DETERMINATION OF OPTICAL PARAMETERS OF AN OPTICALLY THICK HOMOGENEOUS CLOUD LAYER

I.N. Mel'nikova and P.I. Domnin

Scientific-Research Physical Institute at the State University, St. Petersburg
Received July 5, 1996

We present here rigorous analytical formulas derived from asymptotic formulas of the radiation transfer theory for determination of optical thickness and single scattering albedo on the basis of measurements of reflected or transmitted solar radiance or irradiance in stratus clouds. The detailed analysis is carried out of the error of the methodology proposed. The formulas derived are applicable to the interpretation of spectral measurements of solar radiation under continuous cloudiness carried out at the SP–22 *Arctic station in summer 1979. The values of optical thickness and single scattering albedo revealed a pronounced spectral dependence.*

The "excess" absorption of the shortwave solar radiation by clouds (especially stratus clouds) revealed in recent years can significantly affect the energy and dynamic regimes of the atmosphere, and this fact imposes stringent requirements upon the adequacy of the cloud models used in the problems of prediction.[1] So far the optical and radiative models of cloudiness were based on theoretical calculations using the Mie theory and radiation transfer theory, but the aforementioned excess extinction in clouds is not satisfactorily explained in the frameworks of such an approach. For these reasons, the problem of determination of the optical parameters of actual stratus clouds is now very urgent. It was shown that airborne measurements of solar radiations at the boundaries and inside the cloud layer appeared to be quite effective for determining optical parameters (scattering and true absorption coefficients) of the cloud.[2]

In this paper we propose an analytical method for determining the optical thickness and the single scattering albedo of a horizontally extended homogeneous cloud layer on the basis of measurements of reflected radiation (satellite measurements) or transmitted radiation (ground-based measurements). Most serious limitations are imposed on the spatial homogeneity of the cloud field and its stability.

One should emphasize that the interpretation of radiative observations based on the radiation transfer theory needs for spectral measurements. Then one can use measurements of hemispherical upwelling and downwelling radiation fluxes or measurements of the intensity. Performance of the method is based on the asymptotical formulas of the transfer theory,[3] which are the exact solution to the radiation transfer equation for the scattering media of a large optical thickness (characteristic of the stratus cloudiness).

MODEL OF THE STRATUS CLOUD AND BASIC FORMULAS

Let us consider the model of a cloud layer, infinite and homogeneous in the horizontal direction, of a large optical thickness $\tau \gg 1$. The parallel flux of solar radiation πS is incident on the upper boundary at an angle $\arccos\zeta$. The intensity of reflected and transmitted radiation is observed under the sighting angle $\arccos\eta$.

The scattering and absorption properties of the cloud layer are described by the following optical parameters: optical thickness of the layer τ; the single scattering albedo ω_0 and the scattering phase function that is characterized by the asymmetry parameter (mean cosine) g. The cloud layer leans on the underlying surface with the albedo A. Let us neglect the scattering of light in the layer between the ground surface and the cloud layer. The relative intensities of the reflected and transmitted radiation are described by the reflection $\rho(\tau, \eta, \zeta)$ and transmission $\sigma(\tau, \eta, \zeta)$ coefficients which for an optically thick layer are expressed by the asymptotical formulas of the radiation transfer theory

$$\rho(\tau, \eta, \zeta) = \rho_\infty(\eta, \zeta) - \frac{M\,\bar{N}\,u(\eta)\,u(\zeta)\,e^{-2k\tau}}{1 - N\,\bar{N}\,e^{-2k\tau}};$$

$$\sigma(\tau, \eta, \zeta) = \frac{M\,\bar{u}(\eta)\,u(\zeta)\,e^{-k\tau}}{1 - N\,\bar{N}\,e^{-2k\tau}}, \qquad (1)$$

where $\rho_\infty(\eta, \zeta)$ is the reflection coefficient of the semiinfinite medium; the function $u(\zeta)$ describes the angular dependence of the radiation intensity; the values M, N and k are the constants determined by the scattering layer properties and in the case of a weak (as compared to scattering) true absorption ($1 - \omega_0 \ll 1$, that is characteristic of light scattering in clouds in the visible spectral range) are described with expansions in terms of a small parameter s, where $s^2 = (1 - \omega_0)/[3(1 - g)]$.

The reflection of light from the underlying surface with the albedo A is taken into account by the terms marked with the bar above them and described in Ref. 3.

Let one measure the intensity of reflected radiation ρ_1 and ρ_2 at two sighting angles η_1 and η_2, respectively. By applying the first of the formulas (1) at two values η, and considering the ratio $(\rho_\infty(\eta_1, \zeta) - \rho_1)/(\rho_\infty(\eta_2, \zeta) - \rho_2)$ (in what follows we will omit the arguments of the measured values for the sake of reduction of space) and attracting known expansions of the functions $\rho_\infty(\eta_1, \zeta)$, $u(\zeta)$, and $a(\zeta)$ in to a series over the small parameter s

$$u(\zeta) = u_0(\zeta)\,(1 - 1.5\delta s) + u_2(\zeta)\, s^2 + \dots$$

$$a(\zeta) = 1 - 4\, u_0(\zeta)\, s + a_2(\zeta)\, s^2 + \dots$$

$$\rho_\infty(\eta, \zeta) = \rho_0(\eta, \zeta) - 4\, u_0(\zeta)\, u_0(\eta)\, s + \rho_2(\eta, \zeta)\, s^2 + \dots, \quad (2)$$

where $u_2(\zeta) = Q_2 u_0(\zeta) w(\zeta)$ and $w(\zeta) = \zeta^2 + 0.43$, we obtain the expressions for the values s^2 and $\tau' = 3(1 - g)\tau$

$$s^2 = \frac{[\rho_0(\eta_1\zeta) - \rho_1]u_0(\eta_2) - [\rho_0(\eta_2\,\zeta) - \rho_2]\, u_0(\eta_1)}{[\rho(\eta_2\,\zeta) - \rho_2](w(\eta_1) - w(\eta_2))\, Q_2 - \dfrac{a_2(\zeta)}{6\delta}[u_0(\eta_1)a_2(\eta_2) - u_0(\eta_2)a_2(\eta_1)]};$$

$$\tau' = (2s)^{-1} \ln \left\{ \frac{M\, \bar{N}\, u(\eta_1)\, u(\zeta)}{\rho_\infty(\eta_1, \zeta) - \rho_1} + N\, \bar{N} \right\}. \quad (3)$$

These formulas include only the measured values ρ_1 and ρ_2 and the values of the function $u_0(\zeta)$ at the fixed angles ζ, η_1 and η_2, that can be determined from the tables (see Refs. 3 and 4) or calculated by formulas (see Ref. 5). The proposed technique for determining the optical parameters of a cloud layer may be very useful for interpretation of satellite measurements of the solar radiation.

In the case of interpretation *of ground-based measurements of the intensity of the transmitted solar radiation* the use of similar techniques gives rise to the following expressions for s^2 and τ':

$$s^2 = \left[\frac{\sigma_1\, \bar{u}_0(\eta_2)}{\sigma_2\, \bar{u}_0(\eta_1)} - 1\right] \frac{1}{(w(\eta_1) - w(\eta_2))\, Q_2};$$

$$\tau' = s^{-2} \ln \left[\left(\sqrt{\frac{r^2}{N\bar{N}} + 1} + 1 \right) \Big/ r \right],$$

where

$$r = \frac{2\,\sigma_1\, N\, \bar{N}}{M\, \bar{u}(\eta_1)\, u(\zeta)}. \quad (4)$$

Let us note that in order to determine the value s^2 it is sufficient to measure the transmitted radiation flux in relative units, and to reconstruct the values of the optical thickness, the radiative measurements are needed in the unit of the solar radiation flux incident on the upper boundary of the atmosphere, and so the calibration of the instrument is necessary by the direct solar radiation under conditions of cloudless atmosphere.

Errors and the applicability limits. One can derive the formulas for the errors, as well as the errors of indirect measurements, by subsequent taking logarithms and differentiation of basic formulas. When interpreting the transmitted radiation measurements, the formula for the error $\mathrm{d}s/s$ has the form

$$\frac{\mathrm{d}s}{s} = \frac{\mathrm{d}\sigma\, \bar{u}_0(\eta) + \sigma\, \mathrm{d}\bar{u}_0}{\sigma_1\, \bar{u}_0(\eta_2) - \sigma_2\, \bar{u}_0(\eta_1)} + \frac{\mathrm{d}\sigma}{2\sigma} + \frac{\mathrm{d}\bar{u}_0}{2\, \bar{u}_0} + {}$$

$$+ \frac{2\, \mathrm{d}w(\eta)\, Q_2 + \mathrm{d}Q_2\, (w(\eta_1) + w(\eta_2))}{Q_2^2\, (w(\eta_1) + w(\eta_2))^2}. \quad (5)$$

The expression for the relative error of the optical thickness is derived similarly from the basic formula for τ

$$\frac{\mathrm{d}\tau}{\tau} = \frac{\mathrm{d}s}{s} + \frac{1}{\tau} \left\{ \frac{\mathrm{d}r}{r} + \left[\frac{r^2}{N\bar{N}} \left(2\frac{\mathrm{d}r}{r} + \frac{\mathrm{d}N}{N} + \frac{\mathrm{d}\bar{N}}{\bar{N}} \right) \right] \Big/ \left[2\sqrt{\frac{r^2}{N\bar{N}} + 1} \left(\sqrt{\frac{r^2}{N\bar{N}} + 1} + 1 \right) \right] \right\}, \quad (6)$$

where, taking into account the relationship (4), we have

$$\frac{dr}{r} = \frac{d\sigma}{\sigma} + \frac{dN}{N} + \frac{d\bar{N}}{\bar{N}} + \frac{dM}{M} + \frac{d\bar{u}}{\bar{u}} + \frac{du}{u} . \qquad (7)$$

The errors in calculations of N, M and the function $u(\zeta)$ are determined by the value of terms $\sim s^3$ on which the asymptotic expansions are truncated.

The errors in calculating the values of the functions $w(\zeta)$ and $a_2(\zeta)$ are calculated by the formulas

$$dw = \frac{\eta d\eta}{\eta + 0.2} ; \; da_2 = 3\, du_0\, a_2 + \frac{9\, dgu_0}{(1+g)^2} + \frac{9\, gu_0}{(1+g)} d\eta. \qquad (8)$$

The numerical analysis made has show that the first and the last terms of Eq. (5) make the greatest contribution. One can diminish them by selecting the optimum pairs of angles η_1 and η_2.

To improve the accuracy, one should use 2 or 3 pairs of angles and calculate the mean value of the parameter s. It is important that measurements of the transmitted radiation at several sighting angles are useful for the control of homogeneity and optical thickness of a cloud layer under investigation, as well as the measurements at several azimuth angles. The absence of the azimuth dependence of the transmitted radiation and the dependence on the sighting angle, qualitatively corresponding to the function $u(\eta)$, are indicative of the horizontal homogeneity of the cloud layer and its large optical thickness, that allows one to apply the method proposed. The numerical estimates of the terms of Eq. (6) reveal strong effect of the albedo of the underlying surface on the accuracy of determination of the optical thickness. In this case measurements carried out along several sighting directions noticeably decrease the error in reconstructing τ. In the general case it occurs that $d\tau/\tau \approx 10–25$ % at the measurement error about 2% and albedo of the underlying surface from 0.5 up to 0.9.

Let us note that the error in determining the values s and ω_0 are related by the relationship $d\omega_0/\omega_0 = (6s ds(1-g) + 3dgs^2)/(1-3s^2(1-g))$, then $d\omega_0/\omega_0 \sim 0.0001 - 0.0007$. The matter is that the value ω_0 is known *a priori* accurate to 2–3 first digits after decimal point, that determines the so called "*a priori* error." So the reduced value $d\omega_0/\omega_0$ is indicative of the fact that the 3rd and 4th digits after decimal point in the value ω_0 are determined with the error of ~ 5–10 %.

Interpretation of the data of measurements of the hemispheric fluxes of reflected or transmitted radiation. Use of data on the flux measurements carried out at different moments in time (at different zenith angles of the Sun ζ_1 and ζ_2), in the case **of the reflected flux** gives rise to the expression

$$s^2 = \frac{\{[(a(\zeta_1) - F_1^{\uparrow})\, u_0(\zeta_2)]/[(a(\zeta_2) - F_2^{\uparrow})\, u_0(\zeta_1)] - 1\}}{(w(\eta_1) - w(\eta_2))\, Q_2} ;$$

$$\tau' = \frac{1}{2s} \ln \left\{ \frac{M\, \bar{N}\, Q\, u(\zeta)}{a(\zeta) - F^{\uparrow}} + N\, \bar{N} \right\} \qquad (9)$$

and in the case **of the transmitted flux**

$$s^2 = \frac{\{F_1^{\downarrow}\, u_0(\zeta_2)/[F_2^{\downarrow}\, u_0(\zeta_1)] - 1\}}{(w(\eta_1) - w(\eta_2))\, Q_2} ;$$

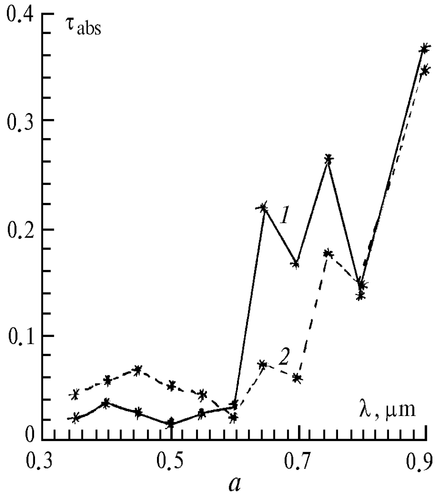
$$\tau' = s^{-1} \ln \left[\frac{\sqrt{r^2/(N\bar{N}) + 1} + 1}{r} \right] , \qquad (10)$$

where the value $r = 2\, F^{\downarrow}\, N\, \bar{N}/(M\, \bar{Q}\, u(\zeta))$ is introduced.

The basis for all the results obtained are the asymptotic formulas of the radiation transfer theory, which have limited applicability, i.e., the optical thickness of the layer should not be less than 10 – 15, and the asymptotical expansions over the small parameter s which imposes the limitations on the true absorption in the layer (the value of the single scattering albedo should be within the limits $\omega_0 \geq 0.98$).

Determination of the mean cosine of the scattering phase function. If one has optimum conditions for observations (stability, homogeneity and large optical thickness of the cloud layer, small albedo of the underlying surface and high accuracy of measurements) providing the minimum possible errors, one can apply the following method for estimating the mean cosine of the scattering phase function g. Let us consider two pairs of measurements of the radiation transmitted through a cloud layer carried out according to the aforementioned procedure. The parameter s^2 is determined by the first pair. The second pair allows one to determine the value $U = [u_2(\zeta_1)/u_0(\zeta_1)] - [u_2(\zeta_2)/u_0(\zeta_2)]$. The numerical analysis of tabulated values of the functions $u_0(\zeta)$ and $u(\zeta)$ presented in Ref. 4 shows that it is possible to approximate the function U by linear dependence on the parameter g and on the difference $\zeta_1 - \zeta_2$ with the accuracy of 2%: $U = 5.57 \times (\zeta_1 - \zeta_2)g$ and to obtain the estimate of the scattering phase function parameter g that earlier was usually set *a priori* on the basis of assumptions on the cloud droplet size and calculations by the Mie theory.

Results of measurements and their interpretation. Ground-based measurements of solar radiation were carried out at the SP–22 Arctic drifting station in summer and fall of 1979 under conditions of continuous stratus cloudiness (August 13, 1979 and October 8, 1979) and under cloudless conditions (June 16, 1979).[6] The latter measurements allowed us to calibrate the instrument using solar radiation, that made it possible to determine the optical thickness of the cloud layer. The angular resolution of the instrument was ~ 2°. The relative error of the intensity measurements of the scattered solar radiation did not exceed 5 %. There are results available for 11 wavelengths. The pairs of the intensity values at each wavelength and at a pair of angles η were processed according to the technique proposed above. The measurements were carried out along 5 sighting directions, that made it possible to compose several pairs of angles. The parameters sought were determined for each pair, and then the mean values s^2 and τ were calculated, and the parameter g estimated. The values of the optical thickness τ and the single scattering albedo ω_0 were calculated taking into account the refined value of g. The results are presented in the Table I and in Figure 1. The true absorption in the layer is characterized by the value of the optical thickness of absorption $\tau_{abs} \sim 0.02–0.06$, that is in agreement with the value obtained earlier when interpreting the airborne measurements.[2] The values ω_0 are indicative of the strong absorption in the cloud layer in the visible spectral range and are an evidence of the presence of "anomalous" absorption. The total optical thickness τ has the value characteristic of the stratus (~ 20) and reveals a well pronounced spectral behavior analogous to that obtained earlier from the analysis of airborne radiative experements.[2]



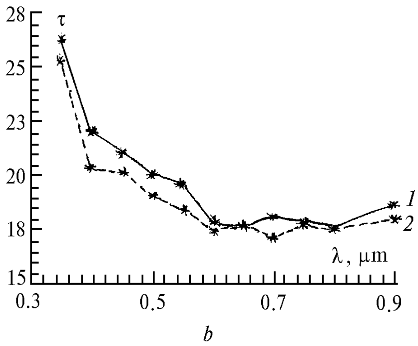


FIG. 1. Spectral dependences of the optical thickness of the true absorption (a) and the optical thickness of scattering (b) obtained from the data of ground-based radiative measurements at the SP–22 *drifting station: 1) August 1 to 13, 1979; 2) October 2 to 8, 1979.*

TABLE I. Optical parameters of the cloud layer determined on the basis of ground-based measurements of the intensity of solar radiation.

	13.08.79			08.10.79		
λ, μm	ω_0	τ	g	ω_0	τ	g
0.35	0.9989	25.5	0.88	1.0	26.2	0.83
0.40	0.9981	22.2	0.85	1.0	20.6	0.85
0.45	0.9987	21.0	0.85	0.9963	20.0	0.82
0.50	0.9994	20.2	0.83	0.9985	19.3	0.82
0.55	0.9990	19.7	0.85	0.9974	18.5	0.80
0.60	0.9985	17.6	0.82	0.9987	17.4	0.79
0.65	0.9907	17.3	0.82	0.9957	17.7	0.81
0.70	0.9930	17.8	0.83	0.9968	17.0	0.77
0.75	0.9894	16.5	0.80	0.9921	17.6	0.80
0.80	0.9919	17.1	–	0.9919	17.4	0.78
0.90	0.9844	14.9	0.81	0.9868	18.0	0.75

CONCLUSION

The method proposed for determining the optical parameters of stratus cloudiness from measurements of solar radiation is based on rigorous analytical formulas of the radiation transfer theory and is convenient for application to the ground-based or satellite radiative monitoring of the cloudy atmosphere.

Analysis of the errors of the method shows that when using measurements with the accuracy of modern instruments, the cloud parameters characterizing the true absorption and scattering of light can be reconstructed with the error of 6 – 15%.

The measurements of the radiation fluxes provide higher accuracy as compared with the intensity measurements, but requires stability of the meteorological conditions.

The possibility is proposed for the first time of estimating the asymmetry parameter of the scattering phase function from measurements of scattered solar radiation.

The results obtained when interpreting radiative measurements in Arctic confirm the presence of significant absorption of light in a cloud, and strong spectral dependence of the optical thickness revealed earlier in other cloud layers by different method from airborne radiative measurements.[2]

REFERENCES

1. R.D. Cess, M.H. Zhang, P. Minnis, et. al., Science **267**, 496–499 (1995).
2. I.N. Mel'nikova and V.V. Mikhailov, J. Atmos. Sci. **51**, No 4. 925–931(1994).
3. I.N. Minin, *Theory of Radiation Transfer in Planet Atmospheres* (Nauka, Moscow, 1988), 264 pp.
4. J.M. Dlugach and E.G. Yanovitskii, Icarus **22**, 66–81 (1974).
5. I.N. Mel'nikova, Atmos. Oceanic Opt. **5**, No. 2, 169–177 (1993).

6. V.F. Radionov, G.G. Sakunov, and V.S. Grishechkin, in: *First Global Experiment PIGAP. Part 2. Polar Aerosol, Extended Cloudiness and Radiation* (Gidrometeoizdat, Leningrad, 1981), pp. 89–91.