THEORY OF LINEAR VISION SYSTEMS. MODELING OF THE LINEAR-SYSTEMS CHARACTERISTICS

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Approaches to the solution of the problems of vision theory are studied, and methods for measuring and calculating the linear-systems characteristics describing the transfer of an image through scattering media are reviewed. Some questions regarding the application of the Monte-Carlo method in problems of seeing through scattering media are discussed. An interpretation is proposed for some experimentally observed distortions of images by the medium.

In the theory of vision a number of approaches are being developed for studying the effect of scattering media on the quality of images of objects observed through such media. In one group of works[1–9] the general tendencies are established based on analysis of the characteristics of images of specific extended test objects. In Refs. 4, 6, and 9–23 some systems characteristics of the image transfer process, in terms of which the solution of particular problems can be expressed, are determined and investigated. Among these approaches we call attention to the method of spatial-frequency characteristics,[16–21] the linear-system approach,[14,25,24,22] and the method of Green's functions[7,9,12,13.15.23.26,28] (the last two approaches are essentially equivalent, if some features[24] of the mechanism by which an image is formed by optical devices are ignored in the solution).

In vision theory the approximation of lowest-order scattering,[29,30,34] the small-angle approximation and its modifications, the method of iteration over the orders of scattering with integration over the characteristics,[16,21] the two-flux and other approximations[1–6,11] and simulation modeling by the Monte-Carlo method[4,8,9,14,22–28,31–36] are widely employed for solving the transfer equation.

Laboratory and field experiments[35,37–46,49] not only supplement or confirm[38,39,42,44,49] existing results but they also generate new results (for example, the t-effect[35,46]) which lead to more fundamental understanding of the theoretical results and stimulate special investigations.[33,44]

Investigators are interested in different aspects of the problem of the distorting effect of the scattering medium on an image: the effect of the medium on the image contrast,[1–8,29,47,48] the spatial resolution[10,11,33,51], and the color characteristics of the image.[9,29] The dependence of image quality on the spatial structure of the object (half-plane, small objects, groups of small objects, rings, etc.) and the reflection properties of the object (for example, nonlambertian nature of the surface) are studied in Refs. 5, 7, 8, 19, 23, and 26. The combined effect of the medium, the structure of objects, and the characteristics of the optical systems on the image is analyzed in Refs. 10, 23, and 47. The reaction of the pulsed characteristics of vision systems to some features of the image formation mechanism of optical receivers is studied in Ref. 24.

In this paper the linear-systems approach to the solution of the problems of vision theory and the application of the Monte-Carlo method for modeling the systems characteristics determining the process of image transfer through scattering media is studied. An interpretation of some experimentally observed distortions of the image by the medium is proposed in the Appendices.

STATEMENT OF THE PROBLEM

Let the object plane be the XOY plane of a Cartesian coordinate system (Fig. 1) and assume that it is characterized by the reflection coefficients (or albedo) $s(\bar{\rho}, \vec{m})$, where $\bar{\rho}$ is the radius vector of a point in the XOY plane and $\vec{m}$ is a unit directional vector ($\vec{m} \in \bar{\Omega}$). The object plane is illuminated through a scattering medium (bounded by the planes $z = z_1 \geq 0$ and $z = z_2 > 0$) by a source of incoherent radiation, which forms a flux of parallel rays, as shown in Fig. 1.

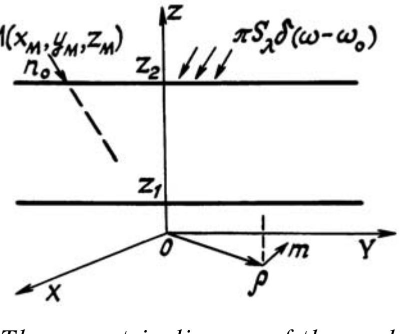

FIG. 1. The geometric diagram of the condition of observation.

A linear detector, which forms the image, is positioned at the point $M(x_M, y_M, z_M)$. The horizontally uniform medium is characterized by the standard collection of optical characteristics appearing in the radiation transfer equation describing the propagation of short-wavelength radiation in turbid media.

We shall call the distribution of the reflection coefficient $s(\vec{\rho}, \vec{m})$ or the intensity (or brightness) $I(\vec{\rho}, \vec{m})$ at the level $z = 0$ the observed object:

$$I(\vec{\rho}, \vec{m}) = E(\vec{\rho})\ s(\vec{\rho}, \vec{m}) \tag{1}$$

where $E(\vec{\rho})$ is the intensity of illumination of the object plane.

We shall call the distribution of intensity (brightness) $I(\vec{r}m, -\vec{n})$ at the point of reception M, where $\vec{n}$ is a unit directional vector, the image of the object. In the case of projective optical systems the point M is fixed, $\vec{n} \in \bar{\Omega}$, i.e., the vector $\vec{n}$ varies within the visual field or the field of view (in the case of scanning systems) of the optical receiver. If the image is formed by the method of spatial scanning, then to construct the image it is sufficient to know $I(\vec{r}_M, -\vec{n})$, for a given region $\vec{r}_M \in \vec{R}_M$ for a fixed value of $\vec{n}$.

Thus the direct problem of the theory of vision reduces to solving the following equation of the theory of radiation transfer:

$$(\vec{\omega}, \mathrm{grad}\, I) = -\beta_{ext} I + \beta_{sc} \int_{\Omega} I(\vec{r}, \vec{\omega}')g(\vec{r}, \vec{\omega}, \vec{\omega}')d\vec{\omega}' + \Phi_0(\vec{r}, \vec{\omega}) \tag{2}$$

under the boundary conditions

$$\begin{cases} I(\vec{\rho}, \vec{\omega}, z=z_1) = E(\vec{\rho}, z=0)s(\vec{\rho}, \vec{\omega}), & (\vec{\omega}, \vec{n}_1)<0, \\ I(\vec{r}, \vec{\omega}) = \pi S_\lambda \delta(\vec{\omega}-\vec{\omega}_0), \quad z=z_2, & (\vec{\omega}, \vec{n}_2)<0, \end{cases} \tag{3}$$

where

$$\beta_{ext},\ \beta_{sc}(\vec{r}) \text{ and } g(\vec{\omega}, \vec{\omega}; \vec{r})$$

are the attenuation coefficient, the scattering coefficient, and the scattering phase function, respectively: πS_λ is the solar constant; and, $\vec{n}_{1,2}$ are the outward normals to the plane $z = z_{1,2}$.

Analysis of the processes accompanying the transfer of an image through scattering media shows[9,23,25,52] that the problem (2) and (3) can be replaced by a collection of simpler (and previously well-known) problems of the theory of radiation transfer. Application of the apparatus of the theory of analysis of linear systems makes it possible to solve the problem (2) and (3) in a more general form irrespective of the form of the functions $s(\vec{\rho}, \vec{m})$, if the object plane is uniform, i.e., $s(\vec{\rho}, \vec{m}) = s_1(\vec{\rho})s_2(\vec{m})$:

$$s(\vec{\rho}, \vec{m}) = \begin{cases} s_1^{(1)}(\vec{\rho})s_2^{(1)}(\vec{m}), & \vec{\rho} \in \vec{R}_1, \\ s_1^{(2)}(\vec{\rho})s_2^{(2)}(\vec{m}), & \vec{\rho} \in \vec{R}_2, \\ \vdots & \vdots \\ s_1^{(n)}(\vec{\rho})s_2^{(n)}(\vec{m}), & \vec{\rho} \in \vec{R}_n \end{cases}$$

We shall divide the process of image formation into the following stages.

1. The formation of light haze I_h owing to the scattering of photons, which have not interacted with the object plane (the ties 1–2 in Fig. 2a), toward the receiver. In the case of illumination by sunlight the boundary conditions for determining I from Eq. (2) are

$$\begin{cases} I_s(\vec{\rho}, \vec{\omega}) = 0, \quad z=z_1, \quad (\vec{\omega}, \vec{n}_1)<0, \\ I_s(\vec{r}, \vec{\omega}) = \pi S_\lambda \delta(\vec{\omega}-\vec{\omega}_0), \quad z=z_2, \quad (\vec{\omega}, \vec{n}_2)<0. \end{cases} \tag{4}$$

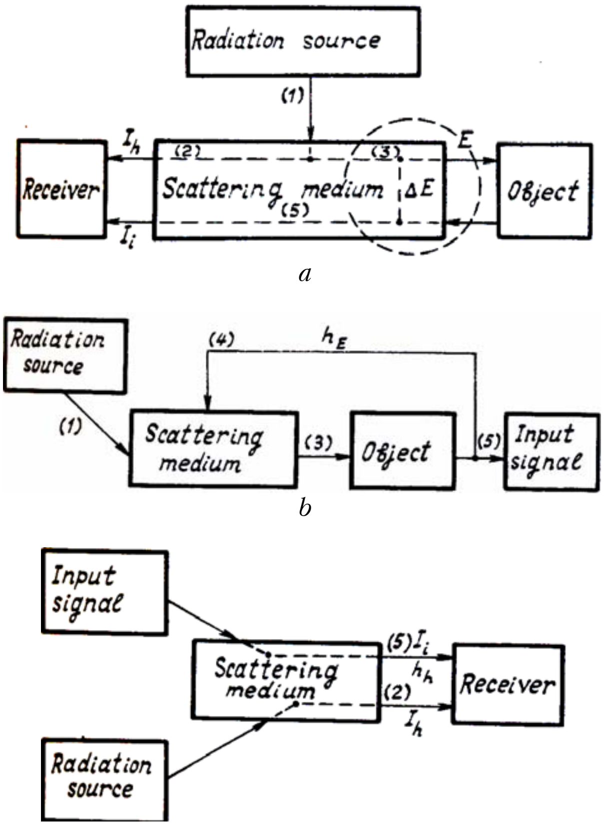

FIG. 2. Block diagram of the process of image transfer through a scattering medium.

2. The formation of the brightness structure of the object (the input signal). In Fig. 2a and b this process corresponds to the ties 1–3–4–5. It includes the following steps:

a) illumination of the object , by directly transmitted and diffusely scattered radiation from the source (ties 1–3);

b) additional (and in the general case nonuniform over the XOY-plane) illumination arising as a result of scattering of reflected (including multiply reflected) photons toward the object plane (ties 3–4).

3. The propagation of the formed input signal through the scattering medium (Fig. 2c).

The problem of 2a above corresponds to Eq. (2) and the conditions (4). The problems 2b and 3 are best solved by using the linearity of the transfer equation with respect to the intensity. This makes it possible to regard the "scattering medium" blocks (Fig. 2) as some linear systems (in particular, in Fig. 2b, with feedback). The methods for analyzing systems are well known. To construct the response of such a system to any perturbation (in this case two-dimensional) $I(x, y)$ it is sufficient to know its response to a special input signal (in this case to a spatial δ-function pulse). Once the unit-pulse response $h(x, y)$ has been determined the response of the system $I(x, y)$ can be written in the form of a convolution integral:

$$I_b(x, y; -\vec{n}) = \iint\limits_{-\infty}^{\infty} h(x, y; -\vec{n})\ I(x-x', y-y')dxdy \tag{5}$$

The expression (5) can be derived based on the general principles for constructing the solutions of differential equations by the method of Green's functions.

In Refs. 16–19 it is proposed that Eq. (2) be regarded under the conditions (3) as an equation with perturbed boundary conditions (in the parameter $s(\bar{\rho}, \vec{m})$). It is shown in Ref. 17 that there is a unique correspondence between the solutions obtained using this approach and the solutions which are formed based on the linear-systems approach in terms of the corresponding unit-pulse response functions $h_h(\cdot)$ (Fig. 2c) and $h_E(\cdot)$ (Fig. 2b). These functions can be found by solving Eq. (2) under the boundary conditions

$$\begin{cases} I(\vec{\rho}, \vec{\omega}) = Es(\vec{\rho}, \vec{m})\delta(\vec{\rho}-\vec{\rho}_0), \quad z=z_1, \quad (\vec{\omega}, \vec{n})<0, \\ I(\vec{r}, \vec{\omega}) = 0, \quad z=z_2, \quad (\vec{\omega}, \vec{n})<0. \end{cases} \tag{6}$$

The function $h_h(\cdot)$ characterizes the so-called interference due to lateral illumination. To construct the brightness image of the object at the level $z = 0$ taking feedback (re-reflection) into account the following iteration process must be performed:[9]

$$I_n(x, y; \vec{m}) = s(x, y; \vec{m}) \iint\limits_{-\infty}^{\infty} h_E(x, y; \vec{m}) \times I_{n-1}(x, y; \vec{m}')dxdyd\vec{m}', \tag{7}$$

$$I = I_0 + I_1 + \ldots \tag{8}$$

The function $h_h(x, y; -\vec{n})$ for the problem 2b is determined for $z = z_2$ and $\vec{n} = \vec{n}_0$ and the ray along the direction $\vec{n}_0$ crosses the plane at the point M_0 with the coordinates $(\bar{x}_0, \bar{y}_0)$ (to simplify the notation in what follows the minus sign in front of the vector $\vec{n}$ in the functions $h_h I(x, y; -\vec{n})$ is dropped). For the problem 3 the solution is sought on the plane $z = 0$. It is obvious that

$$\gamma_\infty = \iint\limits_{-\infty}^{\infty} h_E(x, y)dxdy, \quad (0 \le \gamma_\infty < 1) \tag{9}$$

corresponds to the operator T_{22} in Ref. 23 and characterizes the probability of rereflection or the feedback gain (Fig. 2b). The quantity

$$\eta_\infty = \iint\limits_{-\infty}^{\infty} h_s(x, y, \vec{n})dxdy, \quad (0 \le \eta_\infty < 1), \tag{10}$$

called in Ref. 34 the integral (total) lateral illumination, corresponds to the operator T_{23} in Ref. 23.

The background illumination owing to reflection, rereflection, and scattering, taking into account Eqs. (7)–(10) for a uniform Lambertian surface ($s(x, y; \vec{m}) = \text{const} = \bar{s}$), assumes the form

$$\bar{I}(x, y) = \bar{I} = \eta_\infty \bar{s} \frac{E}{1 - \bar{s}\gamma_\infty} \tag{11}$$

Then the total signal is

$$I_b = I_s + \frac{1}{\pi} \exp^{-\tau_0/\mu_1} E + \bar{I} =$$
$$= I_g + \frac{1}{\pi} \exp^{-\tau_0/\mu_1} \left[\pi S_\lambda (\exp^{-\tau_0/\mu_0} + \eta_\infty)\right] + \eta_\infty \frac{\pi S_\lambda \left[\exp^{-\tau_0/\mu_0} + \eta_\infty\right]}{1 - \bar{s}\gamma_\infty}, \tag{11'}$$

where τ_0 is the optical thickness of the layer, and μ_0 and μ_1 are the cosines of the angles between the OZ axis (Fig. 1) and the direction toward the source and the receiver from the point (x_0, y_0). The theorem of optical reciprocity is employed in the derivation of Eq. (11′).

An expression analogous to Eq. (11) was derived in Ref. 17 in the solution of a boundary-value problem with perturbed boundary conditions. Obviously γ_∞ is the spherical albedo (in Ref. 17 c_0 for a medium illuminated from below by diffuse radiation) and η_∞ characterizes the transmission by a medium taking into account the horizontal diffusion of photons reflected from XOY plane (the function $\vec{W}_0(z, s)$ in Ref. 17).

Following the terminology of the theory of linear systems the Fourier transforms of the functions $h_h(\cdot)$

and $h_E(\cdot)$ in the frequency domain are called, by definition, the transfer functions.

Using Eqs. (7) and (8) for objects with arbitrary $s(x, y; \vec{m})$ we can write

$$I_b(x_0, y_0; \vec{n}_0) = I_s + E(x_0, y_0) s(x_0, y_0; \vec{n}_0) \times$$
$$\times \left\{ h_s(x_0, y_0; \vec{n}_0) + \exp^{-\tau_0/\mu_0} \right\} + \iiint\limits_{-\infty\Omega+}^{\infty} E(x', y') \times$$
$$\times\ s(x', y'; \vec{m}') \times$$
$$\times\ hs\ (x_0 - x', y_0 - y'; \vec{n}_0) dx' dy' d\vec{m}', \tag{12}$$

where $E(x, y) = E_0 + \Delta E(x, y)$, E_0 is the uniform intensity of illumination of the plane XOY (neglecting reflections), and $\Delta E(x, y)$ is in the general case the nonuniform illumination (taking into account rereflections)

$$\Delta E_n(x, y) = \iiint\limits_{-\infty\Omega+}^{\infty} h_E(x', y'; \vec{m}') \times$$
$$\times\ \times \Delta E_{n-1}(x - x', y - y') dx' dy' d\vec{m}';$$
$$\Delta E_0(x, y) = E_0;\ \ \Delta E = \Delta E_1 + \Delta E_2 \ldots + \Delta E_n + \ldots \tag{13}$$

Thus the direct problem of the theory of vision for the conditions examined above has been solved, if the two constants (E; I_h) and the two unit-pulse response functions or the point-spread functions $h_h(x, y; \vec{n}_0)$, $h_E(x, y)$ have been found for an isolated scattering medium with no reflecting surfaces. Similar results and conclusions were obtained in Refs. 16–18 in the solution of the corresponding problem of the theory of transfer with perturbed boundary conditions.

We shall now reformulate the problem. Assume that the objects are not illuminated by a source and that the observed objects are luminous (nighttime viewing conditions). Then the point-spread functions $h_h(x, y; \vec{n}_0)$ (PSF) or the optical transfer function of the medium (OTF) (the Fourier transform of $h(x, y; \vec{n}_0)$ completely describes the process of image transfer in the direction $\vec{n}_0$ in turbid media. Since this cannot be said about the general formulation of the problem we must evidently follow Ref. 17, where it is pointed out that the term "optical transfer function of the atmosphere" is not correct in this case, when the process of distortion of the image is more complicated and is now determined by a collection of characteristics (E, I_h, $h_h(\cdot)$), $h_E(\cdot)$, or (E_0, I_h, η_∞, γ_∞) (in some cases the values of E_0 can be equal to η_∞, if the unscattered component is excluded from E_0). We note that even when luminous objects are being observed the values of the PST for one direction $\vec{n}_0$ may not be sufficient in order to construct the image correctly. This Situation evidently occurs when the receiver consists of an optical system which transforms the angular distribution of the intensity $I(\vec{n})$ at the point M (Fig. 1) into a flat image $G(x, y)$ (here $\vec{n} \in \vec{\Omega}^*$, where $\vec{\Omega}^*$ is the visual field of the receiving system). In this case in order to construct the image it is necessary to determine $h_h(x, y; \vec{n})$ for each direction $\vec{n} \in \vec{\Omega}^*$. In the theory of optical systems[52] the concept of isoplanarity is introduced in order to solve an analogous problem. The image plane or the visual field of the receiver is divided into the regions $\vec{\Omega} = \vec{\Omega}_1 \cup \vec{\Omega}_2 \cup \ldots \cup \vec{\Omega}_n$, within which in constructing the image only the function $h_h(x, y; \vec{n})$ $\vec{n}_i \in \vec{\Omega}_i$, $i = \overline{1, n}$ need be employed.

The region $\vec{\Omega}$ can be divided into zones of isoplanarity $\vec{\Omega}_i$ by different methods. One such method, for example, is based on comparing the brightness image of a point source $I_0(x, y; \vec{n}_i)$ with the unit-pulse response $h_h(x, y; \vec{n}_i)$. We shall assume that $h_{hi}(x, y; \vec{n}_i) = h_h(x, y; \vec{n})$, $\vec{n}_i$, $\vec{n} \in \vec{\Omega}_i$, if

$$\frac{|h_{s1}(x, y; \vec{n}_1) - I_0(x, y; \vec{n}_1)|}{h_{s1}(x, y; \vec{n}_1)} \le \delta,\ \ 0 \le \delta \le 1 \tag{14}$$

or

$$\frac{|\eta_R(\vec{n}_1) - \hat{\eta}_R(\vec{n}_1)|}{\eta_R(\vec{n}_1)} < \varepsilon,\ \ 0 < \varepsilon < 1, \tag{15}$$

where

$$\hat{\eta}_R(\vec{n}_1) = \iint\limits_{-R}^{R} I_0(x, y; \vec{n}_1) dx dy.$$

In other words the separation is performed so that by using the unit-pulse response $h_h(x, y; \vec{n}_i)$ it would be possible to construct within the i-th isozone an image of a point object or to determine the intensity of illumination created by it within this zone with fixed accuracies δ (in Eq. (14)) or ε (in Eq. (15)).

We call attention here to the fact that if the definition of isoplanarity is to be preserved the concept of foreshortening invariance introduced in Ref. 53 cannot be employed to determine the dimensions of the isozones. In Ref. 15 it is pointed out that the image can remain isoplanar even if the condition of foreshortening invariance of the medium is not satisfied.

The clear physical meaning of the unit-pulse response function $h_h(x, y; \vec{n}_i)$ simplifies the planning of the corresponding experimental investigations. Thus existing methods for measuring the responses $h_h(\cdot)$ and the problems involved in organizing these studies are studied in Ref. 37 with the help of numerical and laboratory modeling. The central problem is to measure the wings of the PSF.

A variant of a new method which combines measurement of the unit-pulse response functions with the additional determination of the unit-pulse response function for large values of the argument, for which it is determined based on approximation of the results of numerical computer experiments, is proposed in Ref. 36. We note that together with measurement of the unit-pulse response functions[8,34,37,38,39] attempts have been made[40] to measure the optical transfer functions $F(h_h(\cdot))$ in laboratory experiments. Even in this case, however, the problem of taking into account the effect of lateral illumination from remote regions of the object plane on the OTF cannot be avoided.

The method of angular scanning with determination of the spread function of a point, line, or boundary curve, i.e., the function $I_0(x, y; \vec{n}_i)$ (in the case of the PSF) in Eq. (14), is widely employed in laboratory and field experiments on the theory of vision (as done in some early theoretical works). This is probably a result of transferring the definition of the unit-pulse response from the theory of optical systems, where the PSF is the diffraction image of a point source, directly to the theory of vision. For optical systems this definition of the PSF is admissable because its half-width is very small; in vision systems, however, in the general case this condition is not satisfied. It is shown in Ref. 37 that the approximate method for measuring unit-pulse responses (angular scanning) is applicable only in a limited range. Outside this range the relative errors in measuring the PSF can exceed 100%, especially when layers with high turbidity, not adjoining the object plane, are present in the observation path. The accuracy of such measurements decreases on the wings of the point (line) spread function.

In Refs. 34 and 36 attention is drawn to two properties of the function $h_h(\cdot)$, which can be employed to improve the efficiency of laboratory experiments on the influence of optical geometrical conditions of observation through turbid layers on $h_h(\cdot)$. The first property, which follows from the principle of geometric similarity, is formulated as follows (to simplify the notation we confine our attention to the case of vertical observation in Fig. 1, when $h_h(x, y) = h_h(\rho)$; $\rho = \sqrt{x^2 + y^2}$:

$$h_s(\rho, z_1) = h_0(0, z_1) h_s(\varphi), \tag{16}$$

where $\varphi = \mathrm{arctg}(2\rho/(z_1 + z_2))$, i.e., for any , $z_1^{(1)}$, $z_1^{(2)}$ and fixed geometric thickness of one layer

$$\frac{h(\rho, z_1^{(1)})}{h(\rho, z_1^{(2)})} = \frac{h(0, z_1^{(1)})}{h(0, z_1^{(2)})}.$$

The second property consists of the fact that the functions $h_h(\varphi)$ (for $\varphi \geq 15°$) are virtually independent of the scattering properties of the medium (scattering phase function) and the geometric thickness of the layer, and can be approximated adequately (with an accuracy $\leq$ 20%) by the function

$$\lg h_s(\varphi, \tau) = \alpha(\tau)\,(\varphi - \varphi_0)^2 \tag{17}$$

where $\varphi_0 = 15°$ and x is the optical thickness of the medium.

The property (16) makes it possible to limit significantly the number of measurements in studying the effect of the distance z_i to the layer on the function $h_h(x, y; \vec{n}_i)$, while the property (17) makes it possible to complete the determinationof the wings of the unit-pulse function, which, as a rule, cannot be measured in the experiments. We stress that these measurements give the PSF of the vision system as a whole, i.e., taking into account the optical system. Methods for eliminating its effect on the measurements are traditional, and this is most simply done by assuming that the optical receiver is linear.

SIMULATION MODELING OF THE CHARACTERISTICS E, I_h, h_h, η_∞, h_E, γ_∞

Thus by solving Eq. (2) under the condition (4) it is possible to find the characteristics I_h and E under the condition (5) the functions $h_h(\cdot)$, $h_E(\cdot)$ and the parameters μ_∞, γ_∞. The methods developed for solving this equation with the boundary condition (4) make it possible to determine the intensity of the light haze I_h and the intensity of illumination E without any restrictions on the optical characteristics of the medium, including for the case when the geometry of the problem is spherical. This can be done by approximate[4,5,15] and exact[9,23,26] methods for solving the transfer equation.

It is more difficult to solve Eq. (2) for the unit-pulse response functions $h_h(\cdot)$, $h_E(\cdot)$. The basic results of investigations of the unit pulse responses $h_h(\cdot)$ and the characteristics η_∞, γ_∞ correspond to the assumption that $s(\vec{\rho}, \vec{m}) = \bar{s}_1(\vec{\rho}) s_2(\vec{m}) = \alpha s(\vec{\rho})$ i.e., that the object plane is Lambertian. To determine the unit-pulse response $h_h(\cdot)$ and the integral illumination η_∞ in this case, among approximate methods, the approximation of the lowest orders of scattering[34] and the small-angle approximation[7,12] are most often employed. There is virtually no published information on the function $h_E(\cdot)$ and the parameter γ_∞, even with $s_2(\vec{m}) = \mathrm{const}$. There are only several works (for example, Refs. 6 and 19) where approaches to solving Eq. (2) under the condition $s_2(\vec{m}) \neq \mathrm{const}$ are proposed.

All these problems are eliminated by including the Monte-Carlo method in the linear-systems approach to the problems of the theory' of vision. It is possible to construct quite efficient algorithms for modeling all characteristics determining image transfer in turbid media (I_h, E, η_∞, $h_h(\cdot)$, γ_∞, $h_E(\cdot)$) (Refs. 9, 22–28, 31, 54).

Statistical modeling of the linear-systems characteristics can be performed for any functions

$s_2\bar{m}$, given analytically or tabulated on a uniform or plecewise-uniform object plane; a solution can be obtained with controllable accuracy and it is possible to take into account correctly multiple scattering as well as some features of the formation of flat images of three-dimensional objects by optical systems (i.e., the function $h_h(\cdot)$ and the characteristics I_h and η_∞ can be calculated both taking into account and neglecting the effect of the optical system[24]). The implementation of the algorithms of the method on fast computers (including, for example, on parallel computing systems with overall control by a PS-2000 processor) makes it possible to reduce the labor involved in the calculations. Since the labor intensiveness is the main (and often the only) drawback of the method the works directed toward searching for or choosing the most efficient algorithms for calculating the characteristics of the image transfer process deserve attention.[22–28,31,54] For example, in Ref. 54 the amount of computation involved in calculating the characteristics η_∞, and $h_h(\cdot)$ for the case $\bar{s}_2(\bar{m})$ = const, by direct modeling is compared with that of the method of local computation. The calculations showed that it is more efficient to use specially constructed algorithms for evaluating η_∞ than it is to calculate the functions $h_h(\cdot)$ first and then to integrate Eq. (10) numerically. In turns out that η_∞ can be calculated with the help of a direct-modeling algorithm in the scheme of conjugate random walks. The functions $h_h(\cdot)$ are best modeled with the help of a local algorithm on conjugate trajectories. The local estimate in this case has the form[54]

$$\xi_{1,k} = 2\kappa^k \exp(-\tau_{1,k}) g(\gamma_{1,k}) \mu_{1,k} / (2\pi r^2_{1,k}),$$

where κ is the photon survival probability and $\kappa(\bar{r}) = \kappa$; i is number of the trajectory; k is the number of the collisions; $\tau_{i,k}$ is the optical path length r_{ik} from the k-th collision point to the "reception" point; $\gamma_{i,k}$ is the scattering angle; and, $\mu_{I,k}$ is the cosine of the angle between the inward normal of the layer and the segment $.r_{i,k}$. The estimate $\xi_{I,k}$ determines the probability that a photon will traverse the segment $\tau_{i,k}$.

The characteristics γ_∞ and $h_E(\cdot)$ can be determined by constructing an algorithm for modeling the process of propagation of radiation with a fixed radiation pattern $s_2(\bar{m})$ from a point source. The average number of trajectories crossing the object plane gives an estimate of γ_∞, while the distribution of these crossings over the plane gives the function $h_E(\cdot)$. The functions $h_E(\cdot)$ can also be calculated by the method of local computation.

When the Monte-Carlo algorithms are implemented on parallel computing systems of the PS-type each branch of the calculation must be provided with a pseudorandom sequence of numbers distributed uniformly in the interval [0, 1], and in addition the condition that the numerical sequences in the different branches be uncorrelated must be satisfied (if this condition is not incorporated in the formulation of the problem). The second problem is related with the problem of activation of the processor elements. The characteristics of the statistical modeling algorithms do not permit full use of the computational possibilities of parallel systems. Thus, for example, experience in simulation modeling of the characteristics η_∞ and γ_∞ on the PC-2000 system shows that the efficiency of using 32 processor elements simultaneously is of the order of 20–30%. This, however, turns out to be sufficient to reduce substantially (by a factor of 3 to 5) the amount of computation as compared with the calculations performed on single-processor BESM-6 computers.

In developing simulation algorithms the importance of determining the wings of the unit-pulse responses in order to analyze correctly the effect of the conditions of observation on the image transfer process in turbid media must be taken into account. It is shown in Ref. 7 that neglecting the wings can result in distortion not only of the quantitative but also the qualitative dependences, for example, of the OTF ($F[h_h(\cdot)]$), on the optical and geometric parameters and the characteristics of the observation schemes. This remark can serve as an additional argument in support of local computational methods in the scheme of conjugate random walks for modeling the unit-pulse responses $h_h(\cdot)$, $h_E(\cdot)$ (Refs. 9 and 54).

References 26–28 and 31, where algorithms are proposed for calculating the OTF ($F(h_h(\cdot)]$, and works where the Monte-Carlo algorithms which make it possible to take into account the characteristics of image formation by optical systems, i.e., to model the characteristics I_h, $H_h(\cdot)$, and η_∞ for vision systems (it is obvious that E, $H_E(\cdot)$, and γ_∞ do not depend on the optical systems), illustrate the universal possibilities of the methods of simulation modeling in the theory of vision. In Ref. 55 an algorithm is proposed for modeling the propagation of radiation through all the constructional elements of an optical system. In Ref. 24 methods are proposed for taking into account the scattering spots in the image plane, which arise owing to the finiteness of the depth of field of the space imaged by the instrument and the three-dimensional nature of the scattering medium. This feature can also be taken into account in the algorithms for calculating the OTF ($F[h_h(\cdot)]$).

The characteristics of the process of image transfer through a scattering medium are simulated in order to determine the limits of applicability of approximate computational methods[28,31,34] to predict the range of variation of these characteristics in test or concrete[4,5,9,23,26] applied problems which require that all factors determining the optical-geometric conditions of observation be taken into account correctly, and finally to analyze the effect of these factors on image quality.[9,14,23,26,31] An illustration of the latter is Ref. 35, where it was found that extremal distortion of the image of small objects by a turbid layer occurs when the layer migrates between the object and the observer. In Ref. 44 the existence of this effect was confirmed by an independent laboratory experiment. The Monte-Carlo investigation of the properties of the unit-pulse response function $h_h(\cdot)$ performed in Ref. 34 enabled Belov[33] to establish the reasons and conditions for the appearance of the t-effect in the observation of not only small but also extended objects (two aspects of this effect are studied in an Appendix).

APPENDIX I

The tracing-paper effect. A well-known result of investigations of the dependence of the OTF ($F[h_h(\cdot)]$ on the parameter z_1 (Fig. 1) is that the quality of images of objects observed through a scattering, layer decreases monotonically as the layer approaches the observer. To confirm this result it is proposed in Ref. 15, for example, that an elementary and very graphic experiment be performed: an arbitrary text is screened from the observer with tracing paper. By placing the tracing paper between the observer and the text it can be verified that the text can be seen clearly when the tracing paper is adjacent to the text. As the tracing paper is moved the text rapidly becomes blurry and then vanishes, and there is no longer any position at which the text can be read.

We shall try to find the reason for the contradiction between the results of this experiment and the results described in Refs. 35 and 44, where it is shown that the dependence under study can be, for example, of an extremal character. The reason for the apparent contradiction can be easily established by talcing into account the fact that in Refs. 35 and 44 the experiments were performed with a luminous object while the experiment with the tracing paper is performed under conditions when the object is illuminated by an external source. In the second case the observed signal contains a component I_h (light haze), while in the first case this component is absent. Let us see what changes this can bring about when the image quality is evaluated. Suppose the object is observed with the help of an optical instrument with a fixed threshold contrast sensitivity k_{th} (Fig. 3), which is independent of the external parameter z_1. Consider the contrast between an arbitrary pair of points in cases when the luminous object is observed:

$$k_1 = (I_1 - I_2)/(I_1 + I_2)$$

It follows from laboratory experiments[35,44] and calculations[7,8,33] that it can have an extremal character (curve 1, Fig. 3). We stress that all processes occurring with the propagation of radiation from the object to the observer are taken into account in the intensities $I_{1,2}$. Consider now the same dependence but in the case when an object with reflective capability, analogous to the radiative capability in the first experiment, is observed. Let the intensity of the external source be such that the intensities $I_{1,2}$ are equal in both cases. It is obvious that the contrast factor between the same pair of points will decrease by a constant amount:

$$k_2 = (I_1 - I_2)/(I_1 + I_2 + 2I_s)$$

i.e., $k_2 = k_2(z_1)$ can be represented by the curve 2 (Fig. 3). If $k_{th} > k_2(z_1)$ for $z_1 > z_{thr}$, then the dependence $k(z_1)$ which we observe with the help of the optical instrument will correspond to the curve 3 in Fig. 3. It is precisely this character of the dependence that follows from the experiment with the tracing paper and we are obviously fully justified in calling it apparent. To establish its true character a device with a different, but adequate for this purpose, contrast sensitivity k_{th} must be employed for the observations.

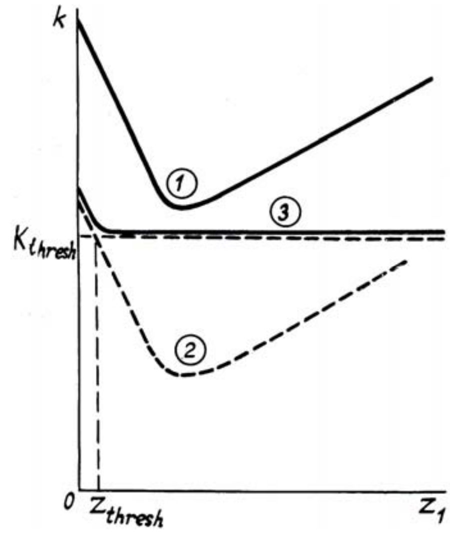

FIG. 3. Explanation of the tracing-paper experiment: 1 — the contrast for observation of luminous objects; 2 — external illumination of reflecting objects; 3 — the apparent dependence $k = k(z_1)$.

APPENDIX II

We shall explain how the t-effect can be explained based on the results of Refs. 12 and 13, namely, that the optical transfer functions $\hat{K}(\omega) = F[h_h(\cdot))]$ depend on the parameter z_1 (Fig. 1). Let the frequency-contrast characteristic be $K(\omega) = \hat{K}(\omega) / \hat{K}(0)$, i.e., to simplify the explanation we shall confine our attention to vision systems with circular symmetry ($\omega = \sqrt{\mu^2 + \beta^2}$, where β and μ are the spatial frequencies in the image plane). Let the object being observed have the spectrum $F_0(\omega)$ (Fig. 4). We shall study three positions of the layer on the line of sight (the layer at the object — a, the layer at the observer — c, and the layer between them — b). It follows from Refs. 12 and 13 that $K(\omega; z_1^{(1)}) \geq K(\omega; z_1^{(2)})$, if $z_1^{(1)} < z_1^{(2)}$. For positions of the layer a–c these functions are shown on the left side of Fig. 4. For linear systems the spectrum of the image $F_1(\omega, z_1) = F_0(\omega)\hat{K}(0)K(\omega; z_1)$. The form of the functions $F_1(\omega; z_1)$ for z_1 corresponding to the positions of the layer a–c is shown on the right side of Fig. 4. Let us compare $F_0(\omega)$ with $F_1(\omega, z_1^{(1)})$ and $F_u(\omega, z_1^{(3)})$.

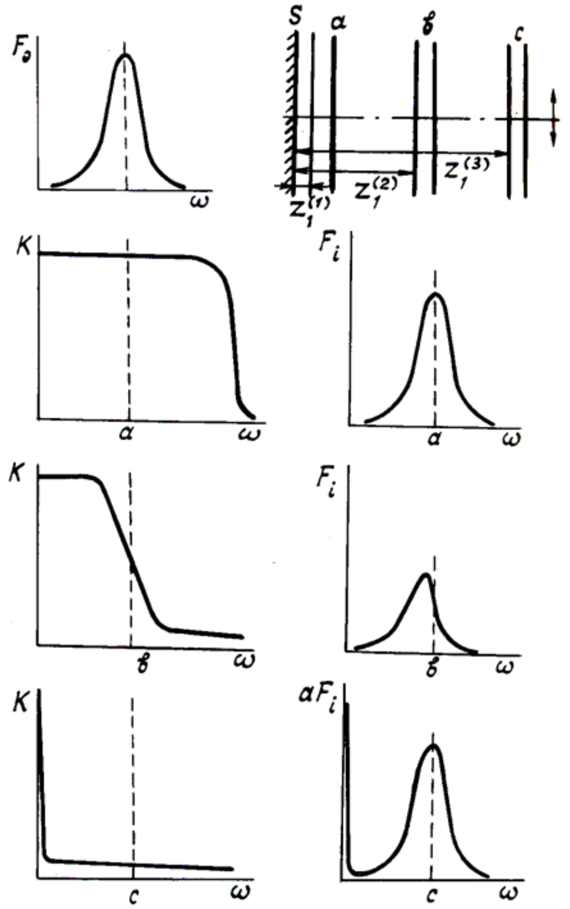

FIG. 4. Explanation of the t-effect. $F_0(\omega)$ is the spectrum of the object; $K(\omega)$ is the frequency-contrast characteristic; and, $F_1(\omega)$ is the spectrum of the image.

In the first case, when the half-width $K(\omega, z_1^{(1)})$ is much greater than the half-width of the spectrum $F_0(\omega)$ the spectrum of the image corresponds to the spectrum of the object and the distortions are minimum. In the case $z_1 = z_1^{(3)}$, on the other hand, $K(\omega; z_1^{(3)}) \to \delta(\omega) + \text{const}$. The existence of a "shelf" for the function $K(\omega)$ is proved, for example, in Ref. 49 and is explained by the unscattered radiation. These two features of $K(\omega, z_1^{(3)})$ lead to the fact that the spectrum $F_1(\omega; z_1)$ is almost everywhere similar to the spectrum $F_0(\omega)$, with the exception of a small neighborhood of the point $\omega = 0$, where a relatively large fraction of all of the energy contained in the image is concentrated. The last remark should lead to the appearance of a quite strong background which is uniform over the area of the frame and on which an image of the object is formed with all spatial features preserved. Laboratory experiments confirm this result;[35,44] for $z = z_1^{(3)}$ undistorted, but low-contrast images of the objects are observed. Consider now the case $z_1 = z_1^{(2)}$. It is obvious that if the frequency at the center of the spectrum $F_0(\omega)$ is equal to the frequency at one-half the intensity $K(\omega; z_1)$, then significant distortions will occur in the form of the spectrum of the object in the image; this will be manifested as a change in its spatial structure, i.e., it will lead to the t-effect. Obviously, if the form of the spectrum $F_0(\omega)$ is made more complicated, then repeating the foregoing arguments we arrive at the conclusion that the t-effect can arise repeatedly (for different regions of the spectrum $F_0(\omega)$, corresponding to different spatial structures of the object) as z_1 varies, i.e., the analogous result obtained in Ref. 33 in the analysis of the effect of the conditions of observation on the resolution in vision systems is confirmed. Thus the t-effect, as one possible type of dependence of the image quality on the parameter z_1, can be explained both in the spatial domain (using the concept of unit-pulse response) and in the frequency domain (in the language of optical transfer functions).

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