ESTIMATION OF TIME DELAYS BY THE METHOD OF ADAPTIVE FILTERING WITH SPLINE SMOOTHING OF SIGNALS

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A theory and scheme of processor design as well as results of numerical experiment on a computer using specially created programs are presented in this paper.

The accuracy in estimating time delays of signal copies detected by various sensors is of importance for the location of isotropic sources of the pulse optical radiation from satellite observation data[1] and for some other applications.[2] One can obtain the high accuracy (fractions of a sample step) with high level of noise (overlapping signal at the first 8–10 steps of a sample) in a scheme of a specially developed FTF–TDE processor (time delay estimator built around a fast transversal filter) using cubic splines for smoothing of the signals to be compared.

The problem for two sensors is formulated by the equations

$$\left.\begin{aligned} x_1(k) &= s(k) + n_1(k)\,, \\ x_2(k) &= a\,s(k+\Delta) + n_2(k)\,, \end{aligned}\right\} \qquad k = 0, 1, \ldots, \qquad (1)$$

where $s(k)$ and $a\,s(k + \Delta)$ are the samples of the signals $s(t)$ and $\alpha\,s(t + \Delta)$, respectively; $n_1(k)$ and $n_2(k)$ are the random noise sequences (white Gaussian noise with zero mean value) independent of one another and of the signal $s(t)$:

$$\overline{n_1(k)} = \overline{n_2(k)} = 0\,,$$
$$\overline{n_i(k)\,n_j(l)} = Q_i\,\delta(k-l)\,\delta_{ij}\,,\ \ i, j = 1, 2\,; \qquad (2)$$

$\delta(k - l)$ is the discrete analogue of the Dirac delta–function; δ_{ij} is the Kronecker symbol; the bar atop the symbol indicates ensemble averaging. The delay Δ must be estimated from the given realization of the random processes $x_1(k)$ and $x_2(k)$.

A classical approach to solving this problem is to calculate the function $R_{x_1x_2}(\tau)$ of cross correlation between x_1 and x_2 and to determine the delay $\tau = \tau_0$ at which the function reaches its maximum.[1] The value τ_0 is taken as the estimate of Δ: $\tau_0 = \hat{\Delta}$. The method of adaptive filtration has come into use for the same purposes recently.

The method implies that the signal copies to be compared, for example, $x_1(k)$ and $x_2(k)$ from Eq. (1), are fed as input and reference signals into an adaptive filter operating in the mode of identification of a linear system connected in parallel. After the adaptation period that lasts m sample steps, the pulse response characteristic of the filter represented by a column vector of p numbers in the nth step

$$h_p(n) = [h_1(n), \ldots, h_p(n)]^{\mathrm{T}},$$

is transformed so that its convolution with the input signal $x_1(k)$ approximates best the reference signal $x_2(k)$ (by the criterion of the least mean–square (LMS) error).

Let us designate

$$x_{1p}(n) = [x_1(n), x_1(n-1), \ldots, x_1(n-p+1)]^{\tau}$$

and

$$x_{2p}(n) = [x_2(n), x_2(n-1), \ldots, x_2(n-p+1)]\,,$$

then

$$x_{2p}(n) = h_p^{\tau}(n) * x_{1p}(n),\ \ n > m\,,$$

where T is the transposition sign, and asterisk is the convolution sign. One determines the delay Δ from the position of maximum of the dependence $h_i(n)$ on i $(1 \le i \le p)$ for $n > m$; for signals of identical structure with low level of noise, the delay is close to the Dirac delta–function $h_i(n) \approx \delta(i - \hat{\Delta})$, where $\hat{\Delta}$ is the estimate of Δ.

John, Ahmed, and Carter[3] proposed to use the Whidrow adaptive filter[6] operating in the mode of identification of unknown system as a time delay estimator (TDE) and described the algorithm LMS TDE. However, although this algorithm solves the problem of TDE with high accuracy and reliability, it is insufficiently fast for operation in real time. The FTF TDE algorithm, proposed and implemented in Ref. 4, is free of this disadvantage. It is based on the adaptive fast transversal filter[5] (FTF) that provides data processing in real time.

The FTF–TDE works well with low noise level, but its characteristics deteriorate rapidly as the noise level increases and fall outside the limits specified by the form of the function $s(t)$ and the sample step. In this paper, we investigate the feasibility of increasing the accuracy and reliability of the FTF–TDE operation when the additive noise background falls outside this limit, by means of including the operations of preliminary filtering of input and reference signals, namely, their approximation by smoothing splines, into the scheme.

One can find the general definition of the approximation spline functions in Ref. 6. Let X and Y be two Hilbert spaces, and T: $X \to Y$ be the linear operator acting from X to Y. Let the system of linear finite functionals k_i $(i = 1, \ldots, n)$ that is supposed linearly independent be set in X. If the element $\sigma \in X$ satisfies two conditions

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1) $k_i(\sigma) \equiv (k_i, \sigma)_X = r_i$, $i = 1, \dots , n$,

2) $(T\sigma, T\sigma)_Y \equiv > T\sigma >^2_Y = \min$,

where $(,)_X$ and $(,)_Y$ are the scalar products in X and Y, respectively, and r_i are the given numbers, then it is referred to as interpolation spline. (The possibility of representation of the functional k_i on the vector σ by a scalar product follows from the Riesz theorem on representation of linear functional in the Hilbert space.) The element $\sigma_\alpha \in X$ is called smoothing spline if it minimizes the square functional

$$\Phi_\alpha(u) = \alpha > T u >^2_Y + \sum_{i=1}^{n} [(k_i, u) - r_i]^2 = \alpha > T u >^2_Y + > K u - r >^2_{R^n} ,$$

$\alpha > 0$.

The discrepancy of equation $Ku = r$ and the energy functional $> T u >^2$ are combined in this functional with the weight $\alpha > 0$. The minimization of $\Phi_\alpha(u)$ yields no spline of a new type. The smoothing spline σ_α is the interpolation spline with the vector of input data $r_\alpha = K \sigma_\alpha$.

In the examined problem of spline approximation of noisy signals in the FTF–TDE scheme, T is the operator of double differentiation, and K is the operator of spur of function being reconstructed on the grid $x_1, \dots, x_n$. The peculiarity of this problem is the fact that redundant smoothing of input data and losses of their fine structure connected with the use of too strong smoothing operators are not awful, if they produce no relative time delay of the dependence to be compared. The experience shows this is the case in the majority of TDE situations arising in practice, i.e., the selection of the specific value of the regularization parameter α at the preliminary filtration stage is not too critical. It makes finding of the optimum (more exactly, quasioptimum) numerical value of the parameter α more easy.

The spline was constructed by the classical Reinsh method.[7,8] The problem is formulated as follows. Let the pairs of numbers x_i, y_i, $i = 0, 1, \dots, n$ be given, with $x_0 < x_1 < \dots < x_n$. It is necessary to find the function that minimizes the expression

$$\int_{x_0}^{x_n} g''(x)\, \mathrm{d}\, x \qquad (3)$$

among all $g(x)$ so that

$$\sum_{i=0}^{n} \left(\frac{g(x_i) - y_i}{\mathrm{d}\, y_i} \right)^2 \le S, \qquad g \in C^2 [x_0, x_n]. \qquad (4)$$

Here $\delta y_i > 0$, $i = 0, 1, \dots, n$; $C^2[x_0, x_n]$ is the class of functions that are continuous together with their second derivatives in the interval $[x_0, x_n]$; and, S is the constant introduced for convenience. This makes it possible to scale the values δy_i when adjusting the smoothing degree. The recommended values S depend on the relative weight δy_i^2. When there is any *a priori* information, one usually takes the standard deviations of the values y_i as δy_i. In this case, the natural values S are within the confidence interval corresponding to the left–hand side of Eq. (4)

$$N - (2N)^{1/2} \le S \le N + (2N)^{1/2}, \qquad N = n + 1 . \qquad (5)$$

Equations (3) and (4) are solved by standard methods of calculus of variations. Introducing the auxiliary variable z and the Lagrange parameter p, we come to the problem of minimizing the functional

$$\int_{x_0}^{x_n} g''(x)\, \mathrm{d}\, x + p \left\{ \sum_{i=0}^{n} \left(\frac{g(x_i) - y_i}{\mathrm{d}\, y_i} \right)^2 + z^2 - S \right\} . \qquad (6)$$

The unambiguous solution is the cubic spline called the Schönberg spline, i.e., the function $f(x)$ consisting of the cubic parabolas

$$f(x) = a_i + b_i\,(x - x_i) + c_i\,(x - x_i)^2 + d_i\,(x - x_i)^3 , \qquad (7)$$

$x_i \le x < x_{i+1}$, $i = 0, 1, \dots , n$,

conjugate through common end points, with f, f', and f' being continuous.

A program (writen in C) has been developed that implements the procedure of calculation of all the coefficients a_i, b_i, c_i, and d_i ($i = 0, \dots, n$) included in Eq. (7) and the parameter p.

Application of the cubic smoothing spline at the stage of preliminary filtration of the input and reference signals complicates the FTF–TDE algorithm not too much,[4] but substantially improves its stability for the additive noise effect. The spline approximation of input and reference signals in the FTF–TDE scheme makes it possible to keep the error in determining the time delay Δ within the limits of a sample interval up to the level of noise whose standard deviation corresponds to the increment s at the first 8–10 steps. The experimental data confirming this conclusion are shown in Figs. 1 and 2 for typical situation described by Eq. (1) with

$$s(t) = A\, t \exp(-t/T) ,$$

where $T = 10$, $A = 3.7$, $a = 0.7$, $\Delta = 4$. Here $h_1(t)$ and $h_2(t)$ are the independent levels of noise produced by a generator of random numbers.

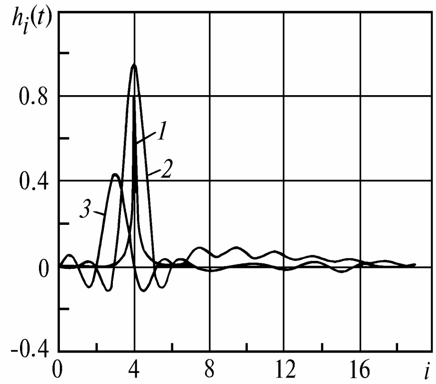

FIG. 1. FTF–TDE response function for unsmoothed signals with different levels of noise.

Figure 1 shows the pulse response characteristics of a filter $h(t)$ interpolated by the Kotel'nikov–Shennon formula (in the experiment, it was represented by a set of numbers h_i, $i = 1, \dots, p$, $p = 20$) in a steady mode of adaptation for signals $x_1(t)$ and $x_2(t)$ without noise (curve *1*) and for the same signals against the additive white noise background with zero mean value and standard deviation $\sigma = 0.2$ (curve *2*) and 0.8 (curve *3*). The position of maximum in the first two curves is very close (within the limits of error ±0.01) to the delay $\Delta = 4$; in the third curve, it is shifted from this value by 1.4. The same curves are shown in Fig. 2 for $\sigma = 0.8$ with spline smoothing of input and reference signals (curve *1*) and without smoothing (curve *2*). As is seen from Fig. 2, the smoothing procedure eliminates the distorting effect of the noise on the processor accuracy characteristics.

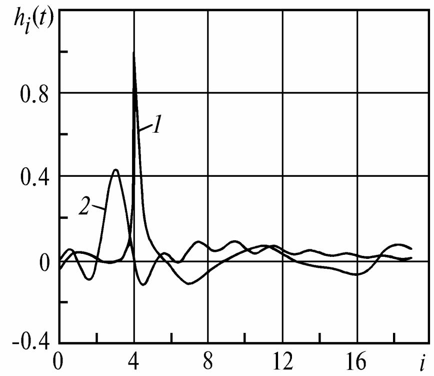

FIG. 2. FTF–TDE response function for noisy signal without (2) and with (1) smoothing.

The results of our investigation enable us to draw the following conclusion. Including the preliminary filtration procedure with the use of smoothing splines into the FTF–TDE scheme essentially improves the processor characteristics and makes it possible to obtain the reliable and highly accurate algorithm for estimating the time delay of signals for solving different problems of applied optics including the problem of determination of the position of isotropic pulse radiation source from the data of satellite observations.

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