Study of the method for determining characteristics of an instantaneous point source from the measured pollutant concentration

B.M. Desyatkov, S.R. Sarmanaev, A.I. Borodulin, and S.S. Kotlyarova

Scientific Research Institute of Aerobiology at the State Scientific Center of Virology and Biotechnology "Vektor," Koltsovo, Novosibirsk Region

Received February 2, 2001

A method for determining the coordinates and power of a point source of pollution from the values of pollutant concentration measured at several points is considered. The method is based on the equation conjugate with the semiempirical equation of turbulent diffusion. The capabilities of the method are studied for the case of the presence of both measurement errors and statistical scatter due to the atmospheric turbulence in measurements. Two series of computations have been performed. In the first series, we used concentrations obtained by numerical simulation of aerosol spreading over Novosibirsk. Actual experimental data were used in the second series. The obtained results on reconstruction of the source's coordinates and power can be considered quite good.

Earlier we have considered the method for determining the coordinates and power of a point source of atmospheric pollution from the values of the pollutant concentration measured at several control points.[1] This method was based on solution of the equation conjugate with the semiempirical equation of turbulent diffusion. The idea of using such equations in problems associated with the environmental protection was first formulated by Marchuk in the early 1970's. Then it was further developed, for example, in Refs. 2–5. With the proper mathematical formulation of the direct and conjugate problems, we can obtain the functional, which relates the sought characteristics to the measured concentration values and the Green's function being the solution of the conjugate problem. As applied to our problem, this functional has the following form:

$$\int\limits_{\Omega} R_* \, D \, \mathrm{d}\Omega = \int\limits_{\Omega} R \, D_* \, \mathrm{d}\Omega;$$

$$D = \int\limits_0^{\infty} C \, \mathrm{d}t, \quad D_* = \int\limits_{-\infty}^{0} C_* \, \mathrm{d}t, \tag{1}$$

where C is the concentration of a pollutant emitted by an instantaneous point source described by the function R; D is the integral concentration of the pollutant, dose; C_* is the Green's function; R_* is the function, which determines the coordinates of the control points, where the pollutant concentration is measured; Ω is a 3D region, in which pollutant spreading is modeled.

Let us take

$$R = R_0 \, \delta(x - x_0, y - y_0, z - z_0, t);$$

$$R_* = \begin{cases} \delta(x - \xi, y - \eta, z - h); & \xi, \eta \in S \\ 0; & \xi, \eta \notin S. \end{cases} \tag{2}$$

Then from Eqs. (1) and (2) we can derive the following equation:

$$J = \int\limits_S \left| R_0 - \frac{D(\xi, \eta, h)}{D_*(x_0, y_0, z_0,; \xi, \eta, h)} \right| \mathrm{d}S. \tag{3}$$

Equation (3) relates the characteristics of the instantaneous point source located at a still unknown point with the coordinates x_0, y_0, and z_0 (at time $t = 0$ this source has emitted some amount of the pollutant R_0 into the atmosphere) and the resulting values of the integral pollutant concentration measured on the plane S at the points with the coordinates ξ, η, and h. The minimum of this functional apparently determines the sought characteristics.

In this work, we studied the capabilities of this method for the case of the presence of both measurement errors and statistical scatter due to atmospheric turbulence in the measured values of the integral concentration.

The total of two series of experiments were conducted. In the first series, spreading of aerosol particles of 10 μm in diameter over Novosibirsk was modeled. The computations allowed for the actual terrain and the type of the underlying surface (urban territory, steppe, forest, water bodies, etc.). First, the direct problem was solved for the source with the coordinates $x_0 = 12$ km, $y_0 = 12.5$ km, $z_0 = 50$ m, where z_0 is the vertical coordinate (Fig. 1*a*). At 15:00 L.T., the source emitted a pollutant with the total mass R_0 into the atmosphere.

The computations were performed for the meteorological conditions typical of Novosibirsk on June 25, namely, southwestern wind, whose speed at the height of a vane situated at the territory of the local meteorological station was 3 m/s. The fields of the wind velocity over the area and, then, the integral concentration of the pollutant were computed with the use of the method described in Ref. 6.

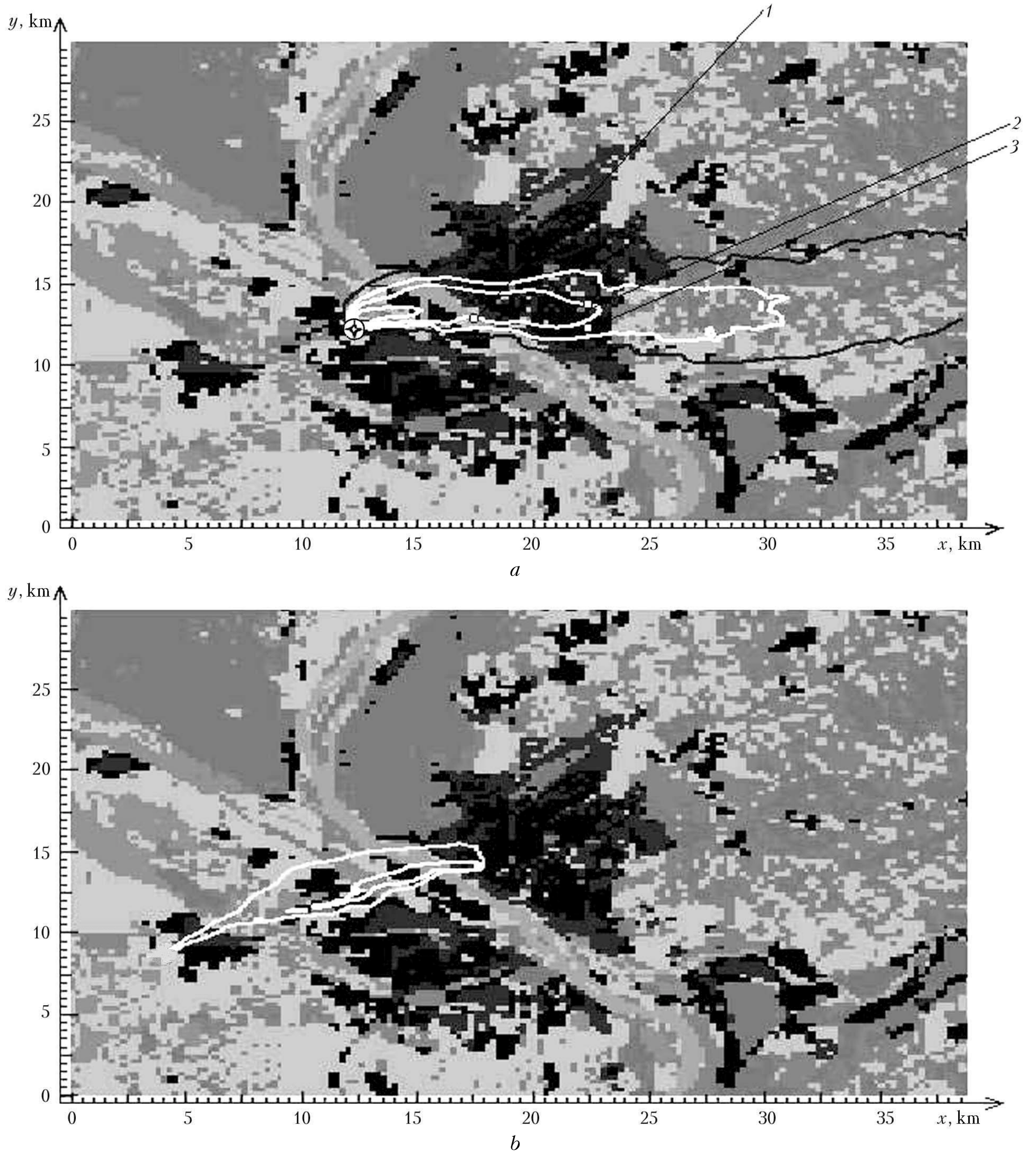

Fig. 1. Isolines of calculated doses (*a*) and functional *J* (*b*); instantaneous point source (⊕), control points, at which the pollution dose was specified (*1*, *2*, and *3*).

Figure 1*a* shows the isolines of the integral concentration obtained for the case under consideration. The inner isoline corresponds to the integral concentration equal to $1\cdot10^5$ rel. units, the other are for $6\cdot10^3$, $5\cdot10^2$, and $1\cdot10^1$ rel. units, respectively. The integral concentration at the control points *1*, *2*, and *3* is equal to $9.2\cdot10^3$, $6.7\cdot10^3$, and $8.8\cdot10^2$ rel. units, respectively. Then, the inverse problem was solved: the values of the integral at the three control points (see Fig. 1) were interpreted as the measured values, the conjugate equation of turbulent diffusion was solved for each control point and the fields of the Green's function were calculated, the functional (3) was minimized and the coordinates of the source and the mass of the emitted pollutant were determined.

With the use of three control points, the determined coordinates coincided with the initially specified coordinates of the source. The reconstructed values of the mass differed from the initial value by less than 3%. Additional computations showed that the increase in the number of control points, as well as the change of their positions almost did not affect the accuracy of determination of the source characteristics. The isolines of the values of the functional J shown in Fig. 1*b* for this case well localize the pollution source. The minimum value of the functional J shown as a white dot in Fig. 1*b* is at the same point, at which the source was while solving the direct problem.

In the second series of computations, we used the data obtained at the State Scientific Center "Vektor" in the experiments on the development of the aerosol technology for insect control in farming.

Figure 2 shows a typical scheme of such experiments. The source of aerosol is shown as a light star, control points, at which the concentration was measured, are shown as circles. The values of the concentration at the points shown as numbered circles were used for solution of the inverse problem.

The total of seven experiments were considered. All the experiments were conducted under conditions of neutral or weakly stable stratification of the atmospheric surface layer. The number of control points used in the computations varied from 3 to 17. As an example, Table 1 gives the measured doses (in rel. units) at the control points in experiment No. 5. It is worth noting a wide scatter of the measured dose.

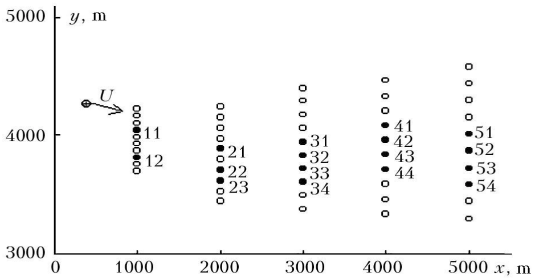

Fig. 2. Experiments: point source (⊕), control points, at which the aerosol concentration was measured (∘).

Table 2 and Figure 3 represent the experimental and calculated values of the coordinates of the point source and the mass of the emitted pollutant in relative units.

Table 1. Experimental values of the dose at control points for experiment No. 5, in rel. units

Point #	11	12	21	22	23	31	32	33	34
Dose, rel. units	0.37	0.15	0.10	0.25	0.56	0.56	1.30	1.50	0.70
Point #	41	42	43	44	51	52	53	54	–
Dose, rel. units	0.60	0.55	0.47	0.18	0.02	0.24	0.37	0.30	–

Table 2. Experimental and calculated values of source parameters

Experiment #	Number of control points	ε, m	$R_{0\exp}$, rel. units	$R_{0\mathrm{calc}}$, rel. units	δ, %
1	3	17	0.88	0.96	9.2
1	4	14	0.88	0.89	0.6
1	7	10	0.88	0.88	0.9
1	15	28	0.88	0.75	15.6
2	3 (a)	34	0.13	0.70	425.0
2	3 (b)	25	0.13	0.53	294.0
2	3 (b)	94	0.13	0.25	88.8
2	4 (a)	19	0.13	0.42	213.0
2	4 (b)	108	0.13	0.27	100.0
2	4 (c)	244	0.13	0.03	78.1
2	5 (a)	91	0.13	0.25	85.8
2	5 (b)	282	0.13	0.10	28.0
2	6 (a)	40	0.13	0.38	184.0
2	6 (b)	40	0.13	0.36	172.0
2	6 (c)	55	0.13	0.32	138.0
3	3	82	0.88	0.80	9.1
3	5	63	0.88	0.80	9.7
3	6	83	0.88	0.88	0.2
4	3	92	0.89	1.17	30.9
4	4	96	0.89	1.92	115.0
4	6	96	0.89	1.88	110.0
5	3	153	0.52	0.40	22.5
5	5	68	0.52	0.07	86.8
5	10	155	0.52	0.15	71.5
5	13	191	0.52	0.29	43.3
5	17	166	0.52	0.16	68.5
6	3	15	0.25	0.45	80.0
6	5	5	0.25	0.19	23.6
7	3	30	0.59	1.04	76.0
Mean value		83			89
Standard deviation		70			96

N o t e . ε is the deviation of the calculated source coordinates from the true ones; $R_{0\exp}$ and $R_{0\mathrm{calc}}$ are the experimental and calculated values of the mass of the emitted pollutant; δ is the relative error in determination of the amount of the emitted pollutant.

One can see that the coordinates of the point source are determined rather well. The mean error in determination of the coordinates is 83 m with the standard deviation of 70 m. The mass of the emitted pollutant is determined with a lower accuracy. The mean relative error and the standard deviation for the mass are equal to 89 and 96%.

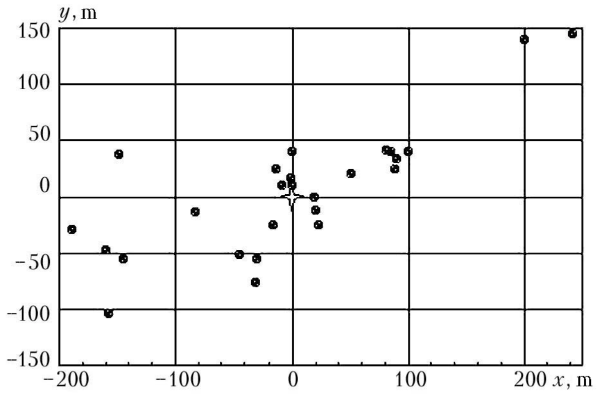

Fig. 3. Calculated coordinates of the source; the true position of the source is marked by ✧.

We have performed computations for different number of control points used (see experiments No. 1 and No. 5), as well as computations implying different arrangement of control points over the experimental area (see experiment No. 2). In this case, the letter "a" in the Table is for the points located near the source, the letter "b" is for the points remote from the source, and the letter "c" is for the points uniformly distributed over the area. The computation revealed no stable dependence of the calculation accuracy on the arrangement of control points. This fact can likely be explained by insufficient accuracy of the measurement data on the wind velocity, temperature, and dose, as well as by the statistical character of the process of pollutant spreading. With the allowance for these notes, the obtained results can be considered as rather good.

Acknowledgments

This work was done under the International Science and Technology Center Contract No. 1217p and partially supported by the Defense Advanced Research Project Agency, the U.S.A.

References

1. B.M. Desyatkov, S.R. Sarmanaev, A.I. Borodulin, S.S. Kotlyarova, and V.V. Selegei, Atmos. Oceanic Opt. **12**, No. 2, 130–133 (1999).
2. G.I. Marchuk, *Mathematical Simulation and Environmental Problems* (Nauka, Moscow, 1982), 320 pp.
3. G.I. Marchuk, Dokl. Akad. Nauk SSSR **227**, No. 5, 1056–1059 (1976).
4. G.I. Marchuk and V.V. Penenko, in: *Proc. of the IFIP. B* (1979), pp. 240–252.
5. V.V. Penenko and V.F. Raputa, Meteorol. Gidrol., No. 2, 59–68 (1983).
6. B.M. Desyatkov, S.R. Sarmanaev, and A.I. Borodulin, Atmos. Oceanic Opt. **9**, No. 6, 517–520 (1996).