HOLOGRAPHIC RECORDING OF OPTICALLY SOFT MICROPARTICLES. CALCULATIONAL PROCEDURE

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The method is proposed being a basis for numerical calculation of the intensity distribution in the hologram plane, in holographic and magnified holographic images. Both transparent and opaque particles of an arbitrary shape can be considered as objects for holographing. The method can easily be adapted to basic holographic schemes.

Holographic methods for recording volume ensembles of microparticles are used for studying aerosols in laboratory experiments,[1] investigating the process of laser pulse interaction with a water drop,[2] for diagnostics of natural aerosol media (mists, fogs, clouds),[3,4] and so on. In practice in all works concerning holographic recording of microparticles, the particles are considered as opaque screens. It is justified since the hologram is normally recorded in a far zone because of small size of particles

$$k\, a^2 \ll z\ . \tag{1}$$

Here $k = 2\pi/\lambda$; λ is the radiation wavelength; z is the distance between a particle and the hologram; a is the particle size. If the condition

$$k\, a\ |n - n_0| \gg 1 \tag{2}$$

holds, the field refracted by a particle can be neglected in the far zone.[5] In this case the hologram is believed to be formed only by the field diffracted by the particle central cross section and by the field of a reference wave as in the case of an opaque screen. Here n and n_0 are the refraction indices of a particle and the medium, respectively.

However, if the condition (2) does not hold, in calculating the intensity distribution in the hologram plane, not only the diffracted field but also the refracted one should be taken into account. The break of condition (2) may happen if (*a*) a recorded particle is small, $a \geq \lambda$, or (*b*) a particle is optically soft, i.e. $n \approx n_0$. The case (*a*) is of no interest since the resolution of holographic methods is of the order of several micrometers (5–10) that corresponds to $a \gg \lambda$ for the visible spectral range. The case (*b*) may take place in the case of suspensions (biological, medical, etc.).

Moreover, the scheme with a transfer of images of microparticles to the hologram plane is most common now. In such a scheme, the condition of far zone (1) does not hold for some particles that also results in the necessity of taking into account the refracted field.

Only few papers[6,7] deal with the peculiarities of holographic recording of transparent and semitransparent particles, and the theoretical description of methods is given only for some particular cases. In this paper we propose a procedure for calculating the intensity in the hologram plane, in reconstructed and magnified holographic images of microparticles. This procedure can be applied to recording of optically soft particles of an arbitrary shape at arbitrary distances from the hologram.

1. BASIC OPTICAL SCHEME

The scheme shown in Fig. 1 is often preferable by experimenters (see, for example, Refs. 2 and 8) by a number of reasons. First, the objective *6* allows the distance to the ensemble under study to be increased that is required in many studies requiring contactless investigation. Second, the use of a mask *7*, which cuts off the radiation passed without scattering by particles, provides for implementing the dark field method and thus improves the contrast of particle images. And, finally, the use of an off–axis scheme allows researchers to adjust the ratio of reference beam intensity to that of an object beam whatever the concentration of the ensemble under study is.

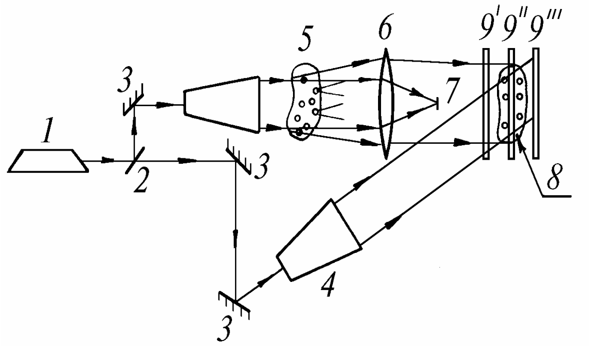

FIG. 1. Basic optical scheme used in calculations: a laser (1), a semitransparent mirror (2), a mirror (3), beam wideners (4), a volume of disperse medium (5), an objective (6), an opaque mask (7), an image of medium volume (8), and possible positions of photorecording material (9', 9'', and 9''').

Moreover, the procedure for calculating the intensity distribution in the plane of hologram recording for this scheme can easily be adapted for an ordinary off–axis hologram. To do this, the objective *6* and the mask *7* are removed from the scheme (and from the calculational algorithm). If the reference beam is also removed, then we have the axial scheme traditional for recording of microparticle ensembles.

The hologram position *9''*, which is usual for holography of focused images, does not suit the case under consideration. This is because at the stage of reconstruction one part of the holographic image of a volume will be virtual whereas another one is real. To magnify the reconstructed microparticle images, the microoptics with a small depth of focus is used what arises some problems in studying virtual holographic images.

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If at the stage of recording the hologram is at the *9‴* position, then at the stage of reconstruction it is exposed to a conjugated reference wave to reconstruct the real image from it. Then the reconstructed image is positioned relative to the hologram as in the case *9′* (see Fig. 1).

In this connection, the scheme shown in Fig. 1 with the hologram being at the position *9′* is the most general one, and it is expedient to construct the calculational procedure just for this case.

3. RECORDING AND RECONSTRUCTION OF A HOLOGRAM

In accordance with the previous section, the distances along the axis are related as follows

$$1/z_0 + 1/(z + \Delta) = 1/F , \qquad (3)$$

where F is the focal length of the optical system performing the image transfer. For simplicity we consider this system as a thin lens keeping in mind that when passing to a more complicated optical system the calculated intensity distributions will not change.

Let the ensemble under study be illuminated with a plane wave of a unit amplitude and $t(x_0, y_0)$ be the transmission coefficient corresponding to the medium layer in the plane (x_0, y_0) (or $z = 0$). Then in the hologram plane the object wave has the following form:

$$u(x_2, y_2) = \int t(x_0, y_0) \exp\{(ik/2\, z_0)[(x_0 - x_1)^2 + (y_0 - y_1)^2]\} \times$$
$$\times P_0(x_1, y_1) \exp[(-\, ik/2\, F)\, (x_1^2 + y_1^2)] \times$$
$$\times \exp\{(ik/2\, z)\, [(x_1 - x_2)^2 + (y_1 - y_2)^2]\}\, dx_0\, dy_0\, dx_1\, dy_1 \, . \qquad (4)$$

In Eq. (4) the constant factor is omitted, and all the below expressions will be written accurate to a constant factor. In Eq. (4) $P_0(x_1, y_1)$ is the function of the objective pupil *2* (Fig. 2), the fourth multiplier in Eq. (4) is the phase factor of a lens, and the expression (4) as a whole is the Kirchhoff integral in the Fresnel approximation for the considered system. Note that the presence or absence of a mask *7* in the optical arrangement shown in Fig. 1 is shown by the function $t(x_0, y_0)$.

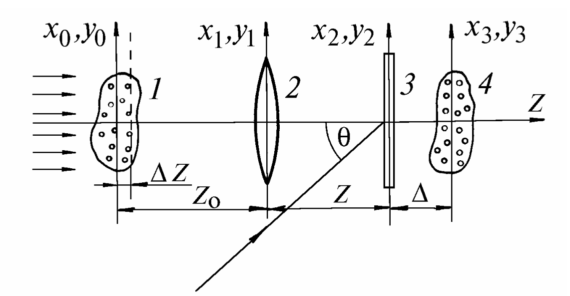

FIG. 2. On the calculation of hologram recording and reconstruction: a volume of a disperse medium (1), an objective (2), a hologram (3), and reconstructed image of a medium volume (4).

The reference wave is considered to be a plane wave (this coincides with the majority of experimental schemes):

$$r = r_0 \exp(ik\, x_2 \sin\theta) , \qquad (5)$$

where θ is the angle between the directions of propagation of reference and object waves (Fig. 2). Then, as known, the intensity in the plane (x_2, y_2) of the hologram recording is described by the expression:

$$I(x_2, y_2) = |u(x_2, y_2)|^2 + |r(x_2, y_2)|^2 + u^*(x_2, y_2)\, r(x_2, y_2) +$$
$$+ u(x_2, y_2)\, r^*(x_2, y_2) \, . \qquad (6)$$

To allow for the extent of medium volume (marked as *1* in Fig. 2) along z axis, it should be divided into monolayers. The division step Δz is governed by size and number density of microparticles. In this case it is assumed that particle mutual screening can be neglected. This assumption is justified for the case of holographic recording. Actually, the particle images reliably distinguishable against speckles are obtained at a small concentration of particles, when ensemble optical density $\sigma N l \le 0.1$ what corresponds to 80% transparency of an ensemble.[8] Here N is the particle number density, l is the layer thickness, and σ is the geometrical area of particle cross section. Then the object wave may be presented as follows:

$$u(x_2, y_2) = \sum_{j=1}^{m} \{u_j(x_2, y_2)\} , \qquad (7)$$

where $u_j(x_2, y_2)$ is determined by Eq. (4) for jth particle, m is the number of particles in the layer, and intensity distribution in the hologram plane is written as

$$I(x_2, y_2) = \left| \sum_{j=1}^{m} u_j(x_2, y_2) + r(x_2, y_2) \right|^2 =$$
$$= \sum_{j=1}^{m} |u_j|^2 + \sum_{j \ne i} u_i\, u_j^* \quad + |r|^2 + r \sum_{j=1}^{m} u_j^* + r^* \sum_{j=1}^{m} u_j \, . \qquad (8)$$

As known, for linear recording of a hologram the condition $|u_j|^2 \ll |r|^2$ must hold, therefore the term $\sum_{j \ne i} u_i\, u_j^*$ can be neglected in some cases, and the expression (8) can be reduced to the sum of m holograms of the type

$$I(x_2, y_2) = \sum_{j=1}^{m} \{ |u_j|^2 + |r_j|^2 + u_j^*\, r_j + u_j\, r_j^* \},$$

where $r_j = r/m$. In this paper we use this simplification in order to obtain analytically some results illustrating the method of calculation. However, in numerical calculations for optically soft particles this term should be taken into account.

When illuminating the recorded hologram by the initial reference wave r, only the wave $ur^*r = ur_0^2$ propagates along z axis, therefore the field in the reconstructed image is calculated by the expression similar to Eq. (7). Thus, the field in the plane corresponding to the image of the plane $z = 0$ is expressed as

$$u(x_3, y_3) = \int t(x_0, y_0) \exp\{(ik/2\, z_0)[(x_0 - x_1)^2 + (y_0 - y_1)^2]\} \times$$
$$\times P_0(x_1, y_1) \exp[(-\, ik/2\, F)\, (x_1^2 + y_1^2)] \times$$
$$\times \exp\{[\, ik/2\, (z + \Delta)]\, [(x_1 - x_3)^2 + (y_1 - y_3)^2]\} dx_0 dy_0 dx_1 dy_1 . \quad 9)$$

Here the linear recording of hologram is assumed. By varying Δ and substituting $t(x_0, y_0)$ into the corresponding plane of a volume studied, the field distribution can be calculated in any plane of a holographic image.

Let us consider the case $P_0(x_1, y_1) = 1$, as an example. Then such calculations can be performed analytically, and for the object plane $z = 0$ we obtain

$$u(x_3, y_3) = t[-(x_3 z_0)/(z + \Delta), -(y_3 z_0)/(z + \Delta)] \times$$
$$\times \exp[ik(x_3^2 + y_3^2)/2(z + \Delta - F)]. \quad (10)$$

As expected, in the image we obtain the initial object with the transmittance t, and the magnification $z_0/(z + \Delta)$ is determined by the properties of the objective *2* (Fig. 2). As to the phase factor, it disappears when passing to the intensity.

3. STUDY OF THE HOLOGRAPHIC IMAGE

To find the parameters of an ensemble under study, the holographic image is studied with the help of a magnifying system *5* (Fig. 3). A microobjective is most commonly used for that, and the magnified image *6* is projected to the eye retina (using an ocular), photographic plate, CCD, or photodiode matrix, depending on the problem to be solved.

In Fig. 3 the plane (x_5, y_5) corresponds to the central plane (x_3, y_3) of the volume holographic image and, consequently, to the plane (x_0, y_0) of an object (or the plane $z = 0$, see Fig. 2). Obviously the distances Δz, Δ, and δ are related to each other. This allows us by varying only one of these distances to automatically change two others.

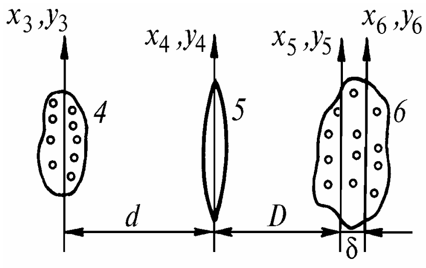

FIG. 3. On the calculation of magnified holographic image: a reconstructed image (4), a magnifying optical system (5), and a magnified image (6).

The field in the plane (x_6, y_6) can be written as

$$u(x_6, y_6) = \int u(x_3, y_3) \exp\{(ik/2d)[(x_3 - x_4)^2 + (y_3 - y_4)^2]\} \times$$
$$\times P(x_4, y_4) \exp[(-i k/2f)(x_4^2 + y_4^2)] \times$$
$$\times \exp\{(ik/2D)[(x_4 - x_5)^2 + (y_4 - y_5)^2]\} \times$$
$$\times \exp\{(ik/2\delta)[(x_5 - x_6)^2 + (y_5 - y_6)^2]\} dx_3 dy_3 dx_4 dy_4 dx_5 dy_5. \quad (11)$$

Here f is the focal length of the system *5* (Fig. 3). Integrating with respect to x_5, y_5 yields the following expression (accurate to the constant factors):

$$u(x_6, y_6) = \int u(x_3, y_3) \exp[(ik/2d)(x_3^2 + y_3^2)] \times$$
$$\times P(x_4, y_4) \exp[(ik/2L)(x_4^2 + y_4^2)] \times$$
$$\times \exp\{(-ik)[x_4[x_3/d + x_6/(D + \delta)] + y_4[y_3/d + y_6/(D + \delta)]]\} \times$$
$$\times \exp\{[ik/2(D + \delta)](x_6^2 + y_6^2)\} dx_3 dy_3 dx_4 dy_4, \quad (12)$$

where

$$1/L = (fD\delta + fD^2 - dD\delta - dD^2 + dfD)/[dfD(\delta + D)]. \quad (13)$$

For $\delta = 0$, $1/L = 1/d - 1/f + 1/D = 0$ and in the plane $(x_6 = x_5, y_6 = y_5)$ we obtain the expression:

$$u(x_6, y_6) = \int u(x_3, y_3) \exp[(ik/2d)(x_3^2 + y_3^2)] P(x_4, y_4) \times$$
$$\times \exp\{(-ik)[x_4(x_3/d + x_6/D) + y_4(y_3/d + y_6/D)]\} \times$$
$$\times \exp\{(ik/2D)(x_6^2 + y_6^2)\} dx_3 dy_3 dx_4 dy_4 = \exp\{(ik/2D)(x_6^2 + y_6^2)\} \times$$
$$\times \int u(x_3, y_3) \exp[(ik/2d)(x_3^2 + y_3^2)] \times$$
$$\times G[(1/\lambda)(x_3/d + x_6/D), (1/\lambda)(y_3/d + y_6/D)] dx_3 dy_3. \quad (14)$$

Assuming, as before, that $(x_3^2 + y_3^2)/\lambda d \ll 1$, the final expression can be written for the field $u(x_6, y_6)$ where the integrand is the field $u(x_3, y_3)$ and Fourier transform of a pupil of the microobjective *5* (Fig. 3) or, in other words, the convolution of the field $u(x_3, y_3)$ reconstructed from the hologram with the pulse response G of a microobjective *5*. In the case $\delta \neq 0$ the expression (14) does not change in structure, but the Fourier transform of a generalized function of a pupil of the objective *5* $P'(x_4, y_4)\exp[(ik/2L)(x_4^2 + y_4^2)]$ will be present in it.

So, the calculational expressions presented in this section allow the field and intensity distributions in the hologram plane to be calculated in the region of reconstructed and magnified images. In so doing, the functions $t(x_0, y_0)$, $P_0(x_1, y_1)$, and $P_1(x_4, y_4)$ should naturally be given in their concrete forms.

It should be noted that these expressions can be used for both opaque "rigid" particles (obeying the condition (2)) and optically soft particles. The type of particles is allowed for by calculating the particle transmission function $t(x_0, y_0)$.

4. CALCULATION OF TRANSMISSION FUNCTION OF A TRANSPARENT PARTICLE

For the case of optically soft particles the reflection from the medium/particle interface can be ignored in calculations. Here we consider the case of a transparent particle, therefore the absorption can be ignored too. For the case of a semitransparent particle the absorption is easily calculated. The main idea of calculational procedure is identical to that in Ref. 7, where the authors have calculated the field scattered by a particle assuming that this field propagates from the central particle cross section (x_0, y_0).

This idea provides a possibility of using the diffraction integral in calculation. The only difference is that in our calculations the phase shift within the particle is calculated directly from the ray geometry, whereas in Ref. 7 it was taken into account via the factor describing the aberrations. This is why the method from Ref. 7 is not applicable to particles of a complificated shape. In addition, in our calculations we assume that the field scattered by a particle propagates from the plane tangent to it (the plane $z = a_2$ in Fig. 4). In the case when, in the selected monolayer of the medium, there are particles of different size, this plane is chosen to be tangent to the largest particle.

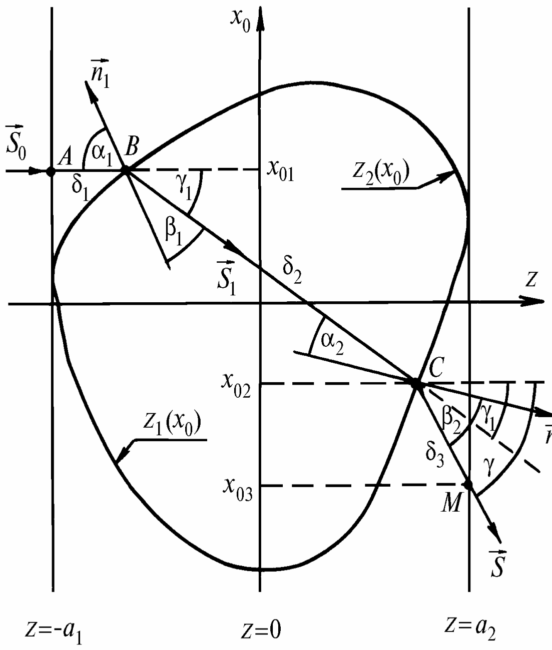

FIG. 4. On the calculation of transmission function of a transparent particle.

Let us perform the calculation for a 2D case, since it can be easily generalized to the 3D case. Shown in Fig. 4 is a particle of an arbitrary shape, n_0 and n are the refraction indices of the medium and the particle, respectively, the particle is illuminated with a plane wave of a unit amplitude and the ray vector $\mathbf{S}_0$. The plane $z = 0$ passes through the central cross section, whereas the planes $z = -a_1$ and $z = a_2$ pass through the outer points of a particle. The plane $z = -a_1$ is, in this case, the wave front of the illuminating wave.

Let us consider the beam passage through such a particle. The segment AB is the path δ_1 passed by an incident wave from the plane $z = -a_1$ to the particle boundary, the segment CM is the path δ_3 passed by the refracted wave from the particle boundary to the plane $z = a_2$, and the segment BC is the path δ_2 passed inside the particle; $\mathbf{S}_0$, $\mathbf{S}$, and $\mathbf{S}_1$ are the corresponding ray vectors.

We present the field at an arbitrary point of the plane $z = a_2$ as $u(x_0, z) = \exp[(i\varphi(x_0)]$, where

$$\varphi(x_0) = \varphi(z = -a_0) + 2\pi/\lambda\,[(\delta_1 + \delta_3)\,n_0 + \delta_2\,n]. \tag{15}$$

Then, to find the phase $\varphi(x_0)$, it is necessary to calculate the distances δ_1, δ_2, and δ_3 knowing the shapes of surfaces (in the considered 2D case, the curves) $z_1(x_0)$ and $z_2(x_0)$(see Fig. 4). Since the plane $z = a_2$ is positioned immediately adjacent to the particle, there are no diffraction effects and we deal with the geometrical approach.

Let us consider the ray passing through the points $B(x_{01}, z_1(x_{01}))$ and $C(x_{02}, z_2(x_{02}))$. It is easy to determine the distance δ_1 as

$$\delta_1 = z_1(x_{01}) - a_1\,. \tag{16}$$

For the case of known $z_1 = z_1(x_0)$ and using standard methods of analytical geometry, we can express the unit vector of the normal to the first surface in the form:

$$\mathbf{n}_1 = \mathbf{i}\sin\alpha_1 + \mathbf{j}\cos\alpha_1\,, \tag{17}$$

then we can determine the ray vector $\mathbf{S}_1$:

$$\mathbf{S}_1(x_{01}) = \mathbf{i}\sin\gamma_1 + \mathbf{j}\cos\gamma_1\,; \tag{18}$$

$$\gamma_1 = \alpha_1 - \beta_1; \qquad \sin\alpha_1 = (n/n_0)\sin\beta_1\,. \tag{19}$$

If γ_1 is known, the distance δ_2 can be easily found:

$$\delta_2^2 = (x_{01} - x_{02})^2 + (z_1(x_{01}) - z_2(x_{02}))^2, \tag{20}$$

$$\delta_2 = (x_{01} - x_{02})/\sin\gamma_1\,. \tag{21}$$

Note that the segment δ_2 can be presented as a function of a single coordinate (x_{01} or x_{02}), since Eqs. (20) and (21) allow one coordinate to be unambiguously expressed via the another one:

$$\sin\gamma_1 = (x_{01} - x_{02})/\sqrt{(x_{01} - x_{02})^2 + (z_1(x_{01}) - z_2(x_{02}))^2}\,, \tag{22}$$

if $\sin\gamma_1$ is known.

With the account for the shape of the curve $z_2(x_0)$ and having determined the normal vector $\mathbf{n}_2$ at the point $(x_{02}, z_2(x_{02}))$, it is possible to calculate the ray vector of a ray emerging from the particle:

$$\mathbf{S}(x_{02}) = \mathbf{i}\sin\gamma + \mathbf{j}\cos\gamma\,, \tag{23}$$

where

$$\gamma = \gamma_1 + (\beta_2 - \alpha_2), \qquad n\sin\alpha_2 = n_0\sin\beta_2\,. \tag{24}$$

Then the segment δ_3 is equal to

$$\delta_3 = (x_{02} - x_{03})/\sin\gamma\,. \tag{25}$$

Moreover, similarly to Eq. (22) the relation between x_{02} and x_{03} can be written as:

$$\sin\gamma = (x_{02} - x_{03})/\sqrt{(x_{02} - x_{03})^2 + (z(x_{02}) - a_2)^2} \tag{26}$$

Let us return now to the 3D case. The proposed calculational procedure allows the field refracted by a particle, $u(x_0, y_0)$, to be found in the plane $(z_0 + a_2)$. Naturally, in this procedure of calculation the factor $ik/2z_0$ in the first exponent in Eq. (4) should be replaced by $ik/2(z_0 - a)$.

Let $t_s(x_0, y_0)$ be the function describing the particle cross section (the particle shadow function):

$$t_s(x_0, y_0) = \begin{cases} 1 & \text{— inside the particle cross section,} \\ 0 & \text{— outside the particle cross section.} \end{cases}$$

Then the transmission function, which should be substituted into the diffraction integral (4), has the following form:

$$t(x_0, y_0) = t_s(x_0, y_0) + u(x_0, y_0)\,. \tag{27}$$

If there is no the opaque mask *7* in the scheme shown in Fig. 1, the expression (27) takes the form:

$$t(x_0, y_0) = 1 - t_s(x_0, y_0) + u(x_0, y_0)\,. \tag{28}$$

Let us illustrate the procedure for calculating the particle transmission function by a simple example. Let the particle have a shape of a hemisphere shown in Fig. 5 and be in the air, i.e. $n_0 = 1$.

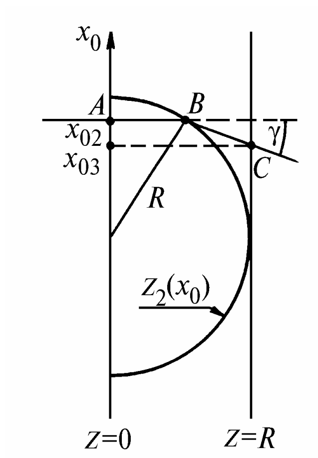

FIG. 5. On the calculation of transmission function of a hemispherical particle.

The procedure of calculation of the field in the plane $z = R$ in this case reduces to the determination of the segment AB

$$\delta_2 = \sqrt{R^2 - (x_{02})^2} \tag{29}$$

and the segment BC

$$\delta_3 = \sqrt{(x_{03} - x_{02})^2 + \left(R - \sqrt{R^2 - (x_{02})^2}\right)^2}, \tag{30}$$

where the coordinates x_{02} and x_{03} are related to each other as

$$\sin\gamma = (x_{02}/R)\left(n\sqrt{1-(x_{02})^2/R^2} - \sqrt{1-(n\,x_{02})^2/R^2}\right); \tag{31}$$

$$\sin\gamma = (x_{03} - x_{02})/\sqrt{(x_{03} - x_{02})^2 + \left(R - \sqrt{R^2 - (x_{02})^2}\right)^2}. \tag{32}$$

Here the expression (31) is obtained based on the law of refraction on spherical surface whereas equation (32) is determined by the geometry of the picture. Having expressed x_{02} via x_{03} and substituting it into Eqs. (29) and (30), we obtain $\delta_2 = R - (x_{03})^2/2R$ and $\delta_3 = (x_{03})^2/2R$. For the field distribution in the plane $z = R$ in paraxial case, we obtain, as could be expected, the following result:

$$u(x_{03}, z) = \exp\left\{ik\left[Rn - \left(\frac{(x_{03})^2}{2R}\right)(n-1)\right]\right\}$$

The distribution obtained is the field of a spherical wave, written in paraxial approximation, converging to the point placed on z axis at a distance $R/(n - 1)$ from the plane $z = R$, i.e. at paraxial focus of the considered lens.

CONCLUSION

The method proposed can be the basis for numerical calculation of intensity distribution in the hologram plane in both holographic and magnified holographic images. The calculations can be performed both for the cases when the particle coordinate is an origin of a coordinate system and when the coordinate of the magnified holographic image of a particle is used for this. Both opaque and transparent particles of an arbitrary shape can be considered. The procedure can easily be adapted to the main holographic schemes. In development of the method we took two simplifying assumptions: (a) only the case of single scattering is considered and (b) the radiation reflected by particles is excluded from the consideration. Both these simplifications are in agreement with the majority of practical situations taking place in holographic recording of microparticles.

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