Method and some results of numerical simulation of the intensity fluctuations of a plane light wave behind a phase screen in a multipath region. Part 1. Average intensity

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The spatial structure of average radiation intensity behind a phase screen including one-dimensional regular and two-dimensional random inhomogeneities is studied. The conditions are formulated, under which the intensity distribution averaged over an ensemble and over a receiver's aperture coincide, as well as the conditions, under which the radiative transfer equation is applicable to calculation of the average intensity distribution. Ill-posedness of the inverse problem is discussed. Restoration of a regular phase distribution on a screen from measurements of the average light intensity in a multipath region is shown to be impossible.

Introduction

The applicability of methods for numerical simulation of light propagation described by quasi-optics equations is restricted because of obvious conditions: a step of the computational grid should be smaller than the minimum spatial scale of light field variation and the size of a computational aperture should be larger than the size of a light beam. These conditions should be fulfilled both in difference and direct schemes of solution employing, for example, the method of phase screens and the method of discrete fast Fourier transform. These methods are successfully used to study transformation of laser beams in turbulent and nonlinear media.[1–3]

However, some problems could not be solved by standard methods used for calculation of wave field transformation because of the above conditions. One of such problems is modeling of stellar scintillations at observation through the Earth's atmosphere from an orbiting platform. Unique data of such observations, their interpretation, and detailed bibliography can be found in Refs. 4 and 5.

This problem is characterized by a wide range of spatial scales of atmospheric inhomogeneities that form the scintillation pattern (from about 1 cm to 10 km in the plane normal to the ray) and even wider range of the scale of variability of light field intensity and phase. If the maximum scales in this case are the same as the scales of refractive index inhomogeneities, that is, about 10 km, then the minimum ones determined by interference of the scattered fields in the multipath region may be about fractions of a millimeter. Thus, the range of spatial scales of the light field is eight to nine orders of magnitude, and this fact complicates considerably implementation of algorithms for field calculation.

In observations it is almost impossible to reveal the information on the spatial structure of light field with the scale less than the size of the receiving aperture, therefore it is excessive. (It should be noted that the size of an aperture for receiving radiation from an individual star cannot be smaller than several centimeters). The aim of this paper is the development of a method to directly simulate the intensity distribution of the light field averaged over a finite-size aperture omitting the stage of calculation of the field itself. This work consists of two parts. In the first one, simple equations are derived for calculation of average scintillations and the conditions of their applicability are formulated. Based on these equations, the relationship between the distribution of the mean refraction angle on the phase screen and the distribution of the average intensity of the light field – one of the main characteristics of scintillations – is studied. The second part presents the results of numerical simulation of scintillations caused by stratified atmospheric inhomogeneities, which can be modeled by a phase screen.

Statement of the problem

Assume that a phase screen is at the plane $x = 0$ and the phase distribution $\varphi(z, y)$ is specified on it in the following form:

$$\varphi(z, y) = k[S_0(z) + S_1(z, y)],$$

where k is the wave number; $S_0(z)$ is the regular eikonal component depending only on one coordinate z; $S_1(z, y)$ is the random component being a function of two coordinates. Division into the regular and random coordinates can be made somewhat arbitrarily. In particular, the regular part can include the phase incursion component introduced by stratified inhomogeneities of a rather large scale, while the

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random one can include the small-scale part of stratified inhomogeneities and a part of the phase incursion introduced by locally isotropic turbulence. The specific restrictions on the scales are formulated below. Assume also that the ensemble-average characteristic $<S_1(z, y)> = 0$, and the regular component $S_0(z) = <S(z, y)>$. The angular brackets denote here averaging over the ensemble of realizations.

As for the fluctuation part, we believe that it is a locally homogeneous function of coordinates z and y obeying normal distribution and having the structure function

$$\tilde{D}(z_1, z_2, y_1, y_2) = <[S_1(z_1, y_1) - S_1(z_2, y_2)]^2> = C(z_1 + z_2)\, D(z_1 - z_2, y_1 - y_2). \quad (1)$$

The diffraction field is measured in the plane separated by L from the phase screen. In the paraxial approximation, its intensity is specified by the following equation:

$$I(z,y) = \left(\frac{k}{2\pi L}\right)^2 \int \exp\{i[\varphi(z_1,y_1) - \varphi(z_2,y_2)]\} \times \exp\left[\frac{ik}{2}\left(\frac{(z-z_1)^2 - (z-z_2)^2 + (y-y_1)^2 - (y-y_2)^2}{L}\right)\right] \times \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}y_1 \mathrm{d}y_2. \quad (2)$$

Hereinafter, the intensity of the incident wave is assumed equal to unity, and the integration limits, unless otherwise specified, are assumed to be $(-\infty, +\infty)$.

Mean intensity at a point receiver

Averaging of Eq. (2) over the ensemble of realizations with the allowance for Eq. (1) gives

$$<I(z)> = \frac{k}{2\pi L} \times \int \exp\left[ik[\Psi_0(\eta,\xi) + \frac{\eta - z}{L}\xi] - \frac{1}{2}k^2 C(\eta) D(\xi,0)\right] \mathrm{d}\eta \mathrm{d}\xi, \quad (3)$$

where

$$\Psi_0(\eta, \xi) = S_0(\eta + \xi/2) - S_0(\eta - \xi/2).$$

Define the coherence length $r_c(\eta)$ at the phase screen as the distance, at which $k^2 C(\eta) D[r_c(\eta)] = 2$. Assume that the structure function D is a power-law function. Then $C(\eta)D(\xi)$ can be written as $C(\eta)D(\xi) = 2|\xi/r_c(\eta)|^\beta$. The vicinity about the point $\xi = 0$ of the radius $r_c(\eta)$ is significant for integration of Eq. (3) over the coordinate ξ. If the size of regular inhomogeneities H is much larger than the coherence length, then the function $\Psi_0(\eta, \xi)$, near the point $\xi = 0$, can be expanded in a power series:

$$\Psi_0(\eta, \xi) = [\gamma(\eta)\xi + B(\eta)\, \xi^3 + \ldots], \quad (4)$$

where

$$\gamma(\eta) = \frac{\mathrm{d}S_0(\eta)}{\mathrm{d}\eta} = S_0'(\eta)$$

is the distribution of the refraction angle on the phase screen;

$$B(\eta) = \frac{1}{24}\frac{\mathrm{d}^3 S_0(\eta)}{\mathrm{d}\eta^3} = \frac{1}{24} S_0'''(\eta). \quad (5)$$

If the size of inhomogeneities is rather large, then in calculating the mean intensity we can restrict our consideration to the first term in the series expansion (4). Through numerical integration it was found that for the power structure function with the exponent $\beta = 5/3$ it is sufficient if

$$R_c(\eta) \geq l_k(\eta), \quad (6)$$

where

$$R_c(\eta) = L/kr_c(\eta); \quad l_k(\eta) = 3L[|B(\eta)|/k^2]^{1/3}.$$

The parameter R_c is the scale of the point spread function[6] (the radius of the effective area in the observation plane, over which the point beam is spread because of the diffraction and random jitter). The parameter l_k is the characteristic spatial size of the near-caustic zone in the absence of random inhomogeneities. If the condition (6) is fulfilled, the integral (3) can be presented in the form

$$<I(z)> = \int G[p(z,\eta),\eta] \mathrm{d}\eta, \quad (7)$$

where

$$p(z, \eta) = \eta + L\gamma(\eta) - z;$$

$$G[p(z,\eta),\eta] = \frac{1}{2\pi}\int \exp\left[-\frac{1}{2}k^2 C(\eta) D(\frac{L}{k}t) + ip(z,\eta)t\right] \mathrm{d}t \quad (8)$$

is the Fourier spectrum of the coherence function of light field (point spread function). The function $G[p(z,\eta),\eta]$ is normalized according to the following rule $\int G(p,\eta)\mathrm{d}p = 1$. With the power structure function,

$$G[p(z,\eta), R_c(\eta)] = \frac{1}{R_c(\eta)} g\left[\frac{p(z,\eta)}{R_c(\eta)}, \beta\right], \quad (9)$$

where

$$g(q,\beta) = \frac{1}{2\pi}\int \exp\left[-|t|^\beta + iqt\right] \mathrm{d}t.$$

It should be noted that the solution (7) with the function $G(p, \eta)$ set by Eq. (8) is the solution of small-scale approximation of the radiative transfer equation in the medium with regular inhomogeneities,[7] if this solution is sought for in the phase screen approximation.

The functions $g(q, \beta)$ can be easily found through numerical integration, tabulated or represented in some other, convenient for integration, form. In particular, at $\beta = 5/3$, which corresponds to the exponent of Kolmogorov turbulence, the function $g(q)$ can be approximated as

$$g(q) = g_1(q)(q \le q_1) + g_2(q)(q_1 < q \le q_2) + + g_3(q)(q > q_2), \quad (10)$$

where

$$g_1(q) = \frac{3}{5\pi}\,\Gamma\left(\frac{3}{5}\right)\exp[-(0.52q)^2],$$

$$g_2(q) = g_1(q_1)\left(\frac{q_1}{q}\right)^3, \quad g_3(q) = g_2(q_2)\left(\frac{q_2}{q}\right)^{8/3},$$

$$q_1 = 3,\ q_2 = 22.$$

The relative error of this approximation in the interval $0 \le q \le 100$ does not exceed 2%.

Thus, calculation of the mean intensity distribution is reduced to calculation of the single integral (7). The areas significant for integration in this case are those with the radius of about R_c near the points, at which $p(z, \eta) = 0$. Let $\eta_n(z)$, $n = 1, 2, \ldots$ be these points. They are the points of stationarity of the regular phase on the screen, the points from which rays come to the observation point with the coordinate z. If the distances between neighboring stationary points exceed R_c and the point z is not singular, that is, $1 + L\gamma'[\eta_n(z)] \ne 0$ at any n, we have, from Eq. (7), an approximate equation for the average intensity

$$\langle I(z)\rangle = \sum_n \frac{1}{|1 + L\gamma'[\eta_n(z)]|}. \quad (11)$$

The distribution $\langle I(z)\rangle$ under these conditions is independent of R_c and, consequently, on the coherence length. If there is only one stationary point, then

$$\langle I(z)\rangle = \frac{1}{1 + L\gamma'[\eta(z)]}. \quad (12)$$

In this case, the height distribution of the mean intensity is the same as the regular intensity distribution in the absence of random inhomogeneities.

In the multipath case, the mean intensity (11) is a sum of intensities of the fields formed by the stationary points on the screen. Its distribution over z naturally differs from the distribution formed in the absence of random inhomogeneities and being a complex interference pattern of the fields coming at the observation point from the vicinity of stationary points. If the condition (6) is fulfilled, turbulence washes out the interference pattern and the complex structure of the near-caustic zones.

Mean intensity on finite-size aperture

The spatial scales of the average intensity distribution formed in the observation plane by large-scale regular inhomogeneities are bounded below by R_c, which can vary from several centimeters at the heights of perigees of light rays (minimum distance to the Earth's surface about 40 km) to tens of meters at the heights of about 10 km. These values are comparable with the size of effective receiving apertures in actual measurements of scintillations. In measurements, the size of the effective aperture is one of the following parameters:

R_{ap} – the radius of the receiving objective, whose standard value is 5–10 cm;

R_t is the distance, at which the receiver moves for time of the signal accumulation. This distance is several tens centimeters in measurements with the measurement rate of about 10 kHz (Refs. 4 and 5) and hundreds of meters in spectroscopic measurements (Ref. 8);

R_d is the dispersion shift of rays at extreme wavelengths of the receiver spectral band, which can reach tens of meters at the height of about 15–10 km in the visible region at the bandwidth of 10 nm (Refs. 4 and 5).

Let the receiver has a rectangular aperture with the halfwidths R_z along the vertical axis and R_y along the horizontal axis. These may be functions of the coordinate z. The double averaged (over the aperture and over realizations) intensity can be determined as

$$\langle I_S(z, R_z, R_y)\rangle = = \frac{1}{4R_zR_y}\left\langle \int_{-R_z}^{R_z}\int_{-Ry}^{Ry} I(z+z', y+y')\,dz'dy'\right\rangle. \quad (13)$$

Taking into account the definition (2), after integration of Eq. (13) over the coordinates z' and y', for the mean intensity distribution on the aperture we obtain the equations:

$$\langle I_S(z, R_z)\rangle = \int G_S[p(z,\eta), R_z, \eta]\,d\eta, \quad (14)$$

where

$$G_S[p(z,\eta), R_z, \eta] = \frac{1}{2\pi}\times$$

$$\times \int \exp\left[-\frac{1}{2}k^2C(\eta)D\left(\frac{L}{k}t\right) + ip(z,\eta)t\right]\frac{\sin(R_zt)}{R_zt}\,dt. \quad (15)$$

The integrand expression in Eq. (15) differs from Eq. (7) by the presence of the aperture factor $\sin(R_zt)/R_zt$. Thus the function G_S is the Fourier spectrum of the product of the coherence function of light field behind the screen by the aperture function. If the effective receiving aperture R_z is much larger than the scale R_c, then the function

$$G_S[p(z, \eta), R_z, \eta] = G_S[p(z, \eta), R_z] = = \frac{1}{2R_z}\,\theta\left[1 - \left(\frac{p(z,\eta)}{R_z}\right)^2\right], \quad (16)$$

where $\theta(x)$ is the unit stepwise function: $\theta(x) = 0$ at $x < 0$, and $\theta(x) = 1$ at $x \ge 0$.

As was found through numerical integration with the allowance made for the cubic term of the series expansion of Ψ_0 function, the condition $R_z \ge 16l_k$ is sufficient for applicability of Eq. (16). In this case, the difference between the calculated function and the stepwise function (16) does not exceed 5%. From a comparison of this condition with the condition (6), it can be seen that at the same parameters of regular inhomogeneities the approximation (8) is fulfilled at far smaller values of the averaging aperture, than the approximation (16).

Beyond the vicinities of singular points of radius R_z, the intensity at the final aperture is determined by the same equation (11), as the point intensity averaged over realizations. In this case, it is independent of the size of the averaging aperture and the averaging method. The differences in the averaging methods manifest themselves in the intensity distribution near singular points.

Through numerical integration of Eqs. (7) and (14) it is easy to find the intensity distribution in the vicinities of singular points at the known distribution of the mean phase on the screen.

Let us consider, as a case study, the model of the phase screen as an exponential distribution with superimposed Gauss perturbation. In this case the height distribution of the refraction angle was specified as

$$\gamma(z) = \frac{dS_0}{dz} =$$

$$= \gamma_0 \{\exp[-(z/H_0)] + 2m \frac{H_0}{H_1} \frac{z}{H_1} \exp[-(z/H_1)^2]\}. \quad (17)$$

Parameters of the model (17) were chosen in the way to model light propagation in the atmosphere through the tropopause zone. They were assumed to be the following: $\gamma_0 = 0.008$ rad, $H_0 = 7$ km, $H_1 = 1$ km, $m = 0.1$. The singular points in the observation plane were $z_1 = -23.482$ km and $z_2 = -8.646$ km. The distance L to the observation plane was taken 2000 km. The coordinates of the stationary points on the screen for this model were $\eta(z_1) = -0.62$ km and $\eta(z_2) = -0.63$ km. The wave intensity at the points in the interval $z_1 < z < z_2$ is determined by three stationary points.

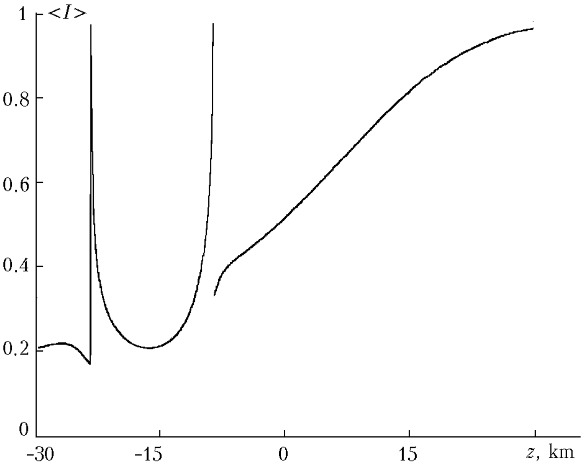

Fig. 1. The mean intensity as a function of the observation height z for the model (17) as calculated by Eq. (11).

Figure 1 shows the distributions of the mean intensity in the observation plane as calculated by the asymptotic equation (11), and Fig. 2 shows the mean intensity distributions near the point z_1 as calculated by Eqs. (7) and (10) at two values of the coherence length $r_c = 0.05$ m ($R_c = 4$ m) and $r_c = 0.005$ m ($R_c = 40$ m), as well as the distributions calculated by Eqs. (14) and (16) at the aperture halfwidth R_z equal to R_c.

The effect of diffraction on random inhomogeneities can be seen from Figs. 1 and 2, and averaging over the receiver aperture leads to limitation of the level of the mean intensity in the caustic zone. The larger the aperture, the larger this limit. One can see the difference in the mean intensity distributions obtained with the two methods of averaging considered.

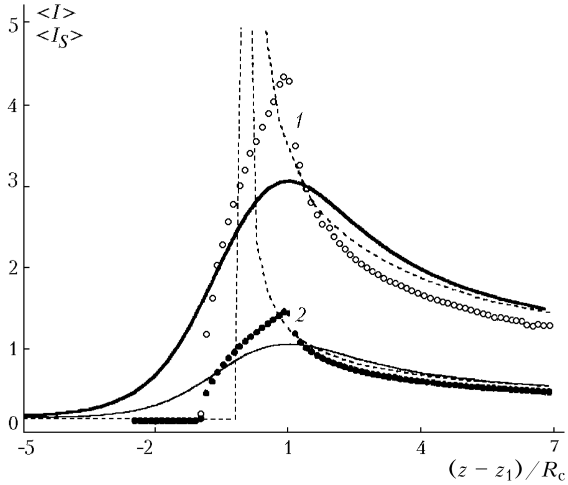

Fig. 2. Mean intensity near the singular point z_1: asymptotic by Eq. (11) (dashed line) and the mean over realizations (solid line) at $R_c = 4$ (*1*) and 40 m (*2*), mean over the aperture with $R_z = R_c$ (dots).

On ill-posedness of the problem on restoration of the mean phase from the mean intensity distribution

In the single-path case, Eq. (12) relating the mean intensity distribution to the regular distribution of the derivative of the refraction angle has an unambiguous solution:

$$L\gamma(\eta) = z(\eta) - \eta, \quad \eta(z) = z_0 + \int_{z_0}^{z} \langle I(z) \rangle \, dz, \quad (18)$$

where z_0 is the initial height that is rather large so that the refraction is insignificant.

If several rays take part in the formation of the intensity distribution, then the problem of restoration of the refraction angle or its derivative from the measured average intensity distribution is ill-posed. Even the number of regular rays taking part in the formation of the average intensity cannot be determined from the mean intensity distribution.

It can be assumed that some information about the regular phase distribution on the screen can be obtained, if restoration is conducted by the single-path equation (18). Figure 3 shows an example of restoration of $\gamma(\eta)$ by this equation from the mean distribution shown in Fig. 1.

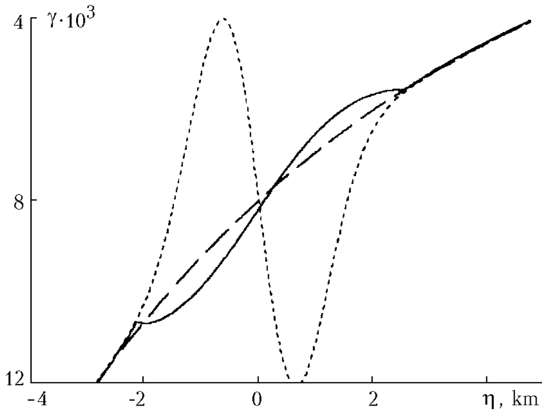

Fig. 3.

The restored distribution is shown by solid curve, the initial distribution $L\gamma(\eta)$ is shown by the short-dash curve, and the unperturbed exponential distribution is shown by the long-dash curve.

From a comparison of the curves, we can see that there is no correspondence between the initial and restored distributions in the multipath region. Even the qualitative behavior of the curves is different. For example, the sign of variations of the restored distribution about the unperturbed (exponential) one is opposite to the sign of specified variations. The amplitude of variations of the reconstructed refraction angle in this example is ten times smaller than the actual one. Thus, the assumption on the possibility of reconstructing the mean perturbation characteristics from the single-path equation proves to be wrong. The interference structure is lost in averaging, but it is mostly just this structure that bears information on multipathing.

Conclusions

The equations have been derived that relate the mean light intensity distribution over a finite aperture to the distribution of the regular refraction angle on the phase screen in the regular multipath region. They correspond to description of wave propagation in the approximation of the radiative transfer equation and are applicable under condition that the size of the effective averaging aperture in the observation plane is larger than the characteristic scale of intensity variations near caustics of the regular ray pattern. Beyond the vicinity of singular points, the average intensity is independent of the size of the averaging aperture. In this case it is a sum of intensities of light from the vicinity of stationary points on the phase screen and is calculated by simple asymptotic equations. Near singular points, the average intensity distribution depends on the averaging method and on the aperture size. It has been shown that the problem of restoring the mean refraction angle on the screen from the mean intensity distribution measured in the regular multipath region is an ill-posed problem.

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