HORIZONTAL STRUCTURE OF STELLAR SCINTILLATIONS FROM OBSERVATIONS FROM SPACE THROUGH THE EARTH'S ATMOSPHERE

A.A. Volkov, G.M. Grechko, A.S. Gurvich, V. Kan, S.K. Krikalev, A.I. Pakhomov, and S.A. Savchenko

Institute of Atmospheric Physics,
Academy of Sciences of the USSR, Moscow
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Results of stellar scintillation observations made from the orbital station "Mir" during horizontal displacement of the sighting beam at heights from 20 to 30 km are given. The horizontal scintillation spectra are calculated from the photocurrent records. By comparing these spectra with the vertical scintillation spectra,[2] the anisotropy coefficient η of the layered inhomogeneities of the air density is obtained ($\eta \sim 160$).

It was shown in Ref. 1 that spectral measurements of stellar scintillations made from onboard an extraterrestrial platform through the Earth's atmosphere car be used to determine the spatial spectra of temperature and air density fluctuations. Scintillation observations[2] make it possible to experimentally evaluate the vertical spectra of the temperature inhomogeneities at heights of from 20 to 40 km. An analysis of the results also showed that inhomogeneities at these heights extend horizontally as well. The recording conditions,[2] however, did not permit us to study the horizontal structure of the stellar scintillation fluctuations while observing them through the atmosphere.

To study the horizontal structure, some specific observations of a star located near the orbital pole were carried out. The angular distance from the pole to this source was such that for an observer onboard the orbital station "Mir" the sighting beam penetrated through the atmosphere down to some minimum height $h_{\min}$ and then, not going below the true horizon, rose up again with the movement of the orbiting station. This paper presents the results of an analysis of such observations made February 28, 1989. The bright star Capella (α-Aur) was used as the scintillation source. The instrumentation and the observation technique were practically the same as those described in Ref. 2, except that the mean interrogation frequency of the photodetector was about 130 Hz.

A plot of the trajectories of the motion of the point of intersection of the sighting beam with a plane passing through the Earth's center and perpendicular to the direction to the star is shown in Fig. 1. The average refraction of the beam was taken into account using the standard atmospheric model in the calculation of these trajectories.

For the sake of convenience we choose a Cartesian coordinate system. Distance along the Earth's surface is plotted on the ordinate axis. It can be seen from Fig. 1 that the beam height changes very slowly in the vicinity of $h_{\min}$. Due to orbital procession $h_{\min}$ changes from orbit to orbit by approximately 7 km. We were able, therefore, to reliably record stellar scintillations in the vicinity of $h_{\min}$ on two orbits on which $h_{\min}$ was 22.8 and 29.5 km (February 28, 1989). During these observations, at the minimum height the beam passed above the points of the Earth's surface with coordinates 10.1°S, 276.8°E and 10.3°S, 244.4°E, respectively.

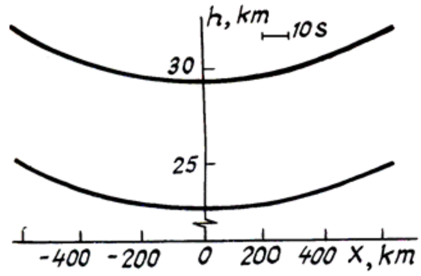

FIG. 1. Trajectory of the passage of the star in the image plane:
$h_{\min} = 22.8$ km, $t_{\min} = 9^{h}06^{m}38^{s}$ µs:
$h_{\min} = 29.8$ km, $t_{\min} = 10^{h}38^{m}16^{s}$ µs.

Using the results of Ref. 1, one can write down an expression for the horizontal scintillation spectrum $V_{\rm I}^{(h)}(\kappa_2)$ in the form

$$V_{1}^{(h)}(\kappa_2) = \int\limits_{-\infty}^{\infty} F_{1}(\kappa_1, \kappa_2)\, d\kappa_1, \qquad (1)$$

where $F_{\rm I}$ is the two-dimensional scintillation spectrum; κ_1 and κ_2 are the spatial angular frequencies with respect to the vertical and the Earth's surface, respectively. Ignoring diffraction by inhomogeneities of the refractive index, $F_{\rm I}(\kappa_1, \kappa_2)$ can be expressed in terms of the three-dimensional spectrum $\Phi_\nu = \Phi_\nu(\kappa_1, \kappa_2, \kappa_3)$ of the relative fluctuations of the air refractive index $\nu = (N - \bar{N}) / \bar{N}$:

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$$F_1 = \frac{2\pi a_e H_0 L^2 \bar{N}^2(h)}{q\left[1 + q^{-2}\kappa_1^2 H_0^2\right]^{1/2}} \cdot$$

$$\cdot\left[\frac{\kappa_1^2}{q} + \kappa_2^2\right]^2 \int\limits_{-\infty}^{\infty} \Phi_\nu\left[\frac{\kappa_1}{q},\ (\kappa_2^2 + \kappa_3^2)^{1/2}\right]\cdot$$

$$\cdot\exp\left\{-\frac{a_e H_0 \kappa_3^2}{1 + q^{-2}\kappa_1^2 H_0^2}\right\} d\kappa_3, \tag{2}$$

where L is the distance from the observer to the beam perigee; a_e is the Earth's radius; H_0 is the height of the homogeneous atmosphere; $\bar{N}(h)$ is the mean value of the refractive index at the beam perigee height h; and, q is the average refractive extinction. In the derivation of Eq. (2) in Ref. 1 account was taken of the fact that the atmosphere is a thin film on a sphere: $\frac{H_0}{a_e} \sim 10^{-3}$, and it was assumed that the random quantity ν is statistically locally isotropic on a sphere centered on the Earth's center and is locally homogeneous with respect to height above the Earth's surface. For a prescribed spectrum Φ_ν formulas (1) and (2) enable one to solve the direct problem, i.e., to calculate F_1 and $V_\mathrm{I}^{(h)}$. The structure of these formulas, however, is such that it is quite difficult to clarify the role of the different scales of the refractive index fluctuations in the atmosphere in the formation of the horizontal spectra $V_\mathrm{I}^{(h)}(\kappa_2)$ for arbitrary Φ_ν when only measurements of $V_\mathrm{I}^{(h)}(\kappa_2)$ are available.

So that we may carry out a qualitative analysis of the relation between $V_\mathrm{I}^{(h)}(\kappa_2)$ and $\Phi_\nu(\kappa_1,\ \kappa_2,\ \kappa_3)$, we shall make an additional assumption about the form of the spectrum Φ_ν, namely, that

$$\Phi_\nu = \Phi_\nu^{(0)}\left[\left[\kappa_1^2 + \eta^2\left[\kappa_2^2 + \kappa_3^2\right]\right]^{1/2}\right], \tag{3}$$

where η is a parameter characterizing the anisotropy of the spectrum. For $\eta \gg 1$ assumption (3) corresponds to highly elongate inhomogeneities of the refractive index along the Earth's surface, which agrees with the results of observations.[2,3]

Note that for spectrum (3) there exists a simple relation between $\Phi_\nu^{(0)}$ and the one-dimensional spectra: the vertical $V_\nu^{(v)}(\kappa_1)$

$$\Phi_\nu^{(0)}(\kappa_1) = -\frac{\eta^2}{2\pi\kappa_1}\frac{dV_\nu^{(v)}(\kappa_1)}{d\kappa_1}, \tag{4}$$

and the horizontal, for example, $V_\nu^{(h)}(\kappa_2)$

$$\Phi_\nu^{(0)}(\kappa_2\eta) = -\frac{\eta^2}{2\pi\eta\kappa_2}\frac{dV_\nu^{(h)}(\kappa_2)}{d\kappa_2}. \tag{5}$$

If we substitute Eq. (3) into formula (2) and then substitute the resultant expression into formula(1), then after transforming to the new integration variables $\tilde{\kappa}$ and φ

$$\kappa_1 = qH_0^{-1}\tilde{\kappa}\cos\varphi, \quad \kappa_3 = \eta^{-1}H_0^{-1}\tilde{\kappa}\sin\varphi, \tag{6}$$

we obtain the following expression for $V_\mathrm{I}^{(h)}(\kappa_2)$:

$$V_\mathrm{I}^{(h)}(\kappa_2) = \frac{8\pi L^2\bar{N}^2(h)a_e}{\eta H_0^5}\int\limits_0^\infty \tilde{\kappa}\, d\tilde{\kappa}\, \Phi_\nu^{(0)}\left[\left[\eta^2\kappa_2^2 + \right.\right.$$

$$\left.\left. + \tilde{\kappa}^2 H_0^{-2}\right]^{1/2}\right]\cdot\left[4q^2\tilde{\kappa}^4 g_2\left[\tilde{\kappa},\ \frac{a_e}{H_0\eta^2}\right] + 2qH_0^2\tilde{\kappa}^2\kappa_2^2 g_1 \times\right.$$

$$\left.\times\left[\tilde{\kappa},\ \frac{a_e}{H_0\eta^2}\right] + \kappa_2^4 H_0^4 g_0\left[\tilde{\kappa},\ \frac{a_e}{H_0\eta^2}\right]\right], \tag{7}$$

where

$$g_m = \int\limits_0^{\pi/2} \frac{(\cos\varphi)^{2m}}{\left[1 + \tilde{\kappa}^2\cos^2\varphi\right]^{1/2}} \exp\left\{-\frac{a_e}{H_0\eta^2}\frac{\tilde{\kappa}^2\sin^2\varphi}{1 + \tilde{\kappa}^2\cos^2\varphi}\right\},$$

$m = 0,\ 1,\ 2.$

As experiments[2,3] show, the asymmetry η is so great that the parameter $a_e H_0^{-1}\eta^{-2}$ which enters into formula (7) can be assumed to be less than unity. Figure 2 represents the weight functions g_m calculated for some possible values of the parameter $a_e H_0^{-1}\eta^{-2}$.

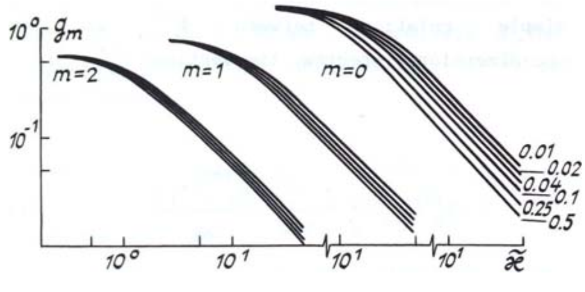

FIG. 2. Weight functions g_m (m = 0, 1, 2). The parameter $a_e H_0^{-1}\eta^{-2}$ varies from 0.5 to 0.01. A decrease of the parameter corresponds to an increase of the values of the weight function for each set of curves.

Calculation of $V_\mathrm{I}^{(h)}(\kappa_2)$ using formula (7) ithout accounting for diffraction is possible if $\kappa_2^2\lambda L \ll 1$, where λ is the light wavelength and the spectrum $\Phi_\nu^{(0)}(\kappa)$ decreases rapidly enough at large frequencies.

If the spectrum $\Phi_{\nu}^{(0)}(\kappa)$ has no large sharp peaks and, starting from some frequency $\kappa_m \ll (\lambda L)^{-1/2}$, decreases faster than κ^{-5}, then in calculating the horizontal scintillation spectrum $V_{\mathrm{I}}^{(h)}(\kappa_2)$ two intervals of the spatial frequency κ_2 can be distinguished: a low-frequency interval $\kappa_2 \ll \kappa_m$, in which the term containing g_2 in expression (7) makes the main contribution, and a high-frequency interval $\kappa_2 \ll \kappa_m$, in which the term containing g is decisive. Calculation of the horizontal scintillation spectrum $V_{\mathrm{I}}^{(h)}(\kappa_2)$ in the range $\kappa_2 \ll \kappa_m$ for sufficiently large η ($\eta^2 H_0/a_e \gg 1$) simplifies since the dependence of g_2 on the parameter $a_e H_0^{-2}\eta^{-2}$ in this case is weak and the approximation

$$g_2 \simeq \frac{3\pi}{16}\left[1 + \left(\frac{9\pi}{32}\tilde{\kappa}\right)^2\right]^{-1/2}. \tag{8}$$

is convenient for the analysis.

Estimates of the vertical scintillation spectra $V_{\mathrm{I}}^{(v)}(\kappa_1)$ obtained from observations of stellar scintillations were used to reconstruct the vertical spectra of the relative fluctuations of the refractive index $V_{\nu}^{(v)}(\kappa_1)$, which can be approximated within the experimental error by the formula[2]

$$V_{\nu}^{(v)}(\kappa_1) = C_{\nu}^2 \kappa_1^{-3}\left[1 + \frac{\kappa_1^2}{\kappa_0^2}\right]^{-1}. \tag{9}$$

If we set $C_{\nu} = 0.84 \cdot 10^{-5}\ \mathrm{m}^{-1}$ and $\kappa_0 = 0.125\ \mathrm{m}^{-1}$, the calculation of $V_{\nu}^{(v)}$ based on formula (9) agrees well with the experimentally obtained values of $V_{\nu}^{(v)}$ over the entire range of measurements $2\pi \cdot 10^{-3}\ \mathrm{m}^{-1} \le \kappa_1 \le 2\pi \cdot 10^{-1}\ \mathrm{m}^{-1}$. Substituting Eq. (9) into Eq. (4), we obtain

$$\Phi_{\nu}^{(0)}(\kappa) = \frac{3}{2\pi}\eta^2 C_{\nu}^2 \kappa^{-5}\left[1 + 5\kappa^2/3\kappa_0^2\right]\left[1 + \kappa^2/\kappa_0^2\right]^{-2}. \tag{10}$$

In making observations from onboard the orbital station one can only approximately take into account the motion of the beam in the vicinity of $h_{\min}$ parallel to the Earth's surface. Calculations of the scintillation spectra over successive intervals, which correspond to 90 km shifts along the surface, show that changes in the shape of the individual spectra are practically unnoticeable at a distance of up to 270 km from the point corresponding to $h_{\min}$. Here the variation of the beam height is about 0.5 km and the velocity of beam uplift varies from 0 to 0.25 km/s. At large deviations from $h_{\min}$ the velocity of beam uplift increases rapidly with simultaneous manifestation of the vertical structure of the refractive index fluctuation field, which leads to a shift of the spectra, calculated over the corresponding intervals, toward higher frequencies and to their deformation in the low-frequency region. Estimates of the vertical structure parameters do not *a priori* agree with the results of Ref. 2 if they are calculated from the spectra in the vicinity of $h_{\min}$ using the well-known actual values of the min vertical velocities of the sighting beam. This is a substantial proof of the fact that the spectra measured in the vicinity of $h_{\min}$ are related to the horizontal, but not the vertical structure of the refractive index field. The characteristic vertical scales calculated in this way in the interval $\pm$ 100 km from $h_{\min}$ differ from the data in Ref. 2, by at least a factor of 2–3.

Observations (February 28, 1989) were made using a narrow-band light filter at the wavelength $\lambda = 0.45\ \mu$m with bandwidth $\Delta\lambda = 0.05\ \mu$m, therefore chromatic refraction with characteristic vertical scales Δh_c equal to 10 and 4 m for the heights $h_{\min} = 22.8$ and 29.5 km, respectively, did not substantially affect the observed scintillations.

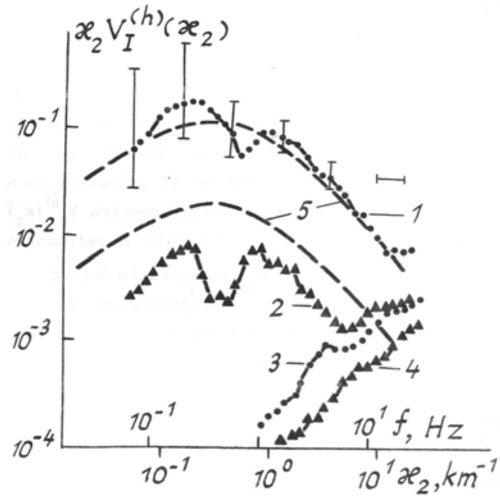

FIG. 3. 1, 2, — horizontal scintillation spectra; 3, 3 — shot noise spectra; 5 — calculation. 1, 3 — $h_{\min}$ = 22.8 km; 2, 4 — $h_{\min}$ = 29.5 km. Vertical straight lines — 90% confidence intervals;[5] horizontal line — width of the smoothing window. rms values of the relative intensity fluctuations: 1 — σ_{I} = 60%; 2 — σ_{I} = 15%.

Figure 3 presents one-sided ($f \ge 0$ and $\kappa_2 \ge 0$) spectra of the relative fluctuations of the intensity calculated over the interval of observations within $\pm$120 km from the point $h_{\min}$ for two values of $h_{\min}$. The experimentally obtained frequency scintillation spectra $V_{\mathrm{I}}^{(h)}(f)$ were transformed into spatial spectra $V_{\mathrm{I}}^{(h)}(\kappa_2)$, where $\kappa_2 = 2\pi f/\nu_h$, f is the frequency, and $\nu_h \approx 7$ km/s is the horizontal velocity of the perigee of the sighting beam in the plane of the terminator. The product of the spatial frequency and the spectral density $\kappa_2 \cdot V_{\mathrm{I}}^{(h)}$ is plotted on the ordinate axis. Smoothing of the estimates of the spectral density was carried out with a rectangular window of constant Q = 2. The same figure shows shot noise spectra of the photodetector obtained from realizations in which the sighting beam passed above the atmosphere and reduced to the observation conditions with the average atmospheric extinction of light flux from the star taken into account. The shot noise spectra give a quantitative measure of the noise level, the frequency characteristics of the entire measuring channel, and the processing scheme. The corresponding shot noise contributions were subtracted from the given scintillation spectra. The scintillation spectra maxima $\kappa_2 \cdot V_I^{(h)}(\kappa_2)$ correspond to horizontal scales $2\pi/\kappa_2$ around 15 km.

Some lifting of the spectrum in the high-frequency range $\kappa_2/2\pi \geq 0.7$ km^{-1}, observed for $h_{\min} = 29.5$ km,can be related to an inadequate account of the shot noise under conditions in which the noise is comparable with the scintillation. However, it may be related to a manifestation of actual features of stratospheric turbulence. The confidence intervals and width of the spectral smoothing window, indicated in Fig. 3, show that a dip near the spectral maximum for $h_{\min} = 29.5$ km can be connected with insufficient statistical stability of the spectral component estimates in the scale range 10 – 20 km, obtained over an observation interval of 240 km. It is possible, however, that the maxima and minima in the measured spectra, which correspond to horizontal scales $2\pi/\kappa_2 \geq 5$ km, are a reflection of quasi periodic wave processes in the stratosphere. For a final explanation of the above-noted peculiarities in the behavior of the horizontal scintillation spectra, we need some additional observations.

We may compare the measured scintillation spectra with calculations of formula (7) based on assumption (3) and estimate, the anisotropy coefficient η which enters into the calculations as a free parameter. Note that assumption (3), which states that the spectral components Φ_ν take the same values on the surface of an ellipsoid of revolution highly compressed along the vertical, in this case does not pretend to be a hydrodynamics model describing a real statistical distribution of air density fluctuations in the stratosphere, but is used rather as an approximation, which enables us to calculate in a rather simple way the horizontal spectra $V_I^{(h)}(\kappa_2)$ on the basis of the vertical spectra of refractive index fluctuations $V_\nu^{(v)}(\kappa_1)$ measured in Ref. 2.

In a qualitative interpretation of the measurements of the horizontal scintillation spectra $V_I^{(h)}(\kappa_2)$ it is necessary to take account of the fact that in the geometric-optics approximation the scintillations are determined by the second derivatives of the refractive index at the air density with respect to the spatial coordinates.[4] Here, because of the strong asymmetry, the inhomogeneities that are observed during horizontal sighting of stellar scintillations are apparently determined for the most part by the horizontal variability of the second derivatives with, respect to the vertical. In any case, as can be seen from formula (7), scintillations due, to strictly horizontal derivatives with weight g_0 are decisive only in the high-frequency region $\kappa_2 \gg \kappa_m$, where because of the rapid fall off of the spectrum Φ_ν, the spectral components of the scintillations $V_I^{(h)}$ are very small and, consequently, measurement errors are great.

As can be seen from Fig. 3, the calculation of $\kappa_2 V_I^{(h)}(\kappa_2)$ based on the spectra $V_\nu^{(v)}(\kappa_1)$ measured in Ref. 2 describes the shape of the spectra fairly well. The maximum values of the calculated spectra $\kappa_2 V_I^{(h)}(\kappa_2)$ are determined by the coefficient C_ν^2 obtained from the experimental vertical spectra $V_\nu^{(v)}(\kappa_1)$. Some overestimate of the calculated maximum of the spectrum $\kappa_2 V_I^{(h)}(\kappa_2)$ compared with the measured one for $h_{\min} = 29.5$ km may be associated with a nearly twice weaker "intensity" of the inhomogeneities in the stratosphere at this height for a given cycle of observations.

The anisotropy coefficient of the inhomogeneities which enters into the calculations can be estimated by fitting the corresponding scale factor, aligning the positions of the peaks of the experimental and calculated scintillation spectra $\kappa_2 V_I^{(h)}(\kappa_2)$ along the logarithmically scaled κ_2 axis, as shown in Fig. 3. These data give the estimate $\eta \simeq 160$ and, hence, the characteristic vertical scale of the air density inhomogeneities $I_v^* \sim 50$ m (Ref. 2) corresponds to a characteristic horizontal scale $I_h^* \sim 8$ km.

In conclusion we note the following. Stellar scintillation observations were made from onboard the orbital station "Mir" with horizontal and nearly horizontal displacement of the sighting beam in the stratosphere at heights of roughly 23 and 30 km. Processing of the observations yielded the horizontal scintillation spectra. These spectra were also calculated theoretically from the measured vertical spectra[2] on the basis of assumption (3). This calculation: does a good job of describing the horizontal scintillation spectra in the scale range 1–100 km. The anisotropy coefficient η of the air density . inhomogeneities in the stratosphere is estimated to be around 160.

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