Optimal polarization characteristics of instruments recording the scattered radiation

V.G. Oshlakov, V.K. Oshlakov, and T.A. Eryomina

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk

Received August 1, 2006

The formulas for determining the optimal polarization characteristics of the radiation source and the receiver are described. Using the numerical method, the effect of polarization parameters of the radiation source and receiver on the signal-to-background ratio at the input of the receiver sensitive element is shown. The use of the sounding signal polarization and the source polarization properties to increase the efficiency of active radio- and optical radars is shown.

Introduction

In development of devices, recording the signals from spatially limited radiation source (optical range), it is necessary in general case to solve the problem of selecting the useful signal against the noise background. An increase of the signal-to-noise ratio at the radiation receiver input can be reached by solving the problem of optimization of the polarization characteristics of the receiver and the source taking into account the background polarization charactetristics.[1,2]

To describe the radiation of the source and the background, we use the corresponding Stokes vectors, and to describe the properties of the radiation propagation medium, we use the scattering phase matrix, keeping the notations stated in Ref. 3.

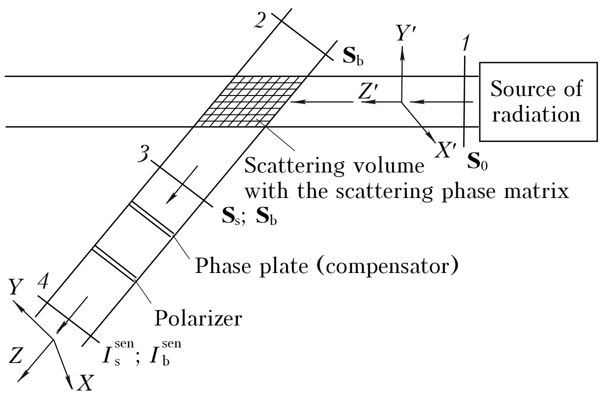

Fig. 1. Optical diagram: $\mathbf{S}_0 = (I_0 Q_0 U_0 V_0)^\mathrm{T}$ is the Stokes vector of the radiation source in the cross section *1*, $\mathbf{S}_\mathrm{b} = (I_\mathrm{b} Q_\mathrm{b} U_\mathrm{b} V_\mathrm{b})^\mathrm{T}$ is the Stokes vector of the background in the cross sections *2* and *3*; $\mathbf{S}_\mathrm{s} = (I_\mathrm{s} Q_\mathrm{s} U_\mathrm{s} V_\mathrm{s})^\mathrm{T}$ is the Stokes vector of the scattered signal in the cross section *3*; $I_\mathrm{s}^\mathrm{sen}$ is the radiation intensity in the cross section *4*; $I_\mathrm{b}^\mathrm{sen}$ is the background intensity in the cross section *4*.

The Stokes vector parameters $\mathbf{S}_\mathrm{s}$ of the scattered signal (Fig. 1) coming to the receiver are determined by the relationship

$$\mathbf{S}_\mathrm{s} = A\mathbf{S}_0, \qquad (1)$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

is the medium scattering phase matrix; $\mathbf{S}_0 = (I_0 Q_0 U_0 V_0)^\mathrm{T}$ are the Stokes vector parameters of the radiation source; $\mathbf{S}_\mathrm{s} = (I_\mathrm{s} Q_\mathrm{s} U_\mathrm{s} V_\mathrm{s})^\mathrm{T}$ are the Stokes vector parameters of the scattered signal; T is the transposition sign.

The intensity of radiation at the input of the receiver is determined as follows:

$$I_\mathrm{s} = a_{11} I_0 + a_{12} Q_0 + a_{13} U_0 + a_{14} V_0. \qquad (2)$$

1. Recording the scattered radiation at the absence of background

If the scattering radiation has been recorded in the absence of background, the use of an unpolarized receiver is most advantageous. It is seen from Eq. (2) that to obtain the I_s maximal value, it is necessary to take into account the medium parameters a_{11}, a_{12}, a_{13}, a_{14} and to select the Stokes parameters of the radiation source by a particular technique.

Calculate the optimal Stokes parameters maximizing the I_s value using the method of Lagrange factors.

If the source radiation is completely polarized, the additional condition is imposed on the Stokes parameters:

$$I_0^2 = Q_0^2 + U_0^2 + V_0^2.$$

The I_0 value always can be made equal to 1 by means of normalization.

0235-6880/07/07 580-06 $02.00

Let us compose the Lagrange function

$$\Phi(Q_0U_0V_0) = a_{11} + a_{12}Q_0 + a_{13}U_0 + a_{14}V_0 + \lambda\left(1 - Q_0^2 - U_0^2 - V_0^2\right) \quad (3)$$

and write the system of equations for determining the parameter λ and the coordinates of the possible extreme points:

$$\begin{cases} \dfrac{\partial\Phi}{\partial Q_0} = a_{12} - 2\lambda Q_0 = 0, \\ \dfrac{\partial\Phi}{\partial U_0} = a_{13} - 2\lambda U_0 = 0, \\ \dfrac{\partial\Phi}{\partial V_0} = a_{14} - 2\lambda V_0 = 0, \\ 1 - Q_0^2 - U_0^2 - V_0^2 = 0. \end{cases} \quad (4)$$

Solving the system of equations (4) relatively to λ, we obtain

$$1 - \frac{a_{12}^2}{4\lambda^2} - \frac{a_{13}^2}{4\lambda^2} - \frac{a_{14}^2}{4\lambda^2} = 0; \ \lambda^2 = \frac{a_{12}^2 + a_{13}^2 + a_{14}^2}{4},$$

$$\lambda_1 = \frac{1}{2}\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}, \ \lambda_2 = -\frac{1}{2}\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}. \quad (4a)$$

Determine the second derivative of the function $\Phi(Q_0\,U_0\,V_0)$ by the formula

$$d^2\Phi = \frac{\partial^2\Phi}{\partial Q_0^2}dQ_0^2 + \frac{\partial^2\Phi}{\partial U_0^2}dU_0^2 + \frac{\partial^2\Phi}{\partial V_0^2}dV_0^2 + 2\frac{\partial^2\Phi}{\partial Q_0\partial U_0}dQ_0\,dU_0 + 2\frac{\partial^2\Phi}{\partial Q_0\partial V_0}dQ_0\,dV_0 + 2\frac{\partial^2\Phi}{\partial U_0\partial V_0}dU_0\,dV_0.$$

In our case

$$d^2\Phi = -2\lambda\left(dQ_0^2 + dU_0^2 + dV_0^2\right).$$

At λ_1 it always follows from Eq. (4a) that $d^2\Phi < 0$, hence, using λ_1 for determination of the Stokes parameters of the radiation source, we will have the maximal intensity at the receiver input.

Write the optimal Stokes parameters of the radiation source in the following form, taking into account the normalization $I_0 = 1$:

$$Q_0 = \frac{a_{12}}{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}}; \ U_0 = \frac{a_{13}}{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}}; \ V_0 = \frac{a_{14}}{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}}. \quad (5)$$

If the source radiation is not polarized and described by the Stokes vector $\mathbf{S}_0 = (1000)^T$, then the signal intensity at the receiver input $(I_s)_{unp} = a_{11}$.

The loss of the signal intensity in comparison with irradiation by the optimal polarized light of the same intensity can be represented as follows:

$$\frac{(I_s)_{unp}}{(I_s)_{opt}} = \frac{a_{11}}{a_{11} + \left(a_{12}^2 + a_{13}^2 + a_{14}^2\right)\Big/\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}} = \frac{1}{1 + \sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}\Big/a_{11}}, \quad (6)$$

where $(I_s)_{opt}$ is the signal intensity at the receiver input in case of the radiation source of optimal polarization. The loss is the greater, the greater is $a_{12}^2 + a_{13}^2 + a_{14}^2$ in comparison with a_{11}^2.

If the Stokes parameters of the radiation source, corresponding to λ_2 and taking into account the normalization $I_0 = 1$ have the form

$$Q_0 = -\frac{a_{12}}{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}}; \ U_0 = -\frac{a_{13}}{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}}; \ V_0 = -\frac{a_{14}}{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}},$$

then the value of the radiation intensity at the receiver input is

$$(I_s)_{pol.\,min} = a_{11} - \sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}.$$

In this case the loss in the signal intensity at the receiver input in comparison with irradiation by unpolarized light described by the Stokes vector $\mathbf{S}_0 = (1000)^T$ is

$$\frac{(I_s)_{pol.\,min}}{(I_s)_{unp}} = 1 - \frac{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}}{a_{11}}.$$

The loss is the greater, the greater is $a_{12}^2 + a_{13}^2 + a_{14}^2$ in comparison with a_{11}^2.

2. Recording the scattered radiation in the presence of background

If the scattered radiation is recorded against the light exposure background, which has the Stokes vector $\mathbf{S}_b = (I_b\,Q_b\,U_b\,V_b)^T$, there appears the problem to reveal the polarization characteristics of the receiver and the Stokes parameters of the source $\mathbf{S}_0 = (I_0Q_0U_0V_0)^T$, which allow us to maximally weaken the background as compared to the signal.

Actually realizable polarization characteristics of the receiver can be obtained by means of compensator with the path difference $\tau = (0 \dots 2\pi)$, the fast axis of which lies at the angle α to the polarizer transmission plane. If the polarizer transmission plane coincides with the axis x, then the intensities of the background I_b^{sen} and the signal I_s^{sen} at the sensitive element are determined in the form[4]:

$$\frac{I_b^{sen}}{C} = I_b + \frac{1 + \cos\tau}{2}Q_b + \frac{1 - \cos\tau}{2}Q_b\cos 4\alpha + \frac{1 - \cos\tau}{2}U_b\sin 4\alpha - V_b\sin\tau\sin 2\alpha,$$

$$\frac{I_s^{sen}}{C} = I_s + \frac{1+\cos\tau}{2}Q_s + \frac{1-\cos\tau}{2}Q_s\cos 4\alpha + $$
$$+\frac{1-\cos\tau}{2}U_s\sin 4\alpha - V_s\sin\tau\sin 2\alpha, \qquad (7)$$

where I_s, Q_s, U_s, V_s are the Stokes vector parameters of the signal entering the receiver:

$$\begin{pmatrix} I_s \\ Q_s \\ U_s \\ V_s \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} I_0 \\ Q_0 \\ U_0 \\ V_0 \end{pmatrix};$$

C is the proportionality coefficient.

Using the elements of the scattering phase matrix A and taking into account the normalization $I_0 = 1$, we can write

$$\frac{I_s^{sen}}{C} = A_1 + A_2Q_0 + A_3U_0 + A_4V_0, \qquad (7a)$$

where

$$A_1 = a_{11} + a_{21}\frac{1+\cos\tau}{2} + a_{21}\frac{1-\cos\tau}{2}\cos 4\alpha + $$
$$+ a_{31}\frac{1-\cos\tau}{2}\sin 4\alpha - a_{41}\sin\tau\sin 2\alpha,$$
$$A_2 = a_{12} + a_{22}\frac{1+\cos\tau}{2} + a_{22}\frac{1-\cos\tau}{2}\cos 4\alpha + $$
$$+ a_{32}\frac{1-\cos\tau}{2}\sin 4\alpha - a_{42}\sin\tau\sin 2\alpha,$$
$$A_3 = a_{13} + a_{23}\frac{1+\cos\tau}{2} + a_{23}\frac{1-\cos\tau}{2}\cos 4\alpha + $$
$$+ a_{33}\frac{1-\cos\tau}{2}\sin 4\alpha - a_{43}\sin\tau\sin 2\alpha,$$
$$A_4 = a_{14} + a_{24}\frac{1+\cos\tau}{2} + a_{24}\frac{1-\cos\tau}{2}\cos 4\alpha + $$
$$+ a_{34}\frac{1-\cos\tau}{2}\sin 4\alpha - a_{44}\sin\tau\sin 2\alpha.$$

Determine the Stokes parameters of the radiation source $\mathbf{S}_0 = (I_0 Q_0 U_0 V_0)^T$ and the receiver parameters τ and α, which provide for the maximum of the ratio I_s^{sen}/I_b^{sen}, by the method of Lagrange factors.

Compose the Lagrange function

$$\Phi(Q_0,U_0,V_0,\tau,\alpha) = I_s^{sen}/I_b^{sen} + \tilde{\lambda}\left(1 - Q_0^2 - U_0^2 - V_0^2\right). \qquad (8)$$

To find its absolute extremum, the system of 5 equations is composed:

$$\frac{\partial\Phi}{\partial\tau} = 0,\ \frac{\partial\Phi}{\partial\alpha} = 0,\ \frac{\partial\Phi}{\partial Q_0} = 0,\ \frac{\partial\Phi}{\partial U_0} = 0,\ \frac{\partial\Phi}{\partial V_0} = 0,$$

from which the values of $\tilde{\lambda}$ and Q_0, U_0, V_0 are determined.

Using Eqs. (7) and (7a), we obtain from the first equation:

$$(B_1 + B_2Q_0 + B_3U_0 + B_4V_0)I_b^{sen} - $$
$$-\left[\frac{\sin\tau}{2}(\cos 4\alpha - 1)Q_b + \right.$$
$$\left. + \frac{\sin\tau}{2}(\sin 4\alpha)U_b - \cos\tau(\sin 2\alpha)V_b\right]I_s^{sen} = 0, \qquad (9)$$

where

$$B_1 = \left[a_{21}(\cos 4\alpha - 1) + a_{31}\sin 4\alpha\right]\frac{\sin\tau}{2} - a_{41}\cos\tau\sin 2\alpha,$$
$$B_2 = \left[a_{22}(\cos 4\alpha - 1) + a_{32}\sin 4\alpha\right]\frac{\sin\tau}{2} - a_{42}\cos\tau\sin 2\alpha,$$
$$B_3 = \left[a_{23}(\cos 4\alpha - 1) + a_{33}\sin 4\alpha\right]\frac{\sin\tau}{2} - a_{43}\cos\tau\sin 2\alpha,$$
$$B_4 = \left[a_{24}(\cos 4\alpha - 1) + a_{34}\sin 4\alpha\right]\frac{\sin\tau}{2} - a_{44}\cos\tau\sin 2\alpha.$$

From the second equation, using Eqs. (7) and (7a), we obtain

$$(C_1 + C_2Q_0 + C_3U_0 + C_4V_0)I_b^{sen} + $$
$$+\left[Q_b(1-\cos\tau)\sin 4\alpha - U_b(1-\cos\tau)\cos 4\alpha + \right.$$
$$\left. + V_b\sin\tau\cos 2\alpha\right]I_s^{sen} = 0, \qquad (10)$$

where

$$C_1 = -a_{21}(1-\cos\tau)\sin 4\alpha + $$
$$+ a_{31}(1-\cos\tau)\cos 4\alpha - a_{41}\sin\tau\cos 2\alpha,$$
$$C_2 = -a_{22}(1-\cos\tau)\sin 4\alpha + $$
$$+ a_{32}(1-\cos\tau)\cos 4\alpha - a_{42}\sin\tau\cos 2\alpha,$$
$$C_3 = -a_{23}(1-\cos\tau)\sin 4\alpha + $$
$$+ a_{33}(1-\cos\tau)\cos 4\alpha - a_{43}\sin\tau\cos 2\alpha,$$
$$C_4 = -a_{24}(1-\cos\tau)\sin 4\alpha + $$
$$+ a_{34}(1-\cos\tau)\cos 4\alpha - a_{44}\sin\tau\cos 2\alpha.$$

Using Eq. (8), we can write the third equation:

$$\frac{\partial\Phi}{\partial Q_0} = \frac{\partial I_s^{sen}}{\partial Q_0}\Big/ I_b^{sen} - 2\tilde{\lambda}Q_0 = CA_2\Big/ I_b^{sen} - 2\tilde{\lambda}Q_0 = 0.$$

Analogously, write the fourth and fifth equations:

$$\frac{\partial\Phi}{\partial U_0} = CA_3/I_b^{sen} - 2\tilde{\lambda}U_0 = 0,$$
$$\frac{\partial\Phi}{\partial V_0} = CA_4/I_b^{sen} - 2\tilde{\lambda}V_0 = 0.$$

Determine $\tilde{\lambda}$ from the system of equations

$$\begin{cases} \dfrac{\partial\Phi}{\partial Q_0} = CA_2/I_b^{sen} - 2\tilde{\lambda}Q_0 = 0, \\ \dfrac{\partial\Phi}{\partial U_0} = CA_3/I_b^{sen} - 2\tilde{\lambda}U_0 = 0, \\ \dfrac{\partial\Phi}{\partial V_0} = CA_4/I_b^{sen} - 2\tilde{\lambda}V_0 = 0, \\ 1 - Q_0^2 - U_0^2 - V_0^2 = 0. \end{cases} \qquad (11)$$

From this system we obtain

$$\tilde{\lambda}^2 = \frac{C^2\left(A_2^2 + A_3^2 + A_4^2\right)}{4\left(I_b^{sen}\right)^2},$$

and, hence,

$$\tilde{\lambda}_1 = \frac{C}{2I_b^{sen}}\sqrt{A_2^2 + A_3^2 + A_4^2},\quad \tilde{\lambda}_2 = -\frac{C}{2I_b^{sen}}\sqrt{A_2^2 + A_3^2 + A_4^2}.$$

Determine the second derivative of the function Φ from the formula

$$d^2\Phi = -2\tilde{\lambda}\left(dQ_0^2 + dU_0^2 + dV_0^2\right).$$

At $\tilde{\lambda} > 0$, $d^2\Phi < 0$, hence:

$$\tilde{\lambda}_1 = \frac{1}{2I_b^{sen}}\sqrt{A_2^2 + A_3^2 + A_4^2}$$

gives the maximum of Φ at any Q_0, U_0, V_0.

Write the Stokes parameters of the source providing for the maximum of the ratio I_s^{sen}/I_b^{sen} in the form

$$Q_0 = \frac{A_2}{2I_b^{sen}\tilde{\lambda}_1} = \frac{A_2}{\sqrt{A_2^2 + A_3^2 + A_4^2}};$$
$$U_0 = \frac{A_3}{2I_b^{sen}\tilde{\lambda}_1} = \frac{A_3}{\sqrt{A_2^2 + A_3^2 + A_4^2}}; \qquad (11a)$$
$$V_0 = \frac{A_4}{2I_b^{sen}\tilde{\lambda}_1} = \frac{A_4}{\sqrt{A_2^2 + A_3^2 + A_4^2}}.$$

It can be concluded from Eq. (11a) that the Stokes parameters Q_0, U_0, and V_0 providing for the maximum of I_s^{sen}/I_b^{sen} are determined by the receiver parameters τ and α and the scattering phase matrix A, and do not depend on $\mathbf{S}_b = (I_b\, Q_b\, U_b\, V_b)^T$. Hence, the only optimal vector can exist at any τ, α, and A

$$\mathbf{S}_0 = \left(1, \frac{A_2}{\sqrt{A_2^2 + A_3^2 + A_4^2}}, \frac{A_3}{\sqrt{A_2^2 + A_3^2 + A_4^2}}, \frac{A_4}{\sqrt{A_2^2 + A_3^2 + A_4^2}}\right),$$

which provides for the maximum of I_s^{sen}/I_b^{sen} independently of $\mathbf{S}_b$. This maximum can be increased (i.e., to obtain the maximum maximorum) by selecting τ and α from the system of equations

$$\begin{cases}\partial\Phi/\partial\tau = 0,\\ \partial\Phi/\partial\alpha = 0.\end{cases}$$

Using Eqs. (9) and (10), we can write this system of equations in the form

$$\begin{cases}(B_1 + B_2Q_0 + B_3U_0 + B_4V_0)I_b^{sen} - \left[\frac{\sin\tau}{2}(\cos 4\alpha - 1)Q_b + \right.\\ \left. + \frac{\sin\tau}{2}(\sin 4\alpha)U_b - \cos\tau(\sin 2\alpha)V_b\right]I_s^{sen} = 0,\\ (C_1 + C_2Q_0 + C_3U_0 + C_4V_0)I_b^{sen} + \left[Q_b(1-\cos\tau)\sin 4\alpha - \right.\\ \left. - U_b(1-\cos\tau)\cos 4\alpha + V_b\sin\tau\cos 2\alpha\right]I_s^{sen} = 0.\end{cases} \qquad (12)$$

If the Stokes parameters of the source Q_0, U_0, and V_0 are related with each other by Eq. (11a) at all τ and α, then $I_s^{sen}/C = A_1 + \sqrt{A_2^2 + A_3^2 + A_4^2}$ and the ratio I_s^{sen}/I_b^{sen}, which is the function of τ and α can be written in the form

$$\frac{I_s^{sen}}{I_b^{sen}} = \left(A_1 + \sqrt{A_2^2 + A_3^2 + A_4^2}\right)\Big/\left(I_b + Q_b\frac{1+\cos\tau}{2} + \right.$$
$$+ Q_b\frac{1-\cos\tau}{2}\cos 4\alpha + U_b\frac{1-\cos\tau}{2}\sin 4\alpha -$$
$$\left. - V_b\sin\tau\sin 2\alpha\right). \qquad (13)$$

3. Example

The scattering phase matrix A is the Rayleigh matrix:

$$A = \begin{pmatrix} 1+\gamma^2 & \gamma^2-1 & 0 & 0\\ \gamma^2-1 & 1+\gamma^2 & 0 & 0\\ 0 & 0 & 2\gamma & 0\\ 0 & 0 & 0 & 2\gamma\end{pmatrix},\quad \gamma\in[-1,1].$$

Let $\gamma = 0.5$, then

$$A = \begin{pmatrix} 1.25 & -0.75 & 0 & 0\\ -0.75 & 1.25 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\quad \begin{matrix} a_{11} = 1.25,\\ a_{12} = -0.75,\\ a_{21} = -0.75,\\ a_{22} = 1.25,\\ a_{33} = 1,\\ a_{44} = 1.\end{matrix}$$

The background has the Stokes vector $\mathbf{S}_b$:

$$(\mathbf{S}_b) = (1; 0.3; 0.5; 0.5)^T;$$
$$I_b = 1;\ Q_b = 0.3;\ U_b = 0.5;\ V_b = 0.5.$$

The relationship

$$I_b^2 \geq Q_b^2 + U_b^2 + V_b^2 = 0.09 + 0.25 + 0.25 = 0.59$$

is fulfilled, hence, $\mathbf{S}_b$ can be the Stokes vector.

Write the maximum possible intensity of the scattered signal $I_{s_{max}}$ at the photoreceiver, taking into account Eqs. (2) and (5) in the form

$$I_{s_{max}} = a_{11}I_0 + a_{12}Q_0 + a_{13}U_0 + a_{14}V_0 =$$
$$= a_{11}\cdot 1 + a_{12}\frac{a_{12}}{\sqrt{a_{12}^2 + a_{13}^2 + a_{14}^2}} = 1.25 + (-0.75)(-1) = 2.$$

If the receiver is not polarization, the maximum ratio $I_{s_{max}}/I_b = 2/1 = 2$ can be obtained.

Conduct the search of τ and α, providing for the maximum of I_s^{sen}/I_b^{sen}, by numerical method of sorting the parameters in Eq. (13) with *a priori* set step of the change of the values of the aforementioned variables in the range $0 \leq \tau \leq 2\pi$ rad, $0 \leq \alpha \leq \pi$ rad.

After finding the maximum value of I_s^{sen}/I_b^{sen} among the node points in the grid of the values of the variables at the given step of value changing, the step was decreased, and the parameters were sorted again in a small area around the found value of the variables, and the next point has been found, at which I_s^{sen}/I_b^{sen} has the maximum value.

The result of the numerical analysis is illustrated by Fig. 2, in which two well pronounced maxima are seen: at the first maximum $I_s^{sen}/I_b^{sen} = 13.67692$ at $\alpha_1 \approx 2.549657$ rad and $\tau_1 \approx 3.86$ rad, and the second maximum $I_s^{sen}/I_b^{sen} = 13.67692$ at $\alpha_2 \approx 0.9788703$ rad and $\tau_1 \approx 2.42$ rad.

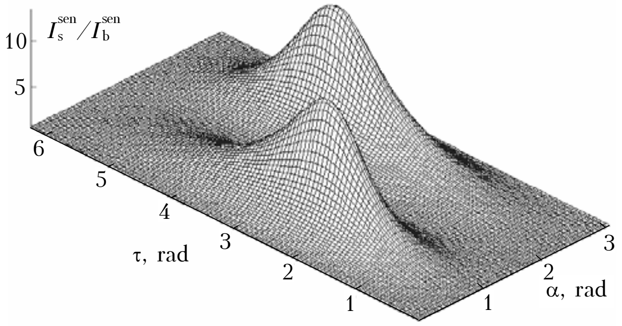

Fig. 2. Dependence of the function I_s^{sen}/I_b^{sen} determined by Eq. (13) in the case $(\mathbf{S}_b) = (1; 0.3; 0.5; 0.5)^T$.

The shapes of the tops of the first and the second maxima along the axis τ are more flat than along the axis α (Tables 1 and 2).

Table 1. The shape of the top of the first maximum I_s^{sen}/I_b^{sen}

α, rad	τ, rad	I_s^{sen}/I_b^{sen}
2.549627	3.863004–3.863725	13.67692
2.549687	3.862944–3.863725	13.67692
2.549747	3.862944–3.863665	13.67692

Table 2. The shape of the top of the second maximum I_s^{sen}/I_b^{sen}

α, rad	τ, rad	I_s^{sen}/I_b^{sen}
0.9787803	2.419733–2.419853	13.67692
0.9788403	2.419433–2.420414	13.67692
0.9789003	2.419433–2.420274	13.67692
0.9789603	2.419613–2.420214	13.67692

The values $\tilde{\tau}$ and $\tilde{\alpha}$, which are solutions of the system (12), allow us to determine

$$\tilde{I}_b^{sen} = C\left(I_b + \frac{1+\cos\tilde{\tau}}{2}Q_b + \frac{1-\cos\tilde{\tau}}{2}Q_b\cos 4\tilde{\alpha} + \right.$$
$$\left. + \frac{1-\cos\tilde{\tau}}{2}U_b\sin 4\tilde{\alpha} - V_b\sin\tilde{\tau}\sin 2\tilde{\alpha}\right).$$

Using $\tilde{Q}_0$, $\tilde{U}_0$, $\tilde{V}_0$, determined from Eq. (11a), as well as the coefficients $\tilde{A}_1$, $\tilde{A}_2$, $\tilde{A}_3$, $\tilde{A}_4$, $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$, $\tilde{C}_1$, $\tilde{C}_2$, $\tilde{C}_3$, $\tilde{C}_4$, calculated at $\tilde{\tau}$ and $\tilde{\alpha}$, we can write Eq. (12) in the form

$$\begin{cases} \tilde{d}_1\tilde{I}_b^{sen} - \tilde{d}_3 c\left(\tilde{A}_1 + \sqrt{\tilde{A}_2^2 + \tilde{A}_3^2 + \tilde{A}_4^2}\right) = 0, \\ \tilde{d}_2\tilde{I}_b^{sen} + \tilde{d}_4 c\left(\tilde{A}_1 + \sqrt{\tilde{A}_2^2 + \tilde{A}_3^2 + \tilde{A}_4^2}\right) = 0, \end{cases} \quad (14)$$

where

$$\tilde{d}_1 = \tilde{B}_1 + \frac{\tilde{B}_2\tilde{A}_2 + \tilde{B}_3\tilde{A}_3 + \tilde{B}_4\tilde{A}_4}{\sqrt{\tilde{A}_2^2 + \tilde{A}_3^2 + \tilde{A}_4^2}},$$

$$\tilde{d}_2 = \tilde{C}_1 + \frac{\tilde{C}_2\tilde{A}_2 + \tilde{C}_3\tilde{A}_3 + \tilde{C}_4\tilde{A}_4}{\sqrt{\tilde{A}_2^2 + \tilde{A}_3^2 + \tilde{A}_4^2}},$$

$$\tilde{d}_3 = \frac{\sin\tilde{\tau}}{2}Q_b(\cos 4\tilde{\alpha} - 1) + \frac{\sin\tilde{\tau}}{2}U_b\sin 4\tilde{\alpha} - \cos\tilde{\tau}V_b\sin 2\tilde{\alpha},$$

$$\tilde{d}_4 = Q_b(1-\cos\tilde{\alpha})\sin 4\tilde{\tau} - U_b(1-\cos\tilde{\tau})\cos 4\tilde{\alpha} +$$
$$+ V_b\sin\tilde{\tau}\cos 2\tilde{\alpha}.$$

Hence, if $\tilde{\tau}$ and $\tilde{\alpha}$ are the points of maximum maximorum, then the following equality is fulfilled

$$\left(\tilde{A}_1 + \sqrt{\tilde{A}_2^2 + \tilde{A}_3^2 + \tilde{A}_4^2}\right)\Big/\left(I_b + \frac{1+\cos\tilde{\tau}}{2}Q_b + \right.$$
$$+ \frac{1-\cos\tilde{\tau}}{2}Q_b\cos 4\tilde{\alpha} + \frac{1-\cos\tilde{\tau}}{2}U_b\sin 4\tilde{\alpha} -$$
$$\left. - V_b\sin\tilde{\tau}\sin 2\tilde{\alpha}\right) = \frac{\tilde{d}_1}{\tilde{d}_3} = -\frac{\tilde{d}_2}{\tilde{d}_4}.$$

The coefficients $\tilde{d}_1$, $\tilde{d}_2$, $\tilde{d}_3$, $\tilde{d}_4$ calculated at the values τ_1 and α_1 of the first maximum are: $\tilde{d}_1^{(1)} = 0.849877$, $\tilde{d}_2^{(1)} = -1.835606$, $\tilde{d}_3^{(1)} = 0.633002$, $\tilde{d}_4^{(1)} = 0.383507$, and at the values τ_2 and α_2 of the second maximum: $\tilde{d}_1^{(2)} = -0.849863$, $\tilde{d}_2^{(2)} = -1.835633$, $\tilde{d}_3^{(2)} = -0.633001$, $\tilde{d}_4^{(2)} = 0.383498$. The following equalities are fulfilled with a good accuracy for the first and the second maxima:

$$\frac{\tilde{d}_1^{(1)}}{\tilde{d}_3^{(1)}} = -\frac{\tilde{d}_2^{(1)}}{\tilde{d}_4^{(1)}} = \frac{\tilde{d}_1^{(2)}}{\tilde{d}_3^{(2)}} = -\frac{\tilde{d}_2^{(2)}}{\tilde{d}_4^{(2)}} =$$
$$= \left(A_1 + \sqrt{A_2^2 + A_3^2 + A_4^2}\right)\Big/\left(I_b + \frac{1+\cos\tau}{2}Q_b + \right.$$
$$+ \frac{1-\cos\tau}{2}Q_b\cos 4\alpha + \frac{1-\cos\tau}{2}U_b\sin 4\alpha -$$
$$\left. - V_b\sin\tau\sin 2\alpha\right) = 13.67692.$$

Hence, τ_1 and α_1, τ_2 and α_2 provide for the maximum maximorum of I_s^{sen}/I_b^{sen}.

Numerical analysis shows that the use of polarized receiver allows improving I_s^{sen}/I_b^{sen} by more than 6 times in comparison with the unpolarized receiver.

Consider the case when the background is the unpolarized radiation with the Stokes vector $\mathbf{S}_b$:

$$(\mathbf{S}_b) = (1; 0, 0, 0)^T; \quad I_b = 1; \; Q_b = U_b = V_b = 0.$$

Determine τ and α providing for the maximum of I_s^{sen}/I_b^{sen} analogously to the numerical method considered above.

The result of numerical analysis is shown in Fig. 3, where two well pronounced maxima are seen: the first maximum has $I_s^{sen}/I_b^{sen} = 4.0$ at $\alpha_1 \approx 0.78$ rad and $\tau_1 \approx 3.14$ rad, and the second has $I_s^{sen}/I_b^{sen} = 4$ at $\alpha_2 \approx 2.36$ rad and $\tau_2 \approx 3.14$ rad.

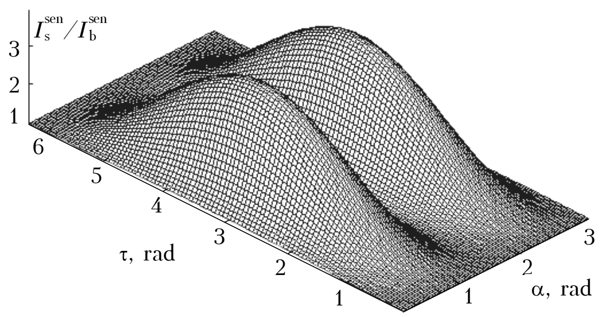

Fig. 3. The dependence of the function I_s^{sen}/I_b^{sen} determined from Eq. (13) in the case $(\mathbf{S}_b) = (1; 0; 0; 0)^T$.

4. Increase of the target contrast in the active radio- and optical location

One of the most important problems of the target finding in radio- and optical location is availability of the optimal techniques for selecting the target signals against the noise background.

The Stokes vector of the target signal $\mathbf{S}_s = (I_s\, Q_s\, U_s\, V_s)^T$ and the Stokes vector of the background $\mathbf{S}_b = (I_b\, Q_b\, U_b\, V_b)^T$ in active radio- and optical location are determined by the Stokes vector of the locator $\mathbf{S}_l = (I_l\, Q_l\, U_l\, V_l)^T$. The scattering phase matrix of the target A_s can be *a priori* known, so the Stokes vector of the target signal is

$$\mathbf{S}_s = A_s\mathbf{S}_l.$$

The Stokes vector of the background is determined analogously:

$$\mathbf{S}_b = A_b\mathbf{S}_l,$$

where A_b is the scattering phase matrix of the underlying surface.

It is more difficult to *a priori* determine the scattering phase matrix of the underlying surface A_b than A_s because of the variety of the underlying surface types, so consider the case when $\mathbf{S}_b$ is unknown.

The characteristics of finding target by the radar (probability of false alarm, probability of missing the target) can be improved through increasing the target contrast, i.e., reaching maximum of the ratio I_s/I_b.

As the optimal value $\mathbf{S}_l$ at any polarization characteristics of the receiver (τ and α) and any scattering phase matrix A_s does not depend on $\mathbf{S}_b$ but is determined only by τ, α, and A_s, then, by sorting τ and α with determining $\mathbf{S}_l$ for them every time, we can select such τ and α, at which I_s/I_b reaches maximum maximorum, i.e., the target contrast is maximum.

Of coarse, the polarization elements realizing similar function in radio- and optical range, differ in design.

Acknowledgements

Authors would like to thank S.M. Chernyavskii for useful discussion and remarks.

References

1. V.G. Oshlakov, in: *Abstract of Reports at 6-th All-Union Symp. on Propagation of Laser Radiation in the Atmosphere* (Tomsk, 1981), Part 2, p. 245.
2. V.G. Oshlakov, V.K. Oshlakov, and T.A. Eryomina, Proc. SPIE **6160**, Part I, 616014-1–616014-10 (2006).
3. C.F. Bohren and D.R. Huffman, *Absorption and Scattering of Light by Small Particles* (Willey, New York, 1986).
4. G.I. Gorchakov, Izv. Akad. Nauk USSR. Ser. Fiz. Atmos. i Okeana, No. 6, 595–605 (1966).