# Optimization of the aerosol deposit density using the "wave" method

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The paper describes the model of the optimum treatment of the process of the successive aerosol deposit formation on the field area by choosing the corresponding aerosol particle size and the source power. The numerical method for calculating the optimal parameters for data processing is proposed based on the conversion of the initial optimization problem to a succession of one-dimensional problems. The numerical simulation is performed of the relation between the optimal aerosol deposit amount and the aerosol particle size.

## Introduction

One of the most promising methods of the use of means for plant protection is their application in the form of aerosols. The aerosol preparations are applied by producing a cloud of particles of a definite size, which then is transported by the side wind to a field to be treated. In this case the possibility exists of the control of a series of parameters enabling one to optimize the preparation amount and to decrease the aerosol transport outside the field and so on.[1–4]

When treating a given area by particles of an optimal diameter a considerable mass of aerosols is often transported outside the area what can result in undesirable environmental impacts. On the other hand, the use of aerosol particles of large diameter to decrease this aerosol transport gives rise to a considerable excess consumption of the preparation and the excess of the given level of the aerosol deposit density. Therefore, it is advisable to consider the problem of processing the data on the sum field area, when there are several fields located along the path of aerosol cloud. In this case the particle diameters and the preparation amount for treating each field are determined. This enables one to reduce the aerosol transport and to obtain more uniform density of the aerosol preparation deposit along the entire area of the fields, Ω.

## Statement of the problem

Assume that the aerosol source moves across the wind. The Cartesian coordinate system $(x, y, z)$ is set as follows: the axis $OX$ is oriented along the wind direction, and the axis $OZ$ is oriented vertically upward. As a goal function, we set a net consumption of aerosol preparation, which is one of the most popular criteria. Then we consider the problem of minimization of the following function

$$\Phi = \sum_{i=1}^{R} Q_i \to \min_{\mathbf{Q},\, \mathbf{d}} \tag{1}$$

at limitations

$$P(\mathbf{x}, \mathbf{Q}, \mathbf{d}) = \sum_{i=1}^{R} Q_i\, w_g(d_i)\, I_i(\mathbf{x}, d_i) \geq P_0; \tag{2}$$

$$Q_i \geq 0,\ i = \overline{1, R},\ (x, y, 0) \in \Omega, \tag{3}$$

where $R$ is the number of fields to be treated; $Q_i$ is the preparation consumption per one meter of the path when moving the aerosol source along the $i$th field; $d_i$ is the particle diameter; $\mathbf{Q} = (Q_1, \ldots, Q_R)$; $\mathbf{d} = (d_1, \ldots, d_R)$; $P(\mathbf{x})$ is the density of aerosol sediment at the point $\mathbf{x}$; $P_0$ is the required level of density of the preparation sediment; $w_g(d)$ is the sedimentation rate of aerosol particles; $I_i(\mathbf{x}, d_i)$ is the pulse of aerosol concentration normalized to a unit linear discharge of the preparation when the aerosol generator moves along the $i$th field

$$I_i(\mathbf{x}, d_i) = \int_0^T c_i(t, \mathbf{x}, d_i)\, \mathrm{d}t,$$

where $c(t, \mathbf{x}, d)$ are the aerosol concentrations in the air; $T$ is the time of passage of aerosol cloud over the large area of fields to be treated.

To describe the concentration fields the following equation[3–5] is used

$$\frac{\partial c_i}{\partial t} + u(z)\frac{\partial c_i}{\partial x} - w_g(d_i)\frac{\partial c_i}{\partial z} = \frac{\partial}{\partial z}\nu(z)\frac{\partial c_i}{\partial z} + \frac{\partial}{\partial y}k_y\frac{\partial c_i}{\partial y} + Q_i f_i(t, \mathbf{x}) \tag{4}$$

with the boundary and initial conditions

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$$\left.\left(\nu \frac{\partial c_i}{\partial z} + (w_g - \beta)\, c_i\right)\right|_{z=0} = 0,\ c_i\Big|_{z=H} = 0,$$

$$c_i\Big|_{t=0} = 0,\ i = \overline{1, R}. \tag{5}$$

Here $u(z)$ is the wind velocity along the direction of $OX$ axis; $k_y$, $\nu(z)$ are the coefficients of turbulent diffusion along the axes $OY$ and $OZ$, respectively; $\beta$ is the coefficient characterizing the interaction of the contamination with the underlying surface; $f_i(t, \mathbf{x})$ are the functions describing the location of the normalized aerosol sources.

## Method of solution

Consider now the case of large area of fields located in succession and having sufficient extension and equal depth $\omega$. Assume that the maximum of concentration pulse is attained inside each field. Under conditions, typical for aerosol treatment of the fields with sufficient depth, this assumption is fully acceptable. Then, taking into account the above assumptions, the limitation (2) can be simplified. It is equivalent to the following set of conditions:

$$\sum_{i=1}^{R} Q_i\, J_i(x_k, d_i) = P_0,\ k = \overline{1, R}, \tag{6}$$

where

$$J_i(x_k, d_i) = w_g(d_i)\, I_i(x_k, d_i),$$

the point $x_k$ is positioned in the back edge of the $k$th field.

The relations (6) enable us at fixed dispersion to calculate recurrently the optimal values of the preparation discharge using the following formula:

$$Q_1(d_1) = P_0/[J_1(x_1, d_1)],$$

$$Q_k(d_1, \ldots, d_k) = \frac{P_0 - \sum_{i=1}^{k-1} Q_i(d_1, \ldots, d_i)\, J_i(x_k, d_i)}{J_k(x_k, d_k)}, \tag{7}$$

$$k = \overline{2, R}.$$

If we substitute the relations (7) in the goal function (1), then, as a result, we come to a cumbersome problem of nonlinear programming to find the optimal vector $\mathbf{d}$.[6] However, we can propose a simple method to determine the optimal diameters by reducing the initial problem to a succession of $R$ one-dimensional ones. It is based on the fact that the preceding treatments gave rise to the background density of the deposit for the subsequent ones, and this method is also based on the linearity of criterion (1) relative to the discharge of $Q_i$, $i = \overline{1, R}$.

At any permissible $(Q_i, d_i)$, $i = \overline{1, R-1}$, from the latter ratio (7) and the criterion (1) it follows that the optimal particle size when the treatment of the $R$th area is determined from the condition of the function maximum $J_R(x_R, d_R)$ with respect to $d_R$. In much the same way, using Eq. (7) sequentially, we can show that the particle dimensions, corresponding to the previous treatment, are determined through the subsequent ones and, as a result, an algorithm is obtained to calculate them.

Thus, the problem on determining the optimal parameters of treatment is reduced to the following operations:

– the solution of the sequence of one-dimensional problems of minimization to determine the optimal values of $d_m$, $m = R, R-1, \ldots, 1$;

– the calculation in the reverse order using the recurrent relations (7) of the optimal discharge $Q_m$, $m = \overline{1, R}$.

## Numerical experiments

Consider now numerical examples of calculating the optimal parameters of treatments under conditions of steady-state atmospheric boundary layer. For a horizontally homogeneous underlying surface and low vegetation the profiles of meteorological elements are described within the framework of the Monin–Obukhov similarity theory by the following parameterization formulas[7,8]:

$$u(z) = \frac{u^*}{\kappa}\left[\ln\frac{z}{z_0} + \beta_u \frac{z - z_0}{L}\right];\ \nu(z) = \frac{\kappa\, u^*\, z}{1 + \beta_u \dfrac{z}{L}}, \tag{8}$$

where $u^*$ is the dynamic velocity, $\kappa = 0.35$ is the Karman constant, $L$ is the Monin-Obukhov length scale; $z_0$ is the height of roughness, $\beta_u = 4.7$.

To calculate the rate of gravitational sedimentation of aerosol particles, the Stokes formula[2]

$$w_g(d) = g\rho d^2/(18\mu),$$

is used, where $\mu$ is the aerodynamic viscosity, $g$ is the acceleration of free fall, $\rho$ is the particle density. It can be assumed that the wind velocity is normal to the line of the generator motion. In this case we are limited to consideration of a two-dimensional stationary transfer equation of the impurity with a steady linear source.[2]

The calculations were made using the following values of the input parameters:

$$u^* = 0.08 \text{ m/s},\ L = 7.8 \text{ m},\ z_0 = 0.05 \text{ m},$$

$$h = 0.5 \text{ m},\ P_0 = 1\ \mu\text{g/m}^2.$$

The results of numerical experiments are given in Tables 1 and 2 and show the qualitative representations of the distribution of optimal consumption and aerosol particle size as the aerosol generator moves deep into the field area. For the solutions, the stepwise variation of the generator power is typical after the passage along the first field. The particle diameter increases more smoothly. The treatment by particles of one optimal diameter $d$ for all the passages in a series of situations is technically appropriate because it results in only a slight excess consumption of the preparation.

**Table 1. Distribution of the optimal discharge of preparations for an individual field and of the summed consumption in aerosol treating of the fields by aerosol particles of the same diameter**

| Field depth, km | Particle diameter, μm | Discharge, g/m | | | | | Total consumption, g/m |
|---|---|---|---|---|---|---|---|
| | | Field number | | | | | |
| | | 1 | 2 | 3 | 4 | 5 | |
| 0.25 | 35 | 0.041 | 0.025 | 0.025 | 0.024 | 0.024 | 0.141 |
| 0.5 | 25 | 0.109 | 0.06 | 0.055 | 0.053 | 0.052 | 0.33 |
| 0.75 | 19 | 0.184 | 0.089 | 0.079 | 0.074 | 0.071 | 0.498 |
| 1 | 19 | 0.235 | 0.123 | 0.11 | 0.104 | 0.1 | 0.672 |

Figure 1*a* shows the calculated results on the sediment density for the area consisting of five fields with equal lengths of 500 m and the optimal particle diameter $d$ = 25 μm. Figure 1*b* shows the curve of the preparation sediment density obtained for the same field area using the particles of diameters optimal for each generator passage given in Table 2.

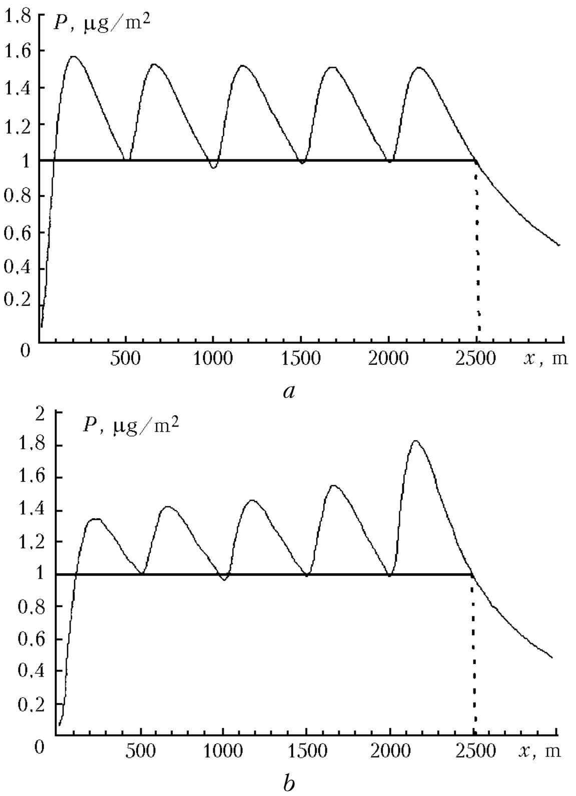


**Fig. 1.** The deposit density when treating the fields of 500 m length by particles: particles of the optimal diameter the same for all the passages of the aerosol generator (*a*); particles of the optimal diameter for each field (*b*).

Analysis of the results indicates that the performance of aerosol treatment of the fields by particles with the diameter optimal for each field with the corresponding discharge of the preparation enables one to obtain uniform sediment density over the entire field area as well as to decrease the aerosol transport and the net preparation consumption (Table 2). It should be noted that systematic treatment of several fields can lead to a decrease of the preparation consumption by more than 50 percent as compared with the treatment of the entire field area by one generator passage using the particles of corresponding optimal size.

**Table 2. Distribution of the optimal discharge of preparations and of aerosol particle diameters**

| Field depth, km | Particle diameter, μm / Discharge, g/m | | | | | Total consumption, g/m |
|---|---|---|---|---|---|---|
| | Field number | | | | | |
| | 1 | 2 | 3 | 4 | 5 | |
| 0.25 | 31 / 0.048 | 32 / 0.024 | 33 / 0.022 | 37 / 0.02 | 39 / 0.02 | 0.136 |
| 0.5 | 21 / 0.129 | 24 / 0.052 | 25 / 0.048 | 27 / 0.045 | 31 / 0.045 | 0.32 |
| 0.75 | 19 / 0.183 | 19 / 0.089 | 20 / 0.077 | 22 / 0.071 | 26 / 0.069 | 0.491 |
| 1 | 19 / 0.235 | 19 / 0.123 | 19 / 0.11 | 19 / 0.1 | 22 / 0.09 | 0.672 |

These conclusions are preliminary since the optimized model does not contain a series of additional limitations due to meteorological conditions, technological peculiarities, the quality of roads and so on. Therefore, the final version of the scheme for treatment of the agricultural area should be chosen with the account for the entire combination of conditions affecting substantially its efficiency.

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