 2, and 6–12.

One-dimensional climate models, the spatial variable in which is altitude, are called the radiation models. The result calculated using these models is the vertical temperature distributions. It is mostly formed due to radiative processes of heating and cooling. The radiation models accounting for convection are called radiative-convective models. These are most often used to study the effect of variable concentrations of minor atmospheric constituents on climate (see, for example, Refs. 13–16). Equations of the radiation model have the form[13]

$$\frac{\partial T(p)}{\partial t} \propto \partial\,[F^{\uparrow}(p) - F^{\downarrow}(p)]/\partial p, \qquad (1)$$

where the frequency-integrated upwelling $F^{\uparrow}(p)$ and downwelling $F^{\downarrow}(p)$ fluxes of the longwave and solar

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radiation are written for the model of a plane-parallel atmosphere.

When solving this problem, the atmosphere is divided into altitude layers and a set of differential equations for layer temperatures $T^{(k)}$ arises instead of the Eq. (1). The vertical temperature profile is, as a rule, obtained by solving the set of equations using pseudoviscosity method.[13] The temperature $T^{(k+1)}(z_i)$ at the time $t_{k+1} = t_k + \Delta t$ is determined as

$$T^{(k+1)}(p_i) = T^{(k)}(p_i) + \partial T/\partial t \Big|_{T^{(k)}} \Delta t, \qquad (2)$$

and the process is reiterated until the preset condition of convergence is fulfilled. In the radiative-convective models, the temperature gradient is replaced in the process of calculation by a given value, if it proves to be larger than this value (convective matching).

Although a large number of works have been done within the framework of radiative-convective models, the attention paid by now to the qualitative study of the altitude behavior of temperature is insufficient.

1. Formulation of the model

A simple one-dimensional model of the temperature regime in the atmosphere, which would be susceptible to qualitative analysis, can be formulated based on the ordinary equations of the radiative model (1), if the temperature dependence of the transmission function is neglected in the equation for radiative fluxes, so that the full temperature dependence is determined by the Planck's function. Standard methods of determining the steady states and the character of their stability are applicable to the obtained ordinary differential equations. In particular, this model allows studying the effect of varying concentrations of minor atmospheric constituents on the altitude behavior of atmospheric temperature, characteristics of stability of the steady-state temperature profile, as well as the times of relaxation to the steady state.

This model includes, as parameters, the transmission functions allowing for absorption of radiation by gases. It is desirable for calculations by use of a simple model to be rather fast and to provide for the capability of studying a large number of situations. Therefore, in a simple model, high accuracy of calculations is not needed, because this model serves, primarily, for detecting qualitative jumps, the position of which will then be refined by a more complex model. Approximating equations[17,18] for the transmission functions due to absorption by water vapor, carbon dioxide, and ozone in the shortwave and longwave regions were used to calculate radiative fluxes.

The steady states were determined by solving algebraic equations for their coordinates, as well as by directly studying the differential equations with obtaining the time dependence of coordinates (recall that the altitude profile of temperature serves as a steady state and the temperatures within the layers serve as coordinates of the steady state). Both methods yield the same results.

The numerical results given below have been obtained using a 33-layer model of a stationary cloudless atmosphere in midlatitude summer. Note that convection is ignored at this stage. As known, this leads to overestimation of the surface temperature.

2. Test calculations

The calculations made for known situations by use of the above-described model convincingly show that it gives qualitatively correct results in spite of some simplifications.

Thus, if the solar radiation is ignored, the natural result is complete cooling of the system. In spite of different methods used for determination of the steady-state temperature profile, our calculations agree well with the pioneering results from Ref. 13 (Fig. 1).

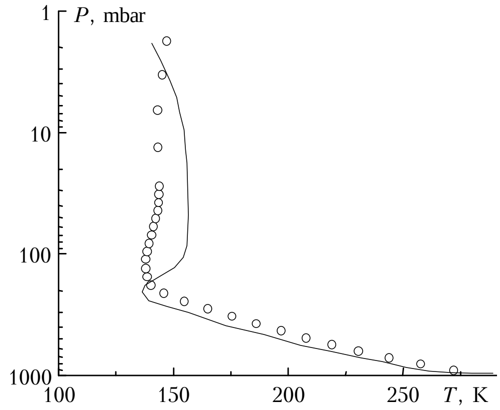

Fig. 1. Temperature distribution at radiant equilibrium neglecting shortwave radiation and convection at the fixed surface temperature (289 K): H_2O, CO_2, and O_3, Ref. 13 (——), H_2O, CO_2, O_3, and water vapor continuum, our calculation (○).

As known, variation of the ozone concentration affects markedly the behavior of temperature above the tropopause. The characteristic minimum at the altitude of the tropopause manifests itself only if the absorption due to ozone is taken into account. The presence of a layer completely absorbing solar radiation at some altitude leads to the decrease of the surface temperature – the phenomenon similar to the "nuclear winter" situation.

The behavior of radiation-equilibrium temperature for different combinations of absorbing gases, which were earlier calculated with a sufficiently detailed radiation model by the pseudoviscosity method,[19] was considered as well. Our curves have been calculated for other values of the zenith angle and albedo. Therefore, the agreement, on the whole, can be accepted as good, Fig. 2 (cp. Fig. 7.2 in Ref. 19).

The calculations show that the approximations used in our simple radiation model (mostly, neglect of the temperature dependence of the transmission functions) provide for a rather realistic behavior of the temperature with height. This allows us to turn to the study of the stability of the altitude behavior of temperature at varying model parameters.

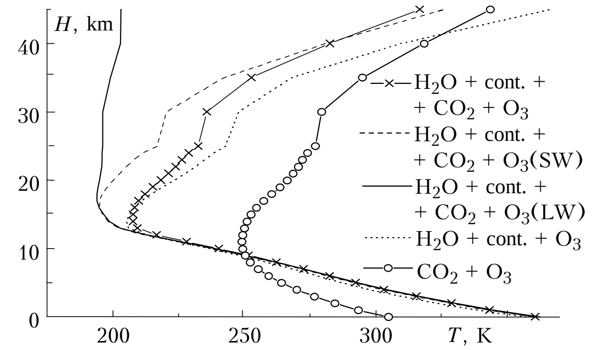

Fig. 2. Altitude profile of temperature for the absorbing gases shown in the figure. O_3(SW) and O_3(LW) mean that ozone absorbs only shortwave or only longwave radiation, respectively.

3. Stability of the altitude behavior of temperature at varying concentrations of atmospheric constituents

The calculations show that the traditionally studied height-uniform 0.5 to 2-fold variations of the concentrations of water vapor and carbon dioxide do not change the stability of the steady state – it remains a stable node. Variations of the concentrations affect the degree of stability by changing the times of relaxation to the steady state, and these changes increase with height. The model allows estimation of these relaxation times from direct solution of differential equations for temperatures of atmospheric layers (Fig. 3, cp. data in Ref. 20).

In further studies, the amplitude of variation of the concentrations was greatly extended. Below we present the calculated altitude behavior of temperature at varying concentrations of the absorbing constituents included in the model. Figure 4 shows the altitude behavior of temperature at proportional variation of the concentrations of carbon dioxide and water vapor in all atmospheric layers, Figure 5 demonstrates the same, for the case of variations in the ozone concentration.

The results obtained allow the following general conclusions to be drawn. Variation of the O_3 and CO_2 concentration affects largely the behavior of temperature in the upper atmosphere, and the increase of the O_3 concentration leads to the increase of temperature in the upper atmosphere, whereas the increase of the CO_2 concentration leads to its decrease. Variation of the surface temperature due to the change of the CO_2 concentration is somewhat higher than that due to the change of the ozone concentration, but still low as compared with that arising due to the change of water vapor concentration. Note again that the steady state remains a stable node at the studied variations of the above constituents concentrations. However, as the O_3 concentration exceeds the current value by ~ 3.5 times, the stability of the temperature behavior is distorted. The saddle sectors likely arise in this 33-dimensional state. However, this problem calls for further investigations.

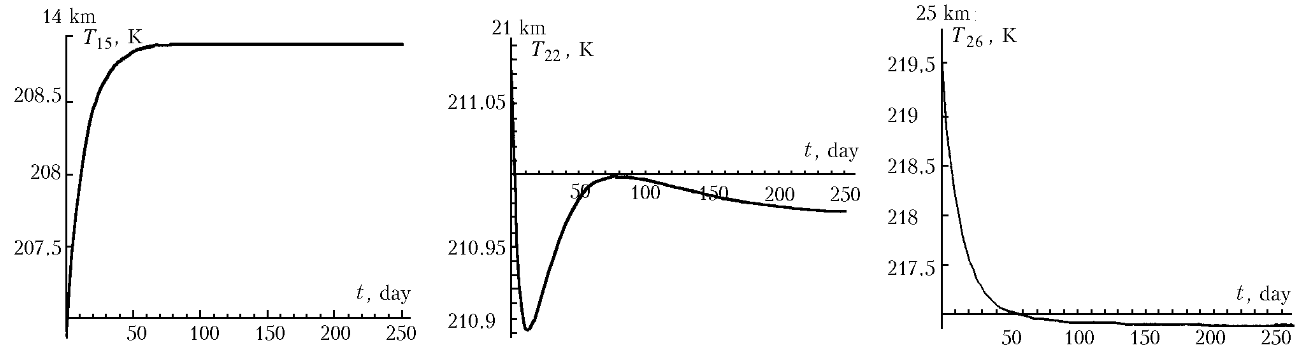

Fig. 3. Relaxation of temperatures of individual atmospheric layers to the steady state at halving of the ozone concentration at the altitudes of 20–25 km.

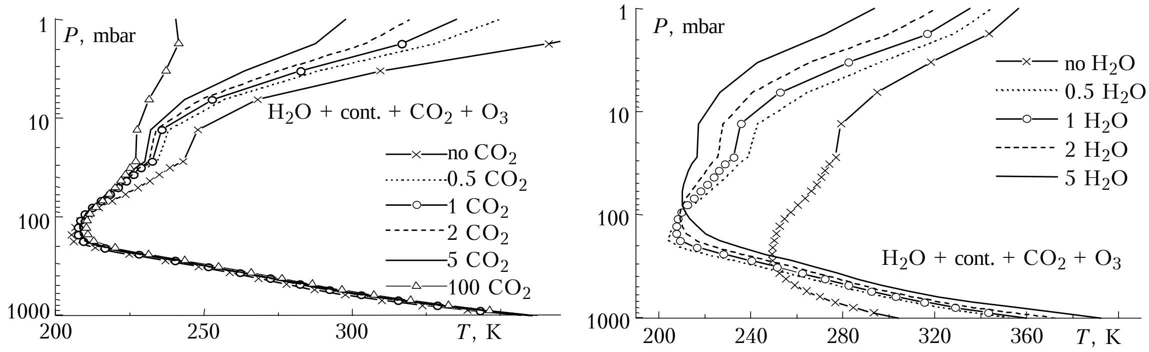

Fig. 4. Change of the temperature profile at proportional variations of the concentrations of carbon dioxide and water vapor.

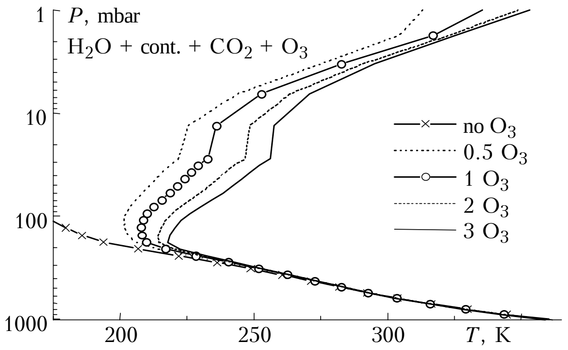

Fig. 5. Change of the temperature profile at proportional variations of the ozone concentration.

Not all possible variations have been considered by now. Thus, introduction of absorption and convection into the equations of temperature dependence can, in principle, make doubtful the conclusion on the ever existing stability of the altitude temperature profile, since this gives rise to terms with another type of nonlinearity in the equations. It turned out that the allowance for the temperature dependence of albedo may lead to the same effect.

4. Stability of the altitude behavior of temperature at variations of the albedo-temperature relation

The one-layer radiation model with the albedo-temperature relation predicts two stable radiation modes separated by an unstable state.[21] The set of equations of the two-layer model[22] "land + atmosphere"

$$dT_{surf}/dt = \sigma\, T_a^4\,(1 - D_w) - \sigma\, T_{surf}^4 + F^{\downarrow}\, D_s\,(1 - \alpha),$$

$$dT_a/dt = \sigma\, T_{surf}^4\,(1 - D_w) - 2\sigma\, T_a^4\,(1 - D_w) + F^{\downarrow}\,(1 - D_s)\,\alpha \quad (3)$$

has one stable node in the physical part of the plane of variables T_{surf} and T_a (T_{surf} is the surface temperature, T_a is the atmospheric temperature, σ is the Stefan-Boltzmann constant). This node does not lose its stability within a wide range of variability of the atmospheric transmission functions for the thermal D_w and solar D_s radiation.

We have introduced the albedo-temperature relation into Eq. (3) by approximating the equation for the albedo $\alpha(T_s)$ in the presence of the ice sheet[23]:

$$\alpha = \alpha_1 + 0.17\,(Z - 0.88) - 0.017\,(T - 283.15) \quad (4)$$

by the function

$$\alpha = (a + \arctan\,[b\,(-T + c)])/d. \quad (5)$$

Variation of the parameter c in Eq. (5) shifts the curve along the vertical axis. Variations of the parameter c in the equation for albedo lead to the temperature behavior and stability assuming the presence of singularities at some values of the parameters. The purposeful search resulted in detection of three steady states at ~250 < c < ~280: two stable nodes separated by a saddle.[24]

The calculations by Eq. (5) for albedo in the 33-layer model show[24] that the structure of the steady states at variations of the albedo-temperature relation remains similar to that obtained in the two-layer model mentioned above (Fig. 6), namely, more than one steady states can exist in a certain variability domain of the parameter c. Thus, for c = 320 two steady states are observed, and some of them can be reached by varying the initial condition (Fig. 7).

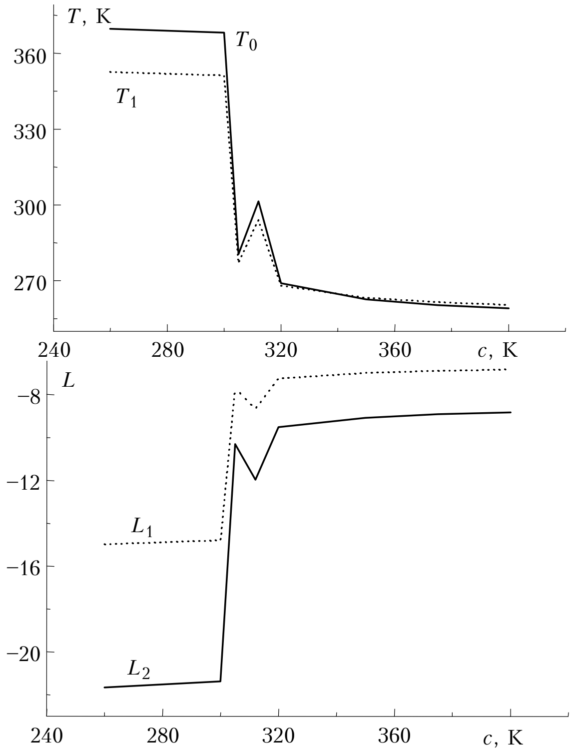

Fig. 6. Behavior of temperature and characteristic numbers L for two lower layers in the 33-layer model at variation of the albedo-temperature relation.

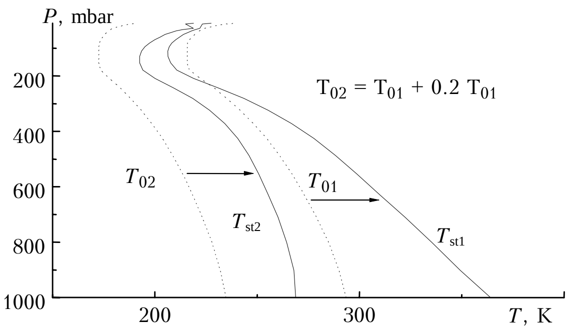

Fig. 7. Different stable temperature profiles T_{st} arising at variations of the albedo-temperature relation in the 33-layer model; T_{01} and T_{02} are initial temperature profiles.

Thus, the allowance for the albedo-temperature relation can result in numerous steady states in the considered system and, under certain conditions, transitions can occur among these states. One of the states is close to the temperature conditions existing nowadays, and another one demonstrates much colder temperatures.

Note that the accurate numerical values do not play a decisive part in this situation. Important is just the fact that the model involves different steady states. Another result is rough determination of the domains of the parameters variability, in which similar states may exist in more realistic models. It is reasonable to expect that numerous steady states are present in all models from the zero-dimensional to the 33-dimensional one. This confirms the assumption that peculiarities of simple models can be conserved in the more complex models. The calculation made in Ref. 25 can be considered as an evidence that the additional steady states can exist in complicated climate models.

In order to find whether the conditions of "white Earth" exist within the used 3D model, Wetherald and Manabe[25] studied the following situation. As the initial approximation, they took the isothermal atmosphere with the temperature of 220 K and the corresponding ice-covered ocean and snowed land, i.e., the "white Earth." The calculation showed that the resulting mean temperature dropped down to 193 K, and, thus, the model climate finding itself in this state failed to leave it.

Conclusion

The above examples of the qualitative study of the altitude behavior of temperature in simple radiation models of climate show that the qualitative study of the climate models themselves and their components allows revealing some general regularities in the behavior of the steady states of a climate system.

The formulated model can be complemented by allowance for the effect of aerosol in calculation of radiative fluxes. The model with the albedo-temperature relation can be applied to analysis of variations of the altitude behavior of temperature due to variations of the cloud cover.

The qualitative analysis of few-dimension models should be a necessary stage before a thorough numerical study. At this stage, the parameter values at which the changes in the system can be radical are determined. Thus, the detailed numerical analysis, as well as thorough theoretical and experimental studies, are then performed just for these values of the parameters.

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