EXPERIMENTAL VERIFICATION OF A MODEL FOR THE SPECTRAL TENSOR OF THE WIND VELOCITY FIELD

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A series of experiments was conducted in order to study in detail the turbulence structure of the lower atmosphere using several computerized acoustic weather stations. The data accumulation was performed in the process of automated measurements using different spatial arrangement of the stations in horizontal and vertical planes. The possibilities of the model description of the spectral tensor of the wind velocity field fluctuations approximated by homogeneous anisotropic turbulence were studied based on the experimental data obtained. We have done model calculations and compared predicted coherence with that measured experimentally.

Detailed structure of small-scale turbulence of lower layers of the atmosphere is interesting from different points of view. There is a wide scope of applied problems requiring adequate description of the spatial spectrum of turbulent pulsations of wind velocity in the surface layer. Information of such a kind is necessary, for instance, in the problems on propagation of optical and radio waves in the atmosphere, in solving problems of acoustic and lidar sounding, for taking account of pollutant dispersal in ecological problems, for estimation of spatial variability of wind pressure in calculating constructional stresses, etc.

According to modern experimental and theoretical data, the three-dimensional spatial spectrum of turbulent pulsations of wind velocity is a tensor of the second rank.

In the approximation of isotropic turbulence,[1] the spectral tensor $\Phi_{ij}(\mathbf{k})$ is completely determined by a single scalar function of the wave number $k = |\mathbf{k}|$

$$\Phi_{ij}(\mathbf{k}) = (4\pi k^2)^{-1}\{\delta_{ij} - k_i\, k_j\, k^{-2}\}E(k)\ , \tag{1}$$

namely, by the energy spectrum $E(k)$ for which the Kolmogorov–Obukhov law of "five thirds": $E(k) \sim \varepsilon^{2/3}k^{-5/3}$, where ε is the dissipation rate of the turbulence kinetic energy, should hold for reasons of dimensionality.

The isotropic approximation in the surface layer sufficiently well works in the inertial interval of the wave number. However, the turbulence becomes anisotropic with the increase of spatial scales and approaching the energy interval of the spectrum.[2] In fact, when the distance between the observation points of wind velocity fluctuations is comparable with the outer turbulence scale, the hypothesis on the spatial isotropy of fluctuations fails. So it is necessary to construct better models for the spectral tensor.

According to the Monin–Obukhov similarity theory, the dimensionless spectra of fluctuations of wind velocity vector components in the surface layer of the atmosphere at the height z are universal functions of the dimensionless wave number kz and the stratification parameter z/L, where L is the Obukhov scale. This means that specified characteristic frequencies corresponding to spectral maxima, low-frequency boundaries of the inertial interval, etc. depend on the stratification.[3] Thus, the similarity theory permits one to make some concrete conclusions about the shape of the spectra under different states of the surface layer stability. Thus, the attempts to take into account some universal regularities in the surface layer by the similarity theory and experimental data are quite urgent in simulation of the spectral tensor.

The kinematic model of turbulence developed in Ref. 4 is of special interest among the investigations on this problem. It is based on the representation of the spatial spectrum of the velocity field fluctuations as an anisotropic tensor $\Phi_{ij}(\mathbf{k})$. The following form is proposed in the above-mentioned paper for the spectral tensor in the approximation of a homogeneous incompressible turbulent flow:

$$\Phi_{ij}(\mathbf{k}) = \sum_{l=1}^{3} A_l(k)\{\delta_{li} - k_l\, k_i\, k^{-2}\}\{\delta_{lj} - k_l\, k_j\, k^{-2}\}. \tag{2}$$

In contrast to the case of isotropic turbulence (1), the description of the tensor (2) needs three independent real scalar functions $A_1(k)$, $A_2(k)$, $A_3(k)$ determining the energy of turbulent vortices along three orthogonal directions assigned by unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{i}_3$.

Variances σ_u^2, σ_v^2, σ_w^2, integral scales l_u, l_v, l_w, and dimensionless parameters μ_u, μ_v, μ_w characterizing the

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spectrum inclination in the transition range from the inertial interval to the energy interval of wave numbers are used in Ref. 4 as the parameters describing the behavior of the spectra of longitudinal, transverse, and vertical components u, v, w of the velocity. Here, the analytical expressions for one-dimensional spatial spectra of fluctuations of the velocity components have the form

$$F_u(k) = \frac{l_u\,\sigma_u^2}{\pi}\left\{1 + \left(\frac{l_u\,k}{a(\mu_u)}\right)^{2\mu_u}\right\}^{-\frac{5}{6\mu_u}},$$

$$F_v(k) = \frac{l_v\,\sigma_v^2}{2\pi}\left\{1 + \frac{8}{3}\left(\frac{l_v\,k}{a(\mu_v)}\right)^{2\mu_v}\right\}\left\{1 + \left(\frac{l_v\,k}{a(\mu_v)}\right)^{2\mu_v}\right\}^{-\frac{5}{6\mu_v}-1}, \quad (3)$$

where $a(\mu) = \{\pi\mu c[5/6\mu]\}/\{c(1/2\mu)c[(1/3\mu)]\}$ is a dimensionless constant determined by the normalization condition $\sigma^2 = \int\limits_{-\infty}^{\infty} F(k)\mathrm{d}k$ for each concrete value; k is the wave number along the direction of the mean horizontal velocity. The expression for $F_w(k)$ is similar to that for $F_v(k)$ given the corresponding change of indices.

Thus, one uses nine parameters as "input" information for constructing the model of tensor (2). The analytical dependences of A-functions on these parameters proposed in Ref. 4 are not presented here because they are too cumbersome. In our opinion, an advantage of the approach considered is that the input parameters of the model can be obtained in two ways: first, by model assignment depending on the conditions of stratification and scaling on the basis of similarity theory (here, one can use data already published in order to reach more high level of generalization); second, by direct determination of the unknown parameters on the basis of approximation of the experimentally measured spectra by expressions (3) according to the least-squares method followed by calculation of A-functions and the tensor (2). The implied possibility of a model description of the spatial structure of homogeneous anisotropic turbulence with the exclusion of the necessity to perform measurements at many points is very attractive from the practical point of view. But the model considered has some shortcomings. In particular, it cannot guarantee positive definiteness of the tensor (2) for arbitrary input parameters.

The coherence function

$$c_{ij}^2(\mathbf{R}, \omega) = |W_{ij}(\mathbf{R}, \omega)|^2 / |W_{ij}(0, \omega)\, W_{ji}(0, \omega)|, \quad (4)$$

where $W_{ij}(\mathbf{R}, \omega) = \dfrac{1}{2\pi}\int\limits_{-\infty}^{\infty} B_{ij}(\mathbf{R}, \tau)\exp(-i\omega\tau)\mathrm{d}\tau$ is the mutual spectrum of fluctuations at two spatially separated points, is one of the most informative characteristics among others being used to describe statistical laws of the spatial structure of the random wind velocity field.

Let the observation points be separated in a horizontal plane by the distance R normally to the mean wind direction. Using the hypothesis of frozen turbulence and expansion into the three-dimensional spatial spectrum for the spatiotemporal correlation function $B_{ij}(\mathbf{R}, \tau)$ and the model representation (2), one can obtain the following expressions[4] for the mutual spectra of longitudinal, transversal, and vertical velocity components:

$$W_{11}(R, \omega) = 2\pi v^{-1}\int\limits_0^{\infty} k^3\,\kappa^{-4}\{k^2 J_0(Rk)\, A_1(\kappa) +$$
$$+ \omega^2 v^{-2}\,[J_0(Rk) - R^{-1}\,k^{-1}\,J_1(Rk)]A_2(\kappa) +$$
$$+ \omega^2 v^{-2}\,R^{-1}\,k^{-1}\,J_1(Rk)A_3(\kappa)\}\mathrm{d}k;$$

$$W_{22}(R, \omega) = 2\pi v^{-1}\int\limits_0^{\infty} k\,\kappa^{-4}\{\,\omega^2 v^{-2}\,k^2[J_0(Rk) -$$
$$- R^{-1}\,k^{-1}\,J_1(Rk)]A_1(\kappa) + [(\omega^4 v^{-4} - 3R^{-2}\,k^2)J_0(Rk) +$$
$$+ 2R^{-1}\,k(\omega^2 v^{-2} + 3R^{-2})J_1(Rk)]A_2(\kappa) +$$
$$+ R^{-2}\,k^2[3J_0(Rk) + (R^2 k^2 - 6)R^{-1}\,k^{-1}\,J_1(Rk)]A_3(\kappa)\}\mathrm{d}k;$$

$$W_{33}(R, \omega) = 2\pi v^{-1}\int\limits_0^{\infty} k\,\kappa^{-4}\{\,\omega^2 v^{-2} R^{-1}\,k\,J_1(Rk)\,A_1(\kappa) +$$
$$+ R^{-2}\,k^2[3J_0(Rk) + (R^2\,k^2 - 6)R^{-1}\,k^{-1}\,J_1(Rk)]A_2(\kappa) +$$
$$+ [(\kappa^4 - 3\,R^{-2}\,k^2)J_0(Rk) - 2(\kappa^2\,k^2 - 3\,R^{-2}\,k^2)\times$$
$$\times R^{-1}\,k^{-1}\,J_1(Rk)]\,A_3(\kappa)\}\mathrm{d}k, \quad (5)$$

where $k = \sqrt{k^2 + \omega^2 v^{-2}}$; $J_n(x)$ are Bessel functions of the nth order.

Thus we can calculate coherence, i.e., obtain information about the spatial structure of an anisotropic wind velocity field as a whole within the frames of the homogeneous turbulence approximation by using the parameters of velocity component autocorrelation spectra measured experimentally at a single point as an input information.

Since most of the investigations are restricted by the use of isotropic turbulence approximation, experimental measurements of coherence are not enough complete to perform detailed comparison with the model prediction.

The aim of the study presented in this paper is to experimentally verify the spectral tensor model[4] which is not isotropic but includes isotropy as a particular case.

The acoustic weather station,[5] which is a compact program-simulated device and enables one to obtain data about each component of the wind velocity, temperature, and their fluctuations at frequency up to 20 Hz and to measure air pressure and humidity, was the principal measuring instrument in the experiments performed. Two to three such weather complexes controlled by a personal computer were used in the experiments, and this made it possible to realize long-term automated measurements and data storage. Different spatial arrangements of the weather stations both in the horizontal and a vertical planes were used in different experiments. The processing of measurement data included the calculation of autocorrelation spectra and coherence spectra for all the three velocity components, friction rate, temperature scale, flows of momentum and heat, Obukhov scale, and Richardson's number what made it possible to monitor the state of the surface layer by stratification conditions. Some preliminary results of the investigations are published in Ref. 6.

In general, the investigations performed confirm the existence of strong anisotropy of the wind velocity field fluctuations in the surface layer and the dependence of the pulsation energy distribution over the space of wave numbers on the shape of the corresponding one-dimensional spectra $F_u(k)$, $F_v(k)$, $F_w(k)$. At the same time, the A-functions calculated in accordance with the model prove to be negative in the region of small wave numbers in some cases for particular parameters of the spectra σ^2, l, μ obtained experimentally. This leads to the break of the positive definiteness condition of the tensor $\Phi_{ij}(\mathbf{k})$, and we obtain coherence greater than unity for the corresponding velocity component.

This discrepancy is caused by the drawbacks of the model related to the fact that, in real experiments, one often observes nonstationarity caused by daily radiation fluctuations and leading to change of the heat flow H depending on time and height. This effect is especially strong under the spacing R considerably exceeding the value of the outer turbulence scale. However, the contribution of negative values of A_3 corresponding to large-size vortices decreases for horizontal spacing equal or somewhat less than l_u, and the model coherence does not exceed unity.

The spectra of wind velocity components measured over the plain underlying surface at midday time in summer under unstable stratification of the surface layer for $z/L = -0.064$ and normalized by the variance are presented, as an example, in Fig. 1. The sample contains 8192 synchronously measured values of three wind velocity components and temperature. The measurements were performed at a rate of 10 Hz during 13.65 min.

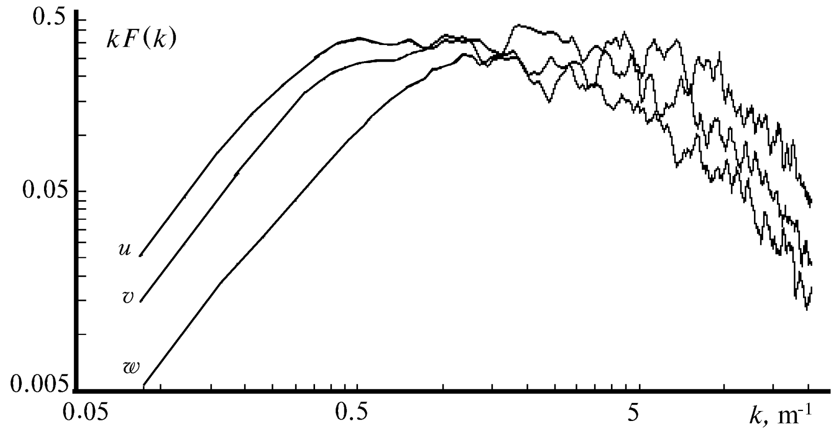

FIG. 1. One-dimensional spectra for the longitudinal, transverse, and vertical wind velocity components, u, v, w, obtained experimentally.

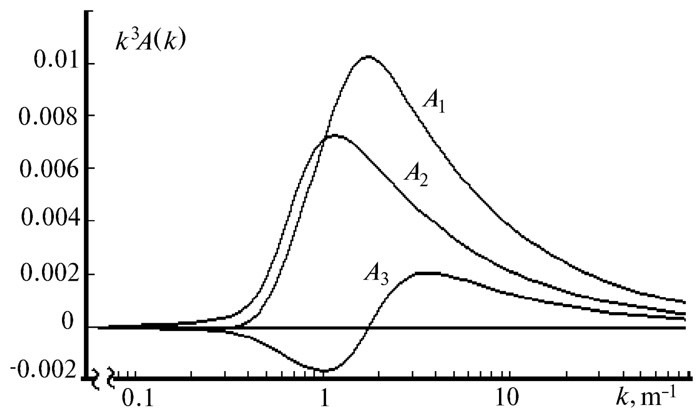

FIG. 2. Three model A-functions determining the spectral tensor, calculated for the spectra depicted in Fig. 1.

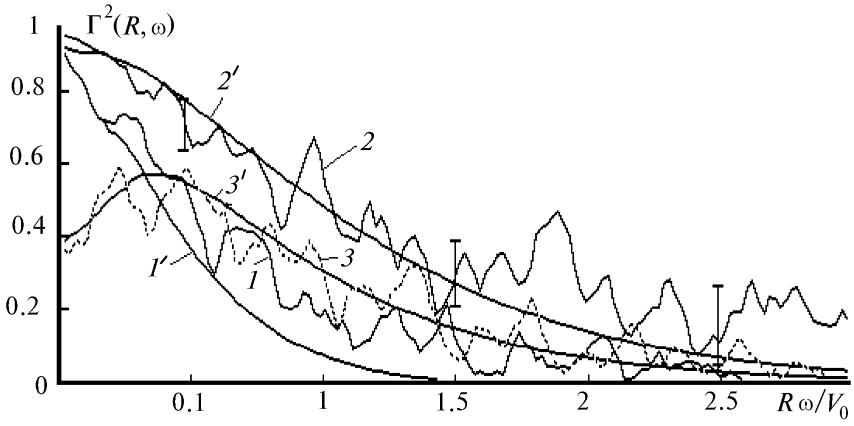

FIG. 3. Coherence spectra for longitudinal (1, 1′), transverse (2, 2′), and vertical (3, 3′) wind velocity components obtained experimentally (curves 1–3) and calculated by the model (1′–3′).

The whole sample was divided into 16 sequential intervals containing 512 points each in order to increase the degree of freedom in the spectral processing. Linear filtration of data was carried out using the approximation with a polynomial of the first degree followed by subtraction from the initial data array. So the trend effects in the wind velocity were excluded. The spectra calculated for each interval were summed up and normalized by the sample variance. The resulted spectra were smoothed over five neighboring points. According to our estimations, the relative error of the spectrum estimation using this processing does not exceed 11%. The values of wind velocity components averaged over measurement time were as follows: $V_u = 1.446$ m/s, $V_v = 0.000$ m/s, $V_w = -0.086$ m/s. Spectral parameters obtained by the least-squares method for these velocities are equal to

$\sigma_u^2 = 0.190$ m^2/s^2 $\sigma_v^2 = 0.145$ m^2/s^2 $\sigma_w^2 = 0.056$ m^2/s^2
$l_u = 1.074$ m $l_v = 1.095$ m $l_w = 0.426$ m
$\mu_u = 1.2$ $\mu_v = 1.5$ $\mu_w = 1.61$

Figure 2 presents three *A*-functions determining the spectral tensor (2) and calculated for the above-mentioned parameters in correspondence with the model.[4] Figure 3 presents the coherence spectra calculated by formulas (4) and (5), and those obtained experimentally at the transverse horizontal separation $R = 0.5$ m. A little excess of the experimental coherence over the calculated one can be explained, on the one hand, by the shortcomings of the model restricted by the hypotheses of frozen and homogeneous turbulence and, on the other hand, by errors in determining input parameters of the model and in performing the model calculations. Since the model considered imposes increased requirements on the correctness of statistical processing of the measurement data, it is necessary to eliminate low-frequency trends in the initial temporal series of data in order to increase the reliability of spectral estimations in the region of low frequencies.

On the basis of a good coincidence of the predicted and the experimentally measured coherence, one can come to a conclusion that the model considered can be useful in estimating spatial statistics of turbulent pulsations using information obtained at a single point of the space when the conditions of the experiment correspond to the accepted hypotheses.

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