SPATIAL STATISTICAL STRUCTURE OF MESOSCALE FIELDS OF TEMPERATURE AND WIND

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In this paper we discuss some results of statistical analysis of spatial structure of mesoscale fields of temperature, zonal and meridional winds carried out based on the data on typically mesometeorological polygons in the near ground layer and at tropospheric levels. Along with a consideration of peculiarities of spatial correlation functions of temperature, zonal and meridional winds the results of their approximation to 560 km *distance are analyzed using analytical expressions derived.*

1. INTRODUCTION

Among numerous important problems of modern mesometeorology the problem of statistical analysis of spatial structure of mesometeorological fields should be solved and, first, the problem of analysis and analytical approximation of empirical correlation functions obtained to a distance of 300 to 500 km, i.e., for the region where the role of mesoscale processes is great. This is conditioned by that the data on statistical mesostructure of meteorological fields and, especially, on spatial correlation functions are needed for when solving applied problems connected, for example, with:

– objective analysis of mesometeorological fields based on the method of optimal interpolation (extrapolation) that is widely used in the numerical schemes of local weather forecast and mesoscale prediction of evolution and transfer of atmospheric pollution[1–5];

– efficient networking of aerological stations on the local scale, the measurement data from which are normally used for ecological monitoring of air basins over limited territories with the horizontal size from tens to hundreds kilometers.

It should be mentioned here that in the numerical schemes of optimal interpolation (extrapolation) of the fields of meteorological quantities we do not normally use spatial correlation functions but only the results of their approximation by different analytical expressions (see, e.g., Refs. 2 and 6).

Another one urgent reason for solving the above problem is that the statistical structures of mesoscale meteorological fields have not been so far properly investigated in contrast to the well–studied large–scale structures of these fields.[2,7] The papers 8 and 9 consider the spatial correlation functions of the daily mean temperature and wind velocity calculated from the data of ground–based observations.

The above–mentioned facts count in favor of making special investigations into the problems of statistical analysis of spatial structure of the mesometeorological fields in order to obtain reliable data on the peculiarities in spatial structure within the limits of limited territories where the effects of mesoscale processes can be substantial (namely, processes with a typical scale from tens to hundreds kilometers[10]) as well as the data on spatial autocorrelation of a meteorological parameter. And the question should also be addressed on the analytical form of that autocorrelation.

In this paper we present such an analysis as applied to mesoscale fields of temperature and wind since these meteorological quantities play a leading part when estimating the dispersal of atmospheric contamination over a limited air basin (for example, over a big city, industrial region or the region as a whole) as well as when solving military geophysical tasks.

2. SOME METHODICAL PROBLEMS AND CHARACTERISTICS OF THE INPUT DATA

It is well known that the atmosphere is a turbulent medium and, consequently, meteorological fields are spatially random. Therefore, to reveal peculiarities of such fields, a statistical approach describing the fields is needed. Use of such an approach calls for refusal from the consideration of individual field properties and to take instead into account only most general regularities characteristic of the entire set of possible realizations of the field under study.

Note that of all the statistical characteristics, used at such an approach, we shall primarily study spatial correlation functions that is caused by the necessity of solving the problem of numerically extrapolating mesometeorological fields to the territory not covered by the observations (the methods and some results obtained using this approach are described in Ref. 11). Moreover, the study of such correlation functions is much easier because the mesometeorological fields are relatively homogeneous.[2] In view of this homogeneity these functions are more representative than the

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corresponding characteristics calculated for the large–scale meteorological fields.

Let us now consider some methodical aspects of determining spatial correlation functions of the meteorological quantities (in our case those are temperature, zonal and meridional components of the wind vector).

Let $\xi^j(\mathbf{r})$ be the jth realization of a random field (here $\mathbf{r}$ is the point radius–vector, the coordinates involve spatial coordinates and time). At N total realizations the statistical estimate of the normalized correlation function, for any pair of points $\mathbf{r}_i$ and $\mathbf{r}_k$, that is being the simplest characteristic of the autocorrelation of a meteorological quantity, can be done by the following expression:

$$\mu_\xi(\mathbf{r}_i, \mathbf{r}_k) = \frac{1}{N}\frac{\sum\limits_{j=1}^{N} [\xi^j(\mathbf{r}_i) - \bar{\xi}(\mathbf{r}_i)]\ [\xi^j(\mathbf{r}_k) - \bar{\xi}(\mathbf{r}_k)]}{\sigma_\xi(\mathbf{r}_i)\ \sigma_\xi(\mathbf{r}_k)}, \quad (1)$$

where $\bar{\xi}(\mathbf{r}_i)$ and $\bar{\xi}(\mathbf{r}_k)$ denote the mean value of the meteorological quantity ξ at the ith and kth points (in our case, observation stations); $\sigma_\xi(\mathbf{r}_i)$ and $\sigma_\xi(\mathbf{r}_k)$ denote the rms deviation of the same meteorological quantity at the same points.

We have used equation (1) for calculating spatial correlation functions of temperature and components of the wind vector. In this case the calculation of correlation functions for each pair of stations of mesometeorological polygon was made only for the cases when the data of aerological observations were available from all the stations.

It is clear that such an approach essentially reduced the initial data set. However, as the data analysis has shown, the total number of remaining synchronous (for all stations) observations N in every season (winter and summer) is no less than 220. This number of observations is sufficient (from the statistical stand–point) for obtaining reliable estimates of spatial correlation functions since, according to Ref. 2, for a qualitative estimates of these parameters it is sufficient to have 50 to 60 field realizations.

As to the procedure itself used for the determination of spatial correlation functions $\mu_\xi(\mathbf{r}_i, \mathbf{r}_k)$ by Eq. (1), it was realized not only for tropospheric levels but also for the levels within the atmospheric boundary layer, where these functions have not yet been calculated earlier.

To approximate the correlation functions obtained, that characterize the autocorrelation of the mesoscale fields of temperature and wind as well as to choose the best analytical functions, we have used the approximating expressions of the following types[2,6,12]:

$$\mu_\xi(\rho) = \exp(-\alpha\rho), \quad (2)$$

$$\mu_\xi(\rho) = \exp(-\alpha\rho^\beta), \quad (3)$$

$$\mu_\xi(\rho) = \{\exp(-\alpha\rho)\} \cos (\beta\rho), \quad (4)$$

$$\mu_\xi(\rho) = \{\exp(-\alpha\rho^\beta)\}\ I_0(\beta\rho), \quad (5)$$

$$\mu_\xi(\rho) = 1 - (\rho/\rho_0), \quad (6)$$

$$\mu_\xi(\rho) = (1 - \alpha\rho) \exp(-\alpha\rho), \quad (7)$$

where $\rho = |\mathbf{r}_i - \mathbf{r}_k|$ is the distance between the points (stations), in thousand km, ρ_0 = 750 km, α and β are the empirical coefficients; $I_0(\beta\rho)$ is the zero–order Bessel function.

Having selected, using Eqs. (2)–(7), the best approximations of spatial correlation functions of temperature, zonal and meridional wind, the analytical functions obtained were compared with the results of similar investigations made previously[2,6,12] for large–scale fields of the same meteorological values.

In conclusion let us briefly characterize characteristics of the input data set.

For statistical evaluation of the spatial correlation function of temperature, zonal and meridional wind we used the data arrays of long–term (1971–1975) radiosonde observations obtained from five aerological stations, namely, Warsaw (52°11'N, 20°58'E), Kaunas (54°53'N, 23°53'E), Brest (52°07'N, 23°41'E), Minsk (53°11'N, 27°32E), and L'vov (49°48'N, 23°57'E), being a typical mesometeorological polygon (its schematic representation is given in Fig. 1).

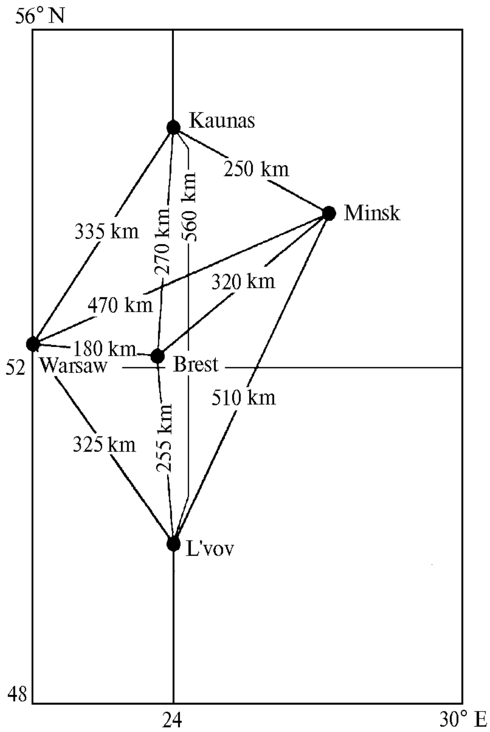


FIG. 1. Schematic representation of a typical mesometeorological polygon used for spatial extrapolation of temperature and wind fields.

All the data characterize two seasons (winter and summer) and the atmospheric layer up to 8 km height. In this case the data are presented in the system of geometrical heights including such levels as 0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 3.0, 4.0, 5.0, 6.0, and 8.0 km. The procedure of reducing these initial data as a system of geometrical heights was performed using linear interpolation of individual values of the meteorological quantities considered from the levels of singular points and basic isobaric surfaces to the heights chosen.

3. PECULIARITIES OF SPATIAL MESOSTRUCTURE OF TEMPERATURE AND WIND FIELDS

In this section we concentrate on the consideration of the results of statistical analysis of spatial mesostructure of temperature and wind fields carried out for a typical mesometeorological polygon. In this case we proceed from the assumption that:

– first, previous statistical investigations of the spatial structure of meteorological fields (see, e.g., Refs. 2, 7–9) mainly dealt with the horizontal correlation of these fields at the level of 500 hPa, more rarely in the ground layer and at other levels in the troposphere (850, 700, 300 hPa), the above statistical investigations have never been extended to the atmospheric boundary layer;

– second, up to now the spatial mesoscale structure of temperature fields, zonal and meridional wind has not practically been studied.

Taking into account both these cases, we consider the characteristics of spatial mesostructure of temperature fields and components of wind vector both in the atmospheric boundary layer and in the free atmosphere (up to the level of 8 km), at high altitude resolution.

Figures 2 to 4 show the spatial correlation functions of temperature, zonal and meridional wind obtained for the six standard heights: 0 (ground level); 0.8; 1.6; 3.0; 5.0; and 8.0 km representing the ground and boundary layer as well as the free atmosphere.

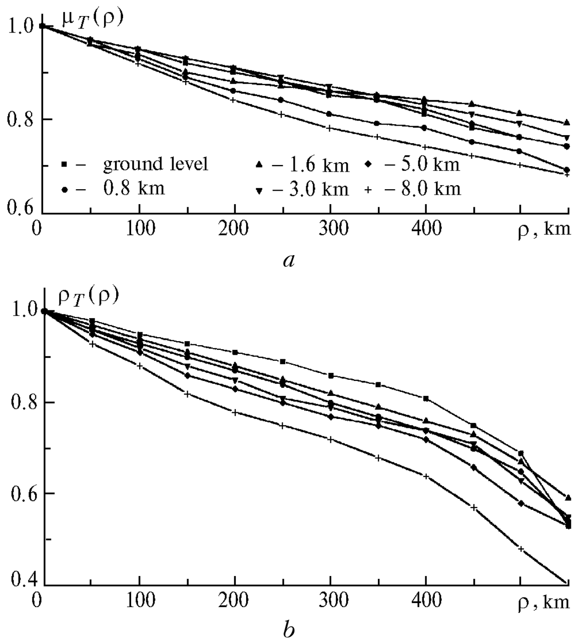


FIG. 2. Space correlation functions of temperature at different altitudes: winter (a), summer (b).

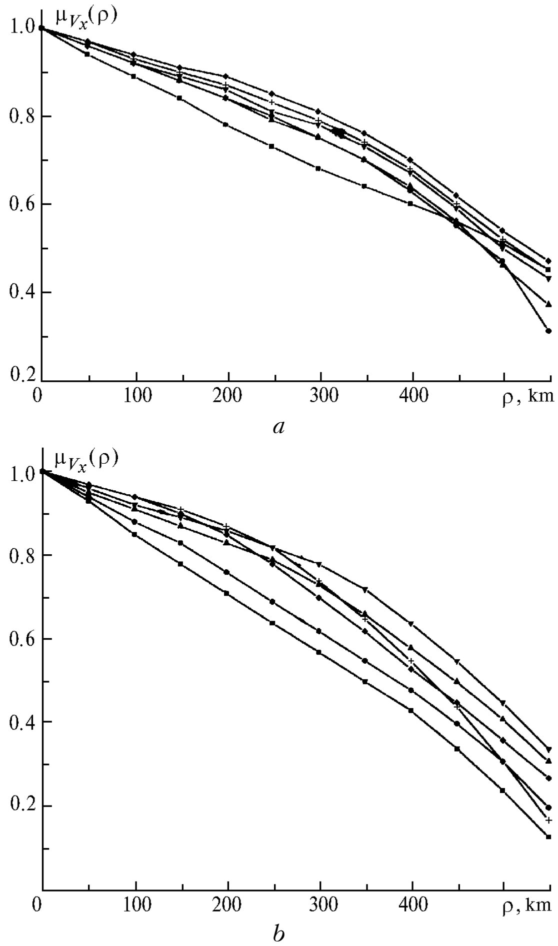


FIG. 3. Spatial correlation functions of zonal wind at different heights. The conventional symbols are given in Fig. 2.

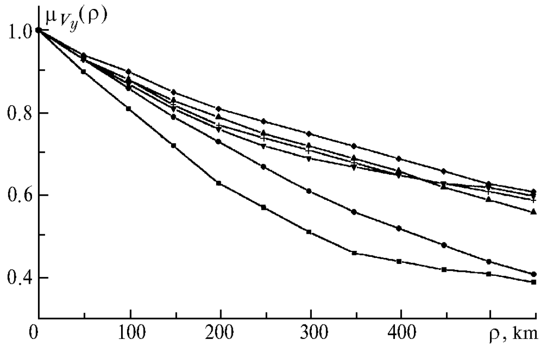
Analysis of these figures and the data obtained indicate that:

– the spatial correlation of temperature, zonal and meridional wind at all heights and independent of season weakens markedly with the increasing distance between the points, however, the correlation even at the largest distance $\rho = 560$ km remains positive;

– the fastest fall off of this correlation with the distance, independent of the investigated level, is observed of the meridional wind in summer when the value $\rho = 200$–350 km corresponds to the value of the correlation function 0.6. The most slow fall off of this correlation is characteristic of temperature, in summer as well (even at the largest distance of 560 km the spatial correlation function $\mu_\tau(\rho) > 0.6$);

– the spatial correlation of zonal and meridional wind at the ground level regardless of season falls with the distance much faster than the correlation of the same parameters in the free atmosphere. However, no such a dependence is observed for temperature;

– certain annual variation is characteristic of the spatial correlation of meteorological quantities being studied; in this case for the temperature, zonal and meridional wind quicker fall off of the correlation with the distance is observed in summer, except for the correlation of zonal wind at the levels in the atmospheric boundary layer.



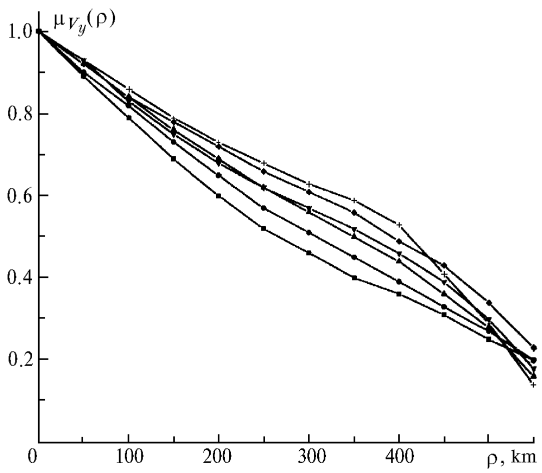
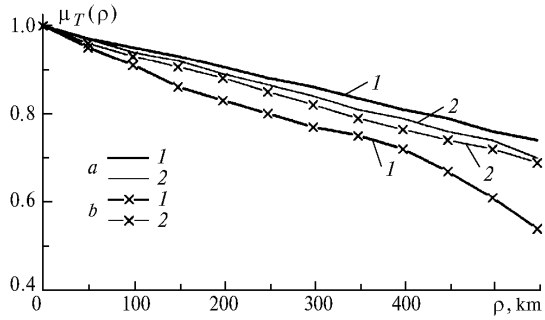


FIG. 4. Space correlation functions of meridional wind at different heights. The conventional symbols are given in Fig. 2.

Thus, certain peculiarities in the behavior of spatial correlation of temperature, zonal and meridional wind, that were determined by the statistical analysis of mesoscale structure of the field of these meteorological quantities.

In conclusion we would compare the obtained correlation functions of temperature, zonal and meridional wind with similar functions constructed previously by V.P. Boltenkov[13] (for temperature) and M.O. Krichak[14] (for the components of the wind velocity vector), when investigating the spatial macrostructure of the corresponding meteorological fields. Consider now Fig. 5 presenting the spatial correlation functions of temperature, zonal and meridional wind for the levels of 500 hPa (~ 5.5 km) calculated on the basis of aerological stations, being a typical mesometeorological polygon, constructed earlier by the authors of Refs. 13 and 14.

Figure 5 shows that in winter the spatial correlation functions of temperature, zonal and meridional wind at the level of 500 hPa obtained in our study (based on the data of mesometeorological polygon) and by the authors of Refs. 13 and 14 are a little bit different, excluding only the coefficients of correlation of zonal wind at distances longer than 400 km, and in summer these functions differ quite strongly. In this case the spatial correlation of the temperature, zonal and meridional wind in summer as estimated with the account for peculiarities in the mesostructure of corresponding fields, is characterized by a faster fall off with the distance compared to that when evaluating the same correlation based on data of macroscale fields of the meteorological quantities studied.



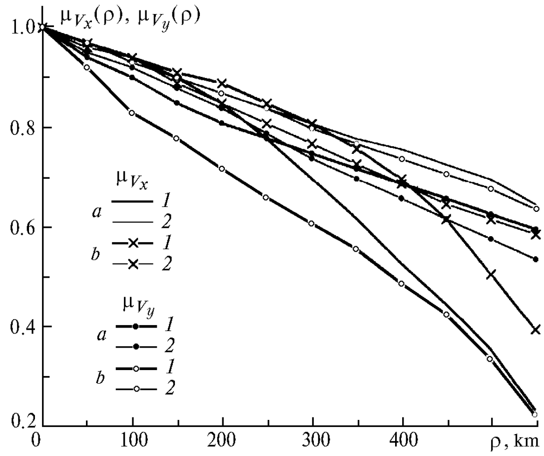


FIG. 5. Space correlation functions of temperature (μ_T), *zonal* (μ_{V_x}) *and meridional* (μ_{V_y}) *wind at the level of* 500 hPa *calculated by the authors (1) and constructed earlier in Refs. 13 and 14 (2); "a" symbol corresponds to the winter season and "b" denote the summer season.*

In our case, for example, the correlation coefficient 0.6 corresponds to the distance of 510 km (for temperature), 360 km (for zonal wind) and 300 km (for the meridional wind). The authors of Refs. 13 and 14 when investigating the spatial macrostructure of the fields of these meteorological quantities, have obtained the same correlation coefficient to a distance of $\rho = 560$ km either is not reached at all (this occurs for temperature and wind) or is observed at a distance of 525 km that is typical for the wind. All this points to the fact that in summer, when we observe fast decrease of correlation (as compared with winter), while constructing the spatial correlation functions to the distance below 500 to 600 km it is necessary to take into account the peculiarities of the mesostructure of meteorological fields.

4. SOME RESULTS OF ANALYTICAL APPROXIMATION OF THE CORRELATION FUNCTIONS OF TEMPERATURE, ZONAL AND MERIDIONAL WIND

It is known that in practice of the objective analysis of meteorological fields using the procedure of optimal interpolation (extrapolation) not the spatial correlation functions $\mu_\xi(\rho)$ themselves are used but their approximation formulas (some of those are given above). Considering this, we tried to find the best analytical functions allowing the description of the parameters of spatial correlation $\mu_\xi(\rho)$ up to the distance of 560 km with a minimum error.

For this purpose we have considered all the above–mentioned expressions (2) to (7) that are used for the approximation. After their careful comparison with the empirical correlation functions $\mu_\xi(\rho)$ we have drawn the following conclusion.

First, the empirical correlation functions of temperature to the distance of 560 km can be approximated throughout the layer of 0–8 km, regardless of season using the analytical formula

$$\mu_T(\rho) = \{\exp(-\alpha\rho)\} \cos(\beta\rho), \quad (8)$$

in this case the parameters of this approximation formula depend on height and can be found from the following expressions:

$$\alpha(h) = 0.436 + 0.051h; \quad (9)$$

$$\beta(h) = 0.864 + 0.005h, \quad (10)$$

where ρ is the distance, in thousand km, and h is the height, m.

Second, the empirical correlation functions of zonal and meridional components of the wind vector to the distance mentioned can be well described at all levels and in both seasons by the same expression

$$\mu_{V_x}(\rho) = \mu_{V_y}(\rho) = (1 - \alpha\rho) \exp(-\rho)^2. \quad (11)$$

At the same time, as in the case with temperature, the parameters of approximation formula (11), describing the empirical correlation functions of zonal and meridional winds, depend on height and can be determined from the expression of the form

$$\alpha(h) = 1.162 \exp(-0.125h). \quad (12)$$

The accuracy of this approximation can be assessed from the Table I, which gives absolute deviations of the values of analytical functions of the form (8) and (11) from the values of the corresponding empirical correlation functions calculated for the ground level, 0.8 and 5.0 km and different distances ρ.

TA” LE I. Absolute values of the difference between the analytical functions of the form (8) and (11) and empirical correlation functions of temperature, zonal and meridional wind at different distances.

Altitude, km	Distance ρ, km									
	50	100	150	200	250	300	350	400	450	500
Winter Temperature										
0.0	0.00	0.01	0.00	0.01	0.00	0.00	0.02	0.02	0.02	0.03
0.8	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00
5.0	0.00	0.00	0.01	0.02	0.03	0.05	0.07	0.07	0.07	0.09
Zonal wind										
0.0	0.00	0.00	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.05
0.8	0.01	0.02	0.03	0.03	0.05	0.05	0.06	0.05	0.03	0.00
5.0	0.00	0.01	0.02	0.02	0.00	0.04	0.07	0.09	0.10	0.13
Meridional wind										
0.0	0.01	0.02	0.03	0.03	0.04	0.05	0.05	0.02	0.03	0.05
0.8	0.02	0.03	0.03	0.03	0.02	0.01	0.01	0.01	0.03	0.05
5.0	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.05
Summer Temperature										
0.0	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.01	0.01
0.8	0.02	0.02	0.02	0.02	0.03	0.03	0.04	0.04	0.06	0.07
5.0	0.02	0.02	0.03	0.03	0.02	0.01	0.00	0.01	0.02	0.03
Zonal wind										
0.0	0.01	0.02	0.02	0.03	0.03	0.03	0.02	0.03	0.03	0.06
0.8	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.03	0.05
5.0	0.00	0.01	0.02	0.03	0.03	0.03	0.05	0.04	0.02	0.03
Meridional wind										
0.0	0.00	0.01	0.01	0.03	0.03	0.05	0.06	0.06	0.07	0.07
0.8	0.00	0.01	0.01	0.02	0.03	0.03	0.03	0.05	0.05	0.05
5.0	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.02	0.07	0.18

We have analyzed the table data and found that the values of differences between analytical and empirical correlation functions regardless of a meteorological quantity, season, and the atmospheric level do not exceed, with the rare exceptions, the values of 0.05–0.07, and up to the distance $\rho \le 250$ km (i.e., the distance to which the problem of spatial extrapolation of mesometeorological fields is solved[11]), these differences varied within 0.00–0.03.

Thus the approximation formulae obtained enable us to effectively describe empirical correlation functions to the distance up to 500–600 km and can be successfully used in the problems of objective analysis and spatial extrapolation of the mesoscale fields of temperature and zonal and meridional components of the wind vector.

REFERENCES

1. P.N. Belov, *The Spatial Methods of the Numerical Weather Forecast* (Gidrometeoizdat, Leningrad, 1967), 335 pp.

2. L.S. Gandin and R.L. Kagan, *Statistical Methods of Meteorological Data Interpretation* (Gidrometeoizdat, Leningrad, 1976), 359 pp.

3. V.V. Kostykov, *The Objective Analysis and the Agreement of Meteorological Fields* (Gidrometeoizdat, Leningrad, 1982), 295 pp.

4. V.A. Gordin, *Mathematical Problems of Hydrodynamic Weather Forecast* (Gidrometeoizdat, Leningrad, 1987), 264 pp.

5. V.S. Komarov, S.A. Soldatenko, and O.M. Sobolevskii, Atmos. Oceanic Opt. **9**, No. 4, 278–281 (1996).

6. D.I. Kazakevich, *The Foundations of Theory of Random Functions and their Application in Hydrometeorology* (Gidrometeoizdat, Leningrad, 1977), 319 pp.

7. L.S. Gandin, V.I. Zakhariev, and R. Tselnai, eds., *Statistical Structure of Meteorological Fields* (Budapest, 1976), 365 pp.

8. A.V. Meshcherskaya, in: *The Application of Statistical Methods in Meteorology* (Gidrometeoizdat, Leningrad, 1971), pp. 164–170.

9. P.M. Korobov, Trudy Gl. Geofiz. Obs, is. 336, 60–67 (1974).

10. P.N. Belov, E.P. Borisenkov, and B.D. Panin, *Numerical Methods of Weather Forecast* (Gidrometeoizdat, Leningrad, 1989), 376 pp.

11. V.S. Komarov, A.V. Kreminskii, and Yu.B. Popov, Atmos. Oceanic Opt. **11**, No. 8, 692–702 (1998).

12. Yu.M. Liberman, *Application of Statistical Methods in Meteorology* (Gidrometeoizdat, Leningrad, 1978), pp. 162–165.

13. V.P. Boltenkov, Trudy Gl. Geofiz. Obs, is. 165, 16–26 (1964).

14. M.O. Krichak, Trudy Gl. Geofiz. Obs, is. 208, 32–40 (1967).