Chaos and order in the model of nonlinear fiber interferometer: wavelet analysis and other methods of investigation

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A mathematical model of processes in a nonlinear optical fiber interferometer is suggested. Phase portraits are constructed, as well as the Fourier and wavelet spectra are calculated. The regime of intermittency is found out. The attempt of automatic identification of phase intermittency is made.

The behavior of light waves in nonlinear ring systems is characterized by their multiple interference with some variable phase shift, depending on light intensity. In general, the result of such interference dissatisfies the superposition principle and defies an analytical solution. This is partly the reason of insufficient theoretical study of nonlinear phenomena in optical ring systems (though K. Ikeda revealed instability in a ring interferometer with nonlinear optical element as early as in 1979). The researches on this subject have been intensified since 1990. S.A. Akhmanov and M.A. Vorontsov, et al. experimentally demonstrated a possibility of generating both optical structures and chaos (so called "dry turbulence") in a nonlinear ring interferometer. New modification of such an instrument is a nonlinear optical fiber interferometer (NOFI), schematically shown in Fig. 1.

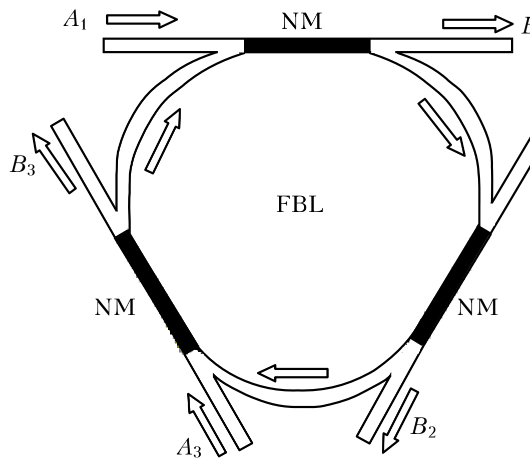

Fig. 1. Scheme of NOFI.

Modeling of processes in the NOFI has shown a possibility of a complex behavior, known as intermittency.

Intermittency is the example of a complex process

In the context of the theory of dynamic systems, this phenomenon was first revealed in 1955 by A.S. Alekseev [Ref. 1, p. 25] and was called *intermittency* in 1980 after publication of the work by P. Manville and I. Pomo. The intermittency is the mode of sequence of almost regular oscillations (so called laminar phase of t_L duration) and chaotic ones (so called turbulence phase of t_T duration) observed just after origination of the deterministic chaos threshold [Ref. 2, p. 149]. The intermittency explains, e.g., unpredictability of relatively seldom but highly damaging earthquakes.

Scheme of the NOFI and the model of light fields dynamics in it

The optical scheme of the NOFI is shown in Fig. 1, where A_i and B_i are the light fields at the interferometer input and output, respectively; NM is the nonlinear medium (semi-conductor-alloyed single-mode optical fiber favoring the Kerr effect). The single-mode fiber free of nonlinear properties (fused silica) forms the feedback loop (FBL).

The dynamics model of light fields in the NOFI is built in the approximation series:

– the fiber is single-mode, low-stress, i.e., the difference between the refraction indices of the core and cladding is small;

– step broadside profile of the refraction index;

– core and cladding of the fiber forming the feedback loop are made of materials with similar dispersion independent of the passing signal intensity;

– nonlinear medium is an optical fiber with the refraction indices of the core and cladding (linearly) dependent on the passing signal intensity;

– the chaotic signal spectrum (in the deterministic chaos mode) falls in the fiber "transparency window";

– loss at joints depends neither on frequency no on amplitude of light in the fiber, light reflection at joints is negligible;

– the fiber polarization dispersion is negligible;

– the light scattering in the fiber, its own noises, diffusion of polarized molecules in optical glass are negligible.

The model describes interrelated variation of the wave amplitude and phase in the fiber core and cladding in the following form:

$$\tau\frac{\partial u_{1,\mathrm{nel}}(t,i)}{\partial t}+u_{1,\mathrm{nel}}(t,i)=u_{10,\mathrm{nel}}(i)+\rho_1(t,i-1)K_{0,\mathrm{core}}(t,i)+$$
$$+\rho_1(t,i-1)\gamma K_{0,\mathrm{core}}(t-t_k,i-1)+$$
$$+2\rho_1(t,i-1)\left[\gamma K_{0,\mathrm{core}}(t,i)K_{\mathrm{core}}(t-t_k,i-1)\right]\times$$
$$\times 0.5\cos\left[\varphi_{\mathrm{nel}}(t-t_k,i-1)+\varphi_{\mathrm{lin}}(i-1)+\right.$$
$$\left.+\varphi(t-t_k,i-1)+\varphi_0(t,i)\right], \quad (1)$$

$$\tau\frac{\partial u_{2,\mathrm{nel}}(t,i)}{\partial t}+u_{2,\mathrm{nel}}(t,i)=u_{20,\mathrm{nel}}(i)+$$
$$+\left[1-\rho_1(t,i-1)\right]K_{0,\mathrm{fac}}(t,i)+$$
$$+\left[1-\rho_1(t,i-1)\right]\gamma K_{0,\mathrm{fac}}(t-t_k,i-1)+$$
$$+2\left[1-\rho_1(t,i-1)\right]\left[\gamma K_{0,\mathrm{fac}}(t,i)K_{\mathrm{fac}}(t-t_k,i-1)\right]\times$$
$$\times 0.5\cos\left[\varphi_{\mathrm{nel}}(t-t_k,i-1)+\varphi_{\mathrm{lin}}(i-1)+\right.$$
$$\left.+\varphi(t-t_k,i-1)+\varphi_0(t,i)\right]; \quad (2)$$

$$\varphi_{\mathrm{nel}}(t)=u_{2,\mathrm{nel}}(t)+\left[u_{1,\mathrm{nel}}(t)-u_{2,\mathrm{nel}}(t)\right]\frac{\partial(V_{\mathrm{nel}}B_{\mathrm{nel}})}{\partial V_{\mathrm{nel}}}, \quad (3)$$

$$\varphi_{\mathrm{lin}}(t)=u_{2,\mathrm{lin}}(t)+\left[u_{1,\mathrm{lin}}(t)-u_{2,\mathrm{lin}}(t)\right]\frac{\partial(V_{\mathrm{lin}}B_{\mathrm{lin}})}{\partial V_{\mathrm{lin}}}, \quad (4)$$

$$\varphi(t,i)=\begin{cases}\dfrac{\pi}{2}-\arctan\left(\dfrac{A_{\mathrm{Re}}}{A_{\mathrm{Im}}}\right), & A_{\mathrm{Im}}<0\\ \dfrac{3\pi}{2}-\arctan\left(\dfrac{A_{\mathrm{Re}}}{A_{\mathrm{Im}}}\right), & A_{\mathrm{Im}}>0\\ 0, & A_{\mathrm{Im}}=0,\ A_{\mathrm{Re}}\ge 0\\ \dfrac{3\pi}{2}, & A_{\mathrm{Im}}=0,\ A_{\mathrm{Re}}<0\end{cases}; \quad (5)$$

$$A_{\mathrm{Re}}=A_0(t,i)\cos\left[\varphi_0(t,i)\right]+\gamma A(t-t_k,i-1)\times$$
$$\times\cos\left[\varphi_{\mathrm{nel}}(t-t_k,i-1)+\varphi_{\mathrm{lin}}(i-1)+\varphi(t-t_k,i-1)\right], \quad (6)$$

$$A_{\mathrm{Im}}=A_0(t,i)\sin\left[\varphi_0(t,i)\right]+\gamma A(t-t_k,i-1)\times$$
$$\times\sin\left[\varphi_{\mathrm{nel}}(t-t_k,i-1)+\varphi_{\mathrm{lin}}(i-1)+\varphi(t-t_k,i-1)\right], \quad (7)$$

where

$$i=1,2,3;\ K_{0,\mathrm{core}}(t)=kl_{\mathrm{nel}}n_{12,\mathrm{nel}}I_0(t),$$
$$K_{0,\mathrm{fac}}(t)=kl_{\mathrm{nel}}n_{22,\mathrm{nel}}I_0(t),$$
$$K_{\mathrm{core}}(t-t_k)=kl_{\mathrm{nel}}n_{12,\mathrm{nel}}I(t-t_k),$$
$$K_{\mathrm{fac}}(t-t_k)=kl_{\mathrm{nel}}n_{22,\mathrm{nel}}I(t-t_k);$$
$$I_0(t)=|A_0(t)|^2,\ I(t-t_k)=|A_0(t-t_k)|^2;$$

τ is the relaxation time of a nonlinear medium; t_k is the time of feedback loop execution by the field; γ is the total loss factor; k is the wave number; l_{nel} and l_{lin} are the (non)linear fiber lengths; $I_0(t)$ and $I(t-t_k)$ are the input and FBL passing light intensities; φ_0 is the field phase at the interferometer input; $\varphi(t-t_k)$ is the phase of the field, resulting from fields summation before nonlinear medium; φ_{nel} and φ_{lin} are the phase variations due to passing (non)linear medium; $u_{1,\mathrm{nel}}(t)$, $u_{2,\mathrm{nel}}(t)$ and $u_{1,\mathrm{lin}}(t)$, $u_{2,\mathrm{linl}}(t)$ are the phase incursions in the core and cladding of (non)linear fiber; $n_{1,\mathrm{nel}}$ and $n_{1,\mathrm{lin}}$ are the refraction indices of the (non)linear fiber core and cladding; V and B are the frequency and phase parameters of the optical fiber; $K_{\mathrm{core}}(t)$ and $K_{\mathrm{fac}}(t)$ are the nonlinearity factors in the fiber core and cladding.

Study of complex behavior in the NOFI model

The results of numerical simulation of processes in the nonlinear fiber interferometer have shown a probability of different types of dynamics, including chaotic. To identify and analyze these types, temporal realization, Fourier spectra, and phase portraits were constructed.

The intermittency mode ("running" chaos) is realizable at some parameters of model (1)–(7) and appears in the NOFI model in multiple changes between the quasiperiodic mode (laminar phase) to the chaotic one (turbulent phase). Such process in the model is observed during the time, corresponding to hundreds of thousands of the feedback loop executions by the light field. Figure 2*a* shows the phase incursion portrait U in the NOFI, operating in the intermittency mode; where signs of quasiperiodic and chaotic modes are seen. Figure 2*b* shows the Fourier spectrum of the filed amplitude, corresponding to the phase incursion portrait in Fig. 2*a*.

As is known, the standard Fourier transform is effective in the analysis of time independent realizations, while it is significantly less effective in the study of transient processes and characteristic frequencies at their different stages. In contrast to this method, the wavelet analysis method allows the appropriate time localization of the process spectrum, its resolution and, hence, identification of the dynamic mode change and characteristic features of the process spectral parameters.

The temporal-frequency dependence of the wavelet spectrum amplitudes for the intermittency mode is shown in Fig. 2*c* (here the value of Y-coordinate, reduced to the reverse scale of the basic function: morlet-wavelet with $\omega_0 = 6$, corresponds to the frequency [Ref. 3, p. 40]). Black color corresponds to the spectrum amplitude maximum and white color is to its minimum.

The approach to the constant of the ratio $t_{\mathrm{L}}/t_{\mathrm{T}}$ of the intermittency phase times of the dynamic system can be assumed in sampling time approaching to the infinity. Therefore, there arises a problem of finding the ratio of laminar and chaotic phase durations for a long observation period. To solve it, an attempt was made to automate the procedure of intermittency phase change identification (Fig. 3).

The authors based on the fact, that the number of signal spectrum components increases with the increase of the complexity and instability of the process, i.e., in the turbulent intermittency phase. Therefore, it is expected that the account and summation of the spectrum maxima will allow the process phases to be differentiated.

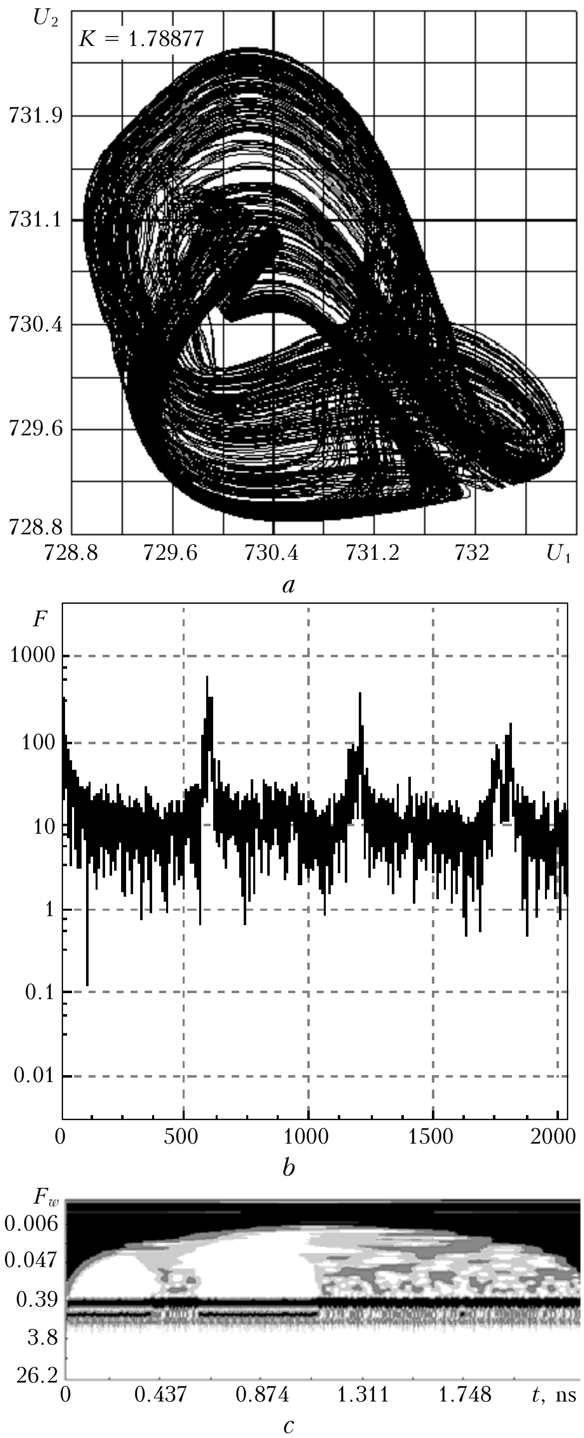

Fig. 2. Phase portrait (a), Fourier spectrum F (b), and wavelet spectrum F_w (c) of field amplitudes in the NOFI.

However, such criterion turned to be ineffective (Fig. 3a), because the phase change affects absolute values of the wavelet spectrum maxima and their position (along the Y-coordinate in Fig. 3c) rather than the number of the maxima.

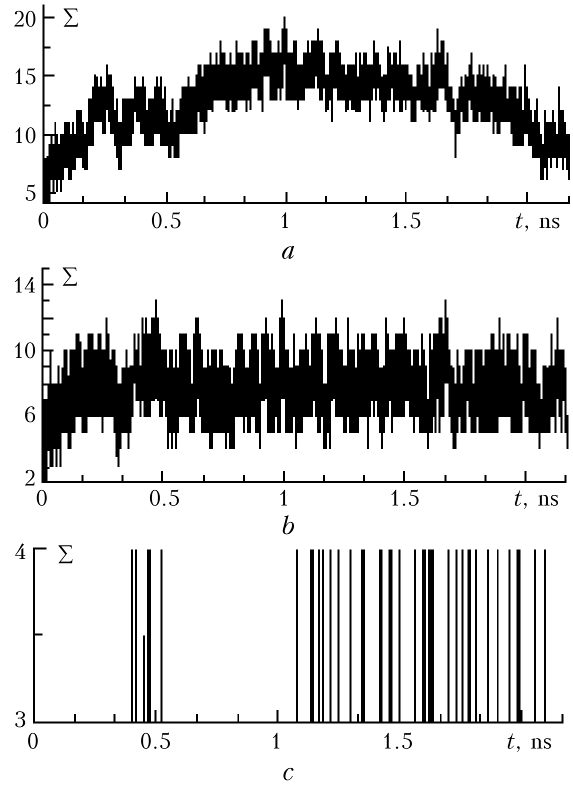

Fig. 3. Temporal variations of the number of maxima $\sum A$ in the wavelet spectrum with accounting for: all the spectrum frequencies (a), central and high frequencies (b), and only central ones (c).

Hence, wavelet spectrum filtration is suggested as the optimizing technique for identifying intermittency phase changes (Figs. 3b and c). The filtration consists in exclusion of one or another wavelet spectrum region from consideration. Figure 3c shows the result, which is in a good agreement with the wavelet spectrum structure from Fig. 2b and is an evidence of the benefit of this approach to study the regularities of intermittency phase changes. Rough estimation of the interferometer operation time shows that the ratio of laminar phase duration to the turbulent one t_L/t_T actually approaches to a certain constant with time.

Conclusion

The intermittency mode has been found out in the suggested NOFI model. According to existing conceptions, the intermittency is a universal phenomenon.

Phase portraits have been constructed, Fourier and wavelet spectra have been calculated. They show the probability of both periodic and chaotic modes in the NOFI model (depending on a set of physical parameters). Wavelet spectra have a structure, characteristic for the intermittency: chaotic oscillations alternate with quasiperiodic ones. The attempt of automatic identification of intermittency phases has been made.

Acknowledgements

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