Cyclotron radiation of charged particles exposed to permanent stochastic perturbation

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The permanent stochastic perturbation influence on charged particles in a stationary magnetic field is under theoretical study. For this purpose, the non-linear Schrödinger equation is solved. It is found that a large number of cyclotron radiation harmonics is excited in weak magnetic fields. The number of harmonics decreases with increase of the field intensity, and radiation can disappear completely at a certain intensity magnitude. Probabilities of populating of for quantum levels induced by oscillators' field are plotted as functions of temperature.

We consider influence of stochastic perturbation on the character of spin-free charged particle motion in a stationary magnetic field.

To analyze the behavior of particles in a magnetic field under permanent stochastic perturbations, the non-linear Schrödinger equation is used, constructed by the method of path integrals for systems that undergo permanent stochastic perturbation.[1,2] If the scalar potential and transit rate of the subsystem in the thermostat in this equation are zero, we obtain

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{1+i\alpha}\left[\frac{1}{2m}\left(\hat{\mathbf{P}} - \frac{q}{c}\mathbf{A}\right)^2 + \chi\right]\psi + \frac{i\alpha}{1+\alpha^2}\langle\psi|\times$$

$$\times\left[\frac{1}{2m}\left(\hat{\mathbf{P}} - \frac{q}{c}\mathbf{A}\right)^2 + \chi\right]|\psi\rangle\psi, \qquad (1)$$

where $\hat{\mathbf{P}}$ is the pulse operator; $\mathbf{A}$ is the vector potential of the external field; q is the particle discharge; m is its mass; c is velocity of light; α is a small positive parameter, whose magnitude depends on the density of the medium surrounding the subsystem; $\chi = kT^*/2$ (k is the Boltzmann constant; T^* is the efficient ambient temperature).

We solve this equation in two steps. At the first step, we find a solution of the stationary reduced equation, in which the non-linear summand is absent. At the second stage, we study the effect of non-linearity of the obtained equation on stability of these states. Introduce the Cartesian coordinate system and suppose that the magnetic field is oriented along the Z-axis. In this case, the vector potential has the following components:

$$A_x = -Hy;\ \ A_y = A_z = 0. \qquad (2)$$

Here H is the magnetic field intensity.

The reduced linear equation following from Eq. 1 has the form

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{1+i\alpha}\left[\frac{1}{2m}\left(\hat{\mathbf{P}} - \frac{q}{c}\mathbf{A}\right)^2 + \chi\right]\psi. \qquad (3)$$

If we represent the possible solution of the equation (3) in the form

$$\psi = \exp\left(-i\frac{E}{\hbar}t\right)\psi(\mathbf{r}),$$

(E is the distribution constant), the stationary equation

$$\tilde{E}\psi = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{q}{c}\mathbf{A}\right)^2\psi, \qquad (4)$$

where

$$\tilde{E} = (1 + i\alpha)E - \chi$$

is valid for the coordinate function $\psi(\mathbf{r})$.

The solution of equation (4) is well-known.[3,4] Representation of the wave function as a product

$$\psi(\mathbf{r}) = \exp\left(\frac{i}{\hbar}(p_x x + p_z z)\right)\varphi(y)$$

permits one, with allowance for Eq. (2), to write the equation

$$\frac{d^2\varphi}{dy^2} + \frac{2m}{\hbar}\left(\tilde{E} - \frac{p_z^2}{2m} - \frac{m}{2}\left(\frac{qH}{mc}\right)^2(y - y_0)^2\right)\varphi = 0, \quad (5)$$

where

$$y_0 = -cp_x/(qH).$$

The equation (5) formally coincides with that for the harmonic oscillator, fluctuating about the point y_0 with the cyclic frequency

$$\omega = |qH/(mc)|.$$

Therefore, wave functions of a particle, which exists in a stationary magnetic field and undergoes permanent stochastic perturbation can be expressed by the Hermite polynomial:

$$\psi_n(\mathbf{r}) = \exp\left(\frac{i}{\hbar}(p_x x + p_z z)\right)\exp\left(-\frac{\xi^2}{2}\right)H_n(\xi),$$

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where

$$\xi = \sqrt{m\omega/\hbar}(y - y_0).$$

For the separation constant $\tilde{E}$, holds

$$\tilde{E}_n = \left(n + \frac{1}{2}\right)\hbar\left|\frac{qH}{mc}\right| + \frac{p_z^2}{2m},$$

where $n = 0, 1, 2, \ldots$, then the initial separation constant in the reduced Schrödinger equation is

$$E_n = \frac{\left(n + \frac{1}{2}\right)\hbar\left|\frac{qH}{mc}\right| + \frac{p_z^2}{2m} + \chi}{1 + i\alpha}.$$

Although the obtained expressions are close to the corresponding relations for the states of a permanently perturbed oscillator,[2] some features are revealed when analyzing the stability of quantum states of a particle in a magnetic field. Let us examine them in more detail.

Represent the solution of equation (1) in the form

$$\psi(\mathbf{r},t) = \sum_{n=0}^{\infty} C_n(t)\psi_n(\mathbf{r}).$$

The coefficients $C_n(t)$ satisfy the system of non-linear equations

$$i\hbar\frac{\partial C_n}{\partial t} = \frac{1}{1 + i\alpha}\left(\tilde{E}_n + \chi\right)C_n + \\ + \frac{i\alpha}{1 + \alpha^2}C_n\sum_k\left(\tilde{E}_k + \chi\right)|C_k|^2. \quad (6)$$

By analogy with Ref. 2, analysis demonstrates that unfilled states are stable at the non-zero α.

Based on Eq. 6, the transition to occupation numbers $P_n(t) = |C_n(t)|^2$ allows us to write

$$\frac{\partial P_n}{\partial t} = \frac{2\alpha}{\hbar\left(1 + \alpha^2\right)}P_n\left(-\left(\tilde{E}_n + \chi\right) + \sum_k\left(\tilde{E}_k + \chi\right)P_k\right). \quad (7)$$

The system of equations (7) shows that a particle existing in a magnetic field in the equilibrium state has only one non-zero occupation number. It means that despite the fact that $\tilde{E}_n$ can formally have a continuous set of values, the energy of a charged particle in a magnetic field corresponds to a fixed quantum number n.

Let us consider the dynamics of population for different quantum states $\psi_n(\mathbf{r}, t)$. Here we take into account the fact that non-occupied states can vary only if

$$\alpha \le \alpha_n = \hbar\delta/(\tilde{E}_n + \chi),$$

where δ is a small positive number.[1,2]

Using the two-level approximation, we write the system (7) in the form

$$\frac{\partial P_n}{\partial t} = \frac{2\alpha}{\hbar(1 + \alpha^2)}P_n\left[-(\tilde{E}_n + \chi) + (\tilde{E}_n + \chi)P_n + (\tilde{E}_l + \chi)P_l\right] = \\ = F(P_n, P_l), \quad (8)$$

$$\frac{\partial P_l}{\partial t} = \frac{2\alpha}{\hbar(1 + \alpha^2)}P_l\left[-(\tilde{E}_l + \chi) + (\tilde{E}_l + \chi)P_l + (\tilde{E}_n + \chi)P_n\right] = \\ = \Phi(P_n, P_l).$$

For definiteness, we assume that initially $P_{n0} = 1$; $P_{l0} = 0$.

The Jacobian of the system of equations (8) is different from zero:

$$\Delta = \frac{4\alpha^2}{\hbar}\left[\left(n + \frac{1}{2}\right)\hbar\left|\frac{qH}{mc}\right| + \frac{p_z^2}{2m} + \frac{kT^*}{2}\right]\left|\frac{qH}{mc}\right|(n - l).$$

This demonstrates that there are no other equilibriums near the fixed points.[5]

The parameter

$$\sigma = F'_{P_n}(P_n, P_l) + \Phi'_{P_l}(P_n, P_l).$$

Its sign determines the stability of trajectories, by which an induced oscillator can be in the non-equilibrium state.[5]

The value of the parameter, to an accuracy of smaller orders of magnitude, is as follows:

$$\sigma = 2\alpha\left|\frac{qH}{mc}\right|\left(2n - l + \frac{1}{2} + \frac{cp_z^2}{2\hbar|qH|} + \frac{kT^*mc}{2\hbar|qH|}\right).$$

For higher intensities of the magnetic field

$$|H| \gg \frac{c}{\hbar|q|}\left(p_z^2 + mkT^*\right).$$

For the state with $n = 0$, the parameter σ becomes negative. In this case, the trajectory is stable and the oscillator returns to its initial state. In other words, the Bose condensation of states must take place for induced oscillators, i.e., when charged particles move in a magnetic field, they do not radiate electromagnetic waves. This possible collapse of cyclotron radiation is caused by the fact that the total mechanic energy is not sufficient to excite oscillators induced by a magnetic field.

Relative intensity of cyclotron radiation harmonics is estimated in this work by the algorithm described in Ref. 2. We take into account the fact that upward transitions from the lower level with the number n are possible for levels, whose numbers satisfy the inequality

$$l \le 2n + \frac{1}{2} + \frac{p_z^2}{2\hbar\omega m} + \frac{kT^*}{2\hbar\omega}.$$

Figures 1–3 present probabilities of occupation densities of oscillation levels for different ratios $y = \omega\hbar/(kT) = 0.5$ at different values of the parameter $z = p_z^2/(mkT)$. For comparison, solid lines present Boltzmann distributions.

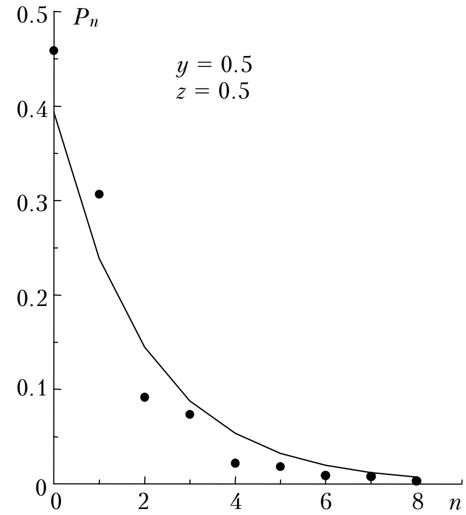

Fig. 1.

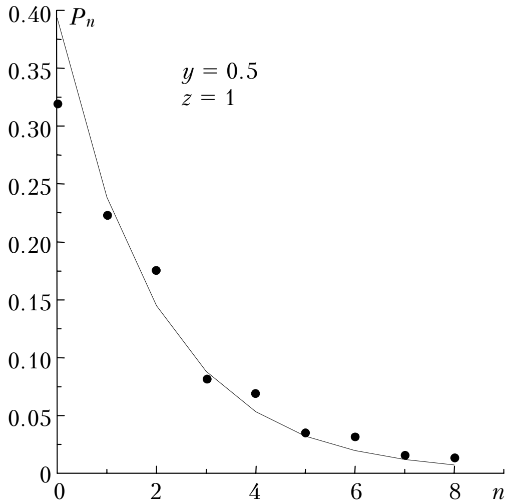

Fig. 2.

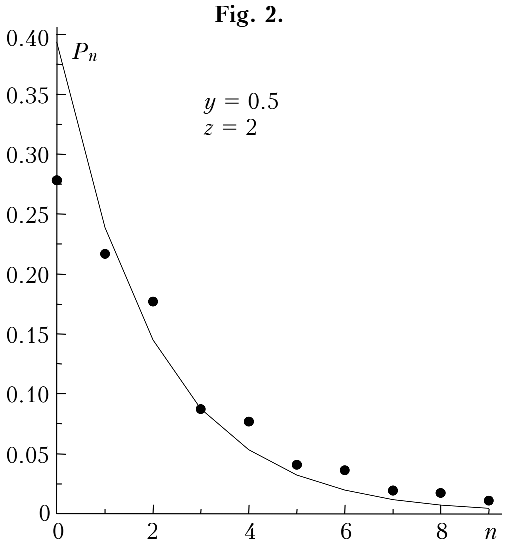

Fig. 3.

As is seen from Figs. 1–3, the above-mentioned dependences are close to each other.

Figures 4–6 present computed intensity distributions for radiation of oscillators induced by a magnetic field for the same values of parameters y and z as in Figs. 1–3.

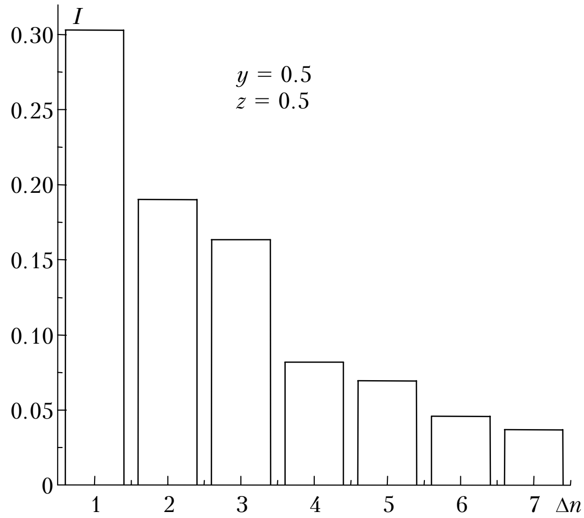

Fig. 4.

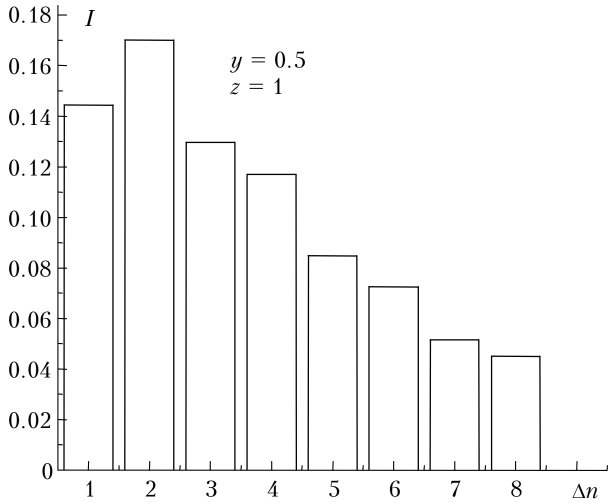

Fig. 5.

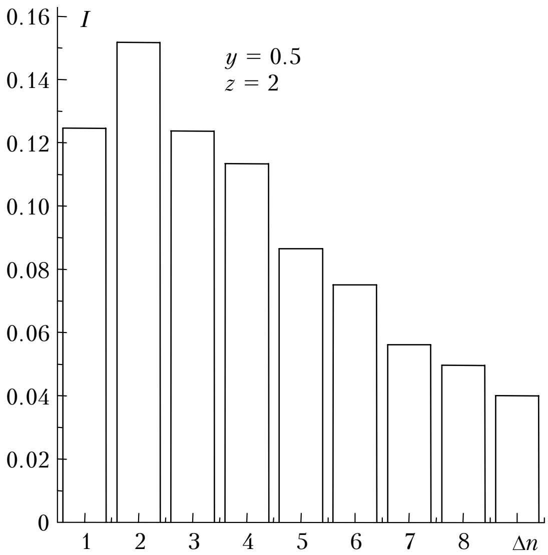

Fig. 6.

Radiation intensities differ from each other to some extent. This is connected with difference in the probability of transition from one energy level to another. For other parameters y and z, the corresponding distributions have the same structure.

The dependences, obtained theoretically, demonstrate that the radiation spectrum contains a large number of intensive harmonics of cyclotron frequency if the intensities of magnetic fields are small. The relative intensity of harmonics decreases with increase of the magnetic field intensity. At the qualitative level, these results agree with the experiment. A large number of cyclotron harmonics can be observed in the aurora borealis (radiation of charged particles in the magnetic field of the Earth, including cyclotron radiation). In strong fields, one can see a small number of cyclotron frequency harmonics.[6]

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