Formation of the lateral shear interferograms in diffusely scattered light fields at double-exposure recording of Fourier hologram with the allowance for high approximation orders

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A shear interferometer with coherent diffusely scattered light fields is analyzed in the third order of approximation for the complex amplitude of light field. It is shown that the range of interferometer sensitivity is restricted by the aberrations in the reference and object channels.

In Refs. 1–3 it was shown that double-exposure recording of a Fourier hologram of a ground glass screen with a positive lens leads to formation of a lateral shear interferogram in the bands of infinite width in coherent diffusely scattered light fields. At the stage of the hologram reconstruction, the interferogram caused by the phase distortions of the reference wave is located in the hologram plane, and the interference pattern characterizing wave aberrations of the lens lies in the far diffraction zone. Spatial filtering of the diffraction field in the hologram plane allows separation of the lateral shear interferograms corresponding to different angles of plane wave incidence on the lens controlled, thus providing its areal control. The mechanism of interferogram formation in coherent diffusely scattered light fields consists in creation of conditions, under which identical subjective speckles of two exposures are matched in the plane of a photographic plate at double-exposure recording of the Fourier hologram of a ground glass screen. In Refs. 1–3, hologram recording and reconstruction conditions were analyzed in the Fresnel approximation, which ignores possible change in the filtered interferogram, when the hologram recording parameters do not fall in the domain of applicability of the parabolic approximation.

In this paper, the double-exposure recording of the Fourier hologram of a ground glass screen is analyzed in the third order of approximation for the complex field amplitude in order to assess possible errors in the control of wave aberrations of a positive lens or objective.

As shown in Fig. 1, a ground glass screen *1* placed in the plane (x_1, y_1) is illuminated with a diverging spherical wave of a coherent radiation having the radius of curvature R. The Fourier hologram of the screen is recorded on a photographic plate *2* in the plane (x_3, y_3) for the first exposure with the controlled positive lens L_1 having the focal length f_1, whose principal plane (x_2, y_2) is at the distance l_1. For this purpose, an off-axis diverging spherical reference wave with the radius of curvature r in the plane (x_3, y_3) is used. Before the second exposure, the tilt angle of the wave front of the radiation used for illumination of the screen is changed by α value in the plane (x, z), and the photographic plate is displaced in its plane by the distance a along the direction parallel to the x axis. At the stage of hologram reconstruction, it is illuminated by a copied reference wave corresponding, for example, to the first exposure, and the interferogram is recorded in the plane (x_4, y_4) with the lens L_2. The use of an opaque screen p with a round aperture in the hologram plane allows spatial filtering of the diffraction field to be performed.

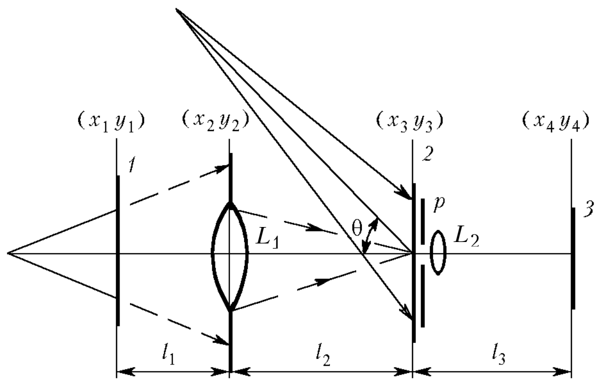

Fig. 1. Optical layout of the recording and reconstruction of the lens Fourier hologram: ground glass screen *1*, photographic plate *2*, plane of interferogram recording *3*, lenses L_1 and L_2, and spatial filter p.

In the third order of approximation, neglecting constant factors, the distribution of the complex amplitude of the object field corresponding to the first exposure in the plane (x_3, y_3) can be presented as

$$u_1(x_3, y_3) \sim \int\int\int\int_{-\infty}^{\infty} t(x_1, y_1) \times$$

$$\times \exp\left\{ik\left[\frac{1}{2R}(x_1^2 + y_1^2) - \frac{1}{8R^3}(x_1^2 + y_1^2)^2\right]\right\} \times$$

$$\times \exp\left\{ik\left\{\frac{1}{2l_1}[(x_1 - x_2)^2 + (y_1 - y_2)^2] -\right.\right.$$

$$- \frac{1}{8l_1^3} [(x_1 - x_2)^2 + (y_1 - y_2)^2]^2 \Big\}\Big\} p(x_2, y_2) \times$$

$$\times \exp\left\{- i \left[\frac{k}{2f_1} (x_2^2 + y_2^2) - \varphi(x_2, y_2)\right]\right\} \times$$

$$\times \exp\Big\{ik \Big\{\frac{1}{2l_2} [(x_2 - x_3)^2 + (y_2 - y_3)^2] -$$

$$- \frac{1}{8l_2^3} [(x_2 - x_3)^2 + (y_2 - y_3)^2]^2 \Big\}\Big\} \mathrm{d}x_1\, \mathrm{d}y_1\, \mathrm{d}x_2\, \mathrm{d}y_2, \quad (1)$$

where k is the wavenumber; l_2 is the separation between the planes (x_2, y_2) and (x_3, y_3); $t(x_1, y_1)$ is the complex transmission amplitude of the ground glass screen that is a random function of coordinates; $p(x_2, y_2) \exp[i\varphi(x_2, y_2)]$ is the generalized pupil function[4] of the controlled lens L_1, which characterizes its wave aberrations.

At $R + l_1 > f_1$ and $0 < l_1 \le f_1$ under the condition that the spectrum of waves scattered by the ground glass screen is bounded, on the spatial frequency scale, by the pupil of the controlled lens, Eq. (1) takes the form

$$u_1(x_3,y_3) \sim \exp\left\{ik\left[\frac{1}{2l_2} (x_3^2 + y_3^2) - \frac{1}{8l_2^3} (x_3^2 + y_3^2)^2\right]\right\} \times$$

$$\times \Big\{t(- \mu x_3, - \mu y_3)\, A(- \mu x_3, - \mu y_3) \times$$

$$\times \exp\Big\{ik \Big[\frac{\mu^2}{2} \left(\frac{1}{R} + \frac{1}{l_1}\right) (x_3^2 + y_3^2) -$$

$$- \frac{\mu^4}{8} \left(\frac{1}{R^3} + \frac{1}{l_1^3}\right) (x_3^2 + y_3^2)^2\Big]\Big\} \otimes \exp\left[- \frac{ikM}{2l_2^2} (x_3^2 + y_3^2)\right] \otimes$$

$$\otimes \Phi_1(x_3,y_3) \otimes \Phi_2(x_3,y_3) \otimes \Phi_3(x_3,y_3) \otimes P(x_3,y_3)\Big\}, \quad (2)$$

where $\otimes$ denotes convolution; $\mu = l_1/l_2$ is the scale coefficient; $M = f_1 l_1 l_2/(f_1 l_1 + f_1 l_2 - l_1 l_2) > 0$ is the parameter determining the spatial extension of the Fourier transform and the scale of the spectrum of spatial frequencies;

$$A(- \mu x_3, - \mu y_3) = \int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty} \exp[i\psi_1(x_1, y_1; x_2, y_2)] \times$$

$$\times \exp\left\{-ik\left[\left(\frac{x_1}{l_1} + \frac{x_3}{l_2}\right) x_2 + \left(\frac{y_1}{l_1} + \frac{y_3}{l_2}\right) y_2\right]\right\} \mathrm{d}x_1 \mathrm{d}y_1 \mathrm{d}x_2 \mathrm{d}y_2$$

is the complex function being the result of calculation at any point of the pupil of the controlled lens;

$$\psi_1(x_1,y_1; x_2,y_2) = - \frac{k}{8l_1^3} (6x_1^2 x_2^2 + 6y_1^2 y_2^2 - 4x_1^3 x_2 -$$

$$- 4x_1^2 y_1 y_2 + 2x_1^2 y_2^2 - 4x_1 x_2^3 - 4x_1 x_2 y_1^2 +$$

$$+ 8x_1 x_2 y_1 y_2 - 4x_1 x_2 y_2^2 + 2x_2^2 y_1^2 -$$

$$- 4x_2^2 y_1 y_2 - 4y_1^3 y_2 - 4y_1 y_2^3)$$

is the phase function due to wave aberrations; $\Phi_1(x_3, y_3)$, $\Phi_2(x_3, y_3)$, $\Phi_3(x_3, y_3)$, $P(x_3, y_3)$ are respectively Fourier transforms of the functions

$$\exp\left[- \frac{ik}{8l_1^3} (x_2^2 + y_2^2)^2\right], \ \exp\left[- \frac{ik}{8l_2^3} (x_2^2 + y_2^2)^2\right],$$

$\exp[i\psi_2(x_2, y_2; x_3, y_3)]$, $p(x_2, y_2) \exp[i\varphi(x_2, y_2)]$ with the spatial frequencies $x_3/\lambda l_2$ and $y_3/\lambda l_2$; λ is the wavelength of the coherent radiation used for hologram recording and reconstruction; $\psi_2(x_2, y_2; x_3, y_3)$ has the same form as $\psi_1(x_1, y_1; x_2, y_2)$ with the corresponding change of variables and l_1 substituted by l_2.

Let, as in Refs. 1 and 2, $R = [f_1 l_2/(l_2 - f_1)] - l_1$. Then, based on the integral representation of the operation of convolution with the function $\exp[- ikM(x_3^2 + y_3^2)/2l_2^2]$, the distribution of the complex field amplitude in the third order of approximation in the plane of the photographic plate within its diameter $D \searrow dl_2/M$, where d is the pupil diameter of the controlled lens, is described by the equation

$$u_1(x_3,y_3) \sim \exp\left\{ik\left[\frac{1}{2r_2} (x_3^2 + y_3^2) - \frac{1}{8l_2^3} (x_3^2 + y_3^2)^2\right]\right\} \times$$

$$\times \Big\{F\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right] \otimes \Phi_1'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right] \otimes$$

$$\otimes \Phi_2'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right] \otimes \Phi_3'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right] \otimes$$

$$\otimes \Phi_1(x_3,y_3) \otimes \Phi_2(x_3,y_3) \otimes \Phi_3(x_3,y_3) \otimes P(x_3,y_3)\Big\}, \quad (3)$$

where $r = l_2^2/(l_2 - M)$ is the radius of curvature of the spherical wave;

$$F\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right], \ \Phi_1'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right],$$

$$\Phi_2'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right], \ \Phi_3'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right]$$

are Fourier transforms of the functions

$$t(x_1, y_1), \quad \exp\left[- \frac{ik}{8l_1^3} (x_1^2 + y_1^2)^2\right],$$

$$\exp\left[- \frac{ik}{8R^3} (x_1^2 + y_1^2)^2\right], \ \exp\left[i\psi_1\left(x_1, y_1; \frac{M}{l_2} x_3, \frac{M}{l_2} y_3\right)\right]$$

with the spatial frequencies $Mx_3/\lambda l_1 l_2$ and $My_3/\lambda l_1 l_2$.

It follows from Eq. (3) that within this area of the plane (x_3, y_3) the complex amplitude of the field corresponds to the Fourier transform of the function $t(x_1, y_1)$ with every point being broadened up to the size of a subjective speckle determined by the width of the function

$$\Phi_1'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right] \otimes \Phi_2'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right] \otimes$$

$$\otimes \Phi_3'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes \Phi_1(x_3, y_3) \otimes$$

$$\otimes \Phi_2(x_3, y_3) \otimes \Phi_3(x_3, y_3) \otimes P(x_3, y_3).$$

As compared to the diffraction limit, speckle broadening is caused by the axial and off-axial aberrations in the object channel and distortions introduced by the controlled lens to the scattered wave. Besides, the phase distribution of a diverging quasi-spherical wave with the radius of curvature r is superimposed on the subjective speckle field, and $r = \infty$ for $l_1 = f_1$ (Ref. 1).

Assume that spatial filtering is conducted, as, e.g., in Ref. 5, both in the channel for illumination of the ground glass screen and in the channel of the reference wave in order to exclude phase distortions caused by optical elements. Then in the approximation used, the distribution of the complex amplitude in the plane of the photographic plate corresponding to the first exposure for the spatially bounded reference wave takes the form

$$u_{01}(x_3, y_3) \sim \exp\left\{ik\left\{\frac{1}{2r}\left[(x_3 + c)^2 + y_3^2\right] - \right.\right.$$
$$\left.\left. - \frac{1}{8r^3}\left[(x_3 + c)^2 + y_3^2\right]^2\right\}\right\}, \qquad (4)$$

where the designation $c = r\sin\theta$ is introduced for brevity; θ is the angle between the axis of the spatially bounded reference beam and the normal to the plane of the photographic plate.

The distribution of the complex amplitude of the object field corresponding to the second exposure can be written in the form

$$u_2(x_3, y_3) \sim \int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty} t(x_1, y_1) \times$$
$$\times \exp\left\{ik\left\{\frac{1}{2R}\left[(x_1 - R\sin\alpha)^2 + y_1^2\right] - \right.\right.$$
$$\left.\left. - \frac{1}{8R^3}\left[(x_1 - R\sin\alpha)^2 + y_1^2\right]^2\right\}\right\} \times$$
$$\times \exp\left\{\frac{ik}{2l_1}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]\right\} \times$$
$$\times \exp\left\{-\frac{ik}{8l_1^3}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]^2\right\} p(x_2, y_2) \times$$
$$\times \exp\left\{-i\left[\frac{k}{2f_1}(x_2^2 + y_2^2) - \varphi(x_2, y_2)\right]\right\} \times$$
$$\times \exp\left\{\frac{ik}{2l_2}\left[(x_2 - x_3 + a)^2 + (y_2 - y_3)^2\right]\right\} \times$$
$$\times \exp\left\{-\frac{ik}{8l_2^3}\left[(x_2 - x_3 + a)^2 + (y_2 - y_3)^2\right]^2\right\} \times$$
$$\times \mathrm{d}x_1\,\mathrm{d}y_1\,\mathrm{d}x_2\,\mathrm{d}y_2. \qquad (5)$$

If the condition $\sin\alpha = aM/l_1l_2$ is fulfilled, then Eq. (5) takes the form

$$u_2(x_3, y_3) \sim$$
$$\sim \exp\left\{ik\left\{\frac{1}{2r}\left[(x_3 - a)^2 + y_3^2\right] - \frac{1}{8l_2^3}(x_3^2 + y_3^2)^2\right\}\right\} \times$$
$$\times \left\{F\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes \Phi_1'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes\right.$$
$$\otimes \exp[-ikaR\,(M/l_1\,l_2)^2\,x_3]\,\Phi_2'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes$$
$$\otimes \Phi_3'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes \exp(-ikaMx_3/l_2^2) \times$$
$$\times \Phi_1(x_3, y_3) \otimes \exp[-ika\,(M - l_2)\,x_3/l_2^2] \times$$
$$\times \Phi_2(x_3, y_3) \otimes \exp[-ika\,(M - l_2)\,x_3/l_2^2] \times$$
$$\left.\times \Phi_3(x_3, y_3) \otimes \exp(-ikaMx_3/l_2^2)\,P(x_3, y_3)\right\}. \qquad (6)$$

According to Eq. (6), the structure of the subjective speckle field in the object channel in the plane (x_3, y_3) is identical to the structure observed at the first exposure. Only tilt angles for some components of the subjective speckle change.

For the second exposure, the distribution of the complex amplitude of the reference wave in the plane of the photographic plate is determined by the equation

$$u_{02}(x_3, y_3) \sim \exp\left\{ik\left\{\frac{1}{2r}\left[(x_3 + c - a)^2 + y_3^2\right] - \right.\right.$$
$$\left.\left. - \frac{1}{8r^3}\left[(x_3 + c - a)^2 + y_3^2\right]^2\right\}\right\}. \qquad (7)$$

Under condition that the double-exposure hologram is recorded within the linearity range of the photographic material blackening curve and that the diffracting waves are spatially separated,[5] one can find the distribution of the complex amplitude $u(x_3, y_3)$ in its plane for the component corresponding to the (−1)st diffraction order. Based on Eqs. (3), (4), (6), and (7), it takes the following form:

$$u(x_3, y_3) \sim \exp(-ikx_3\sin\theta) \times$$
$$\times \exp\left\{ik\left\{\frac{1}{8r^3}\left[(x_3 + c)^2 + y_3^2\right] - \frac{1}{8l_2^3}(x_3^2 + y_3^2)^2\right\}\right\} \times$$
$$\times \left\{F\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes \Phi_1'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes\right.$$
$$\otimes \Phi_2'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes \Phi_3'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes$$
$$\otimes \Phi_1(x_3, y_3) \otimes \Phi_2(x_3, y_3) \otimes \Phi_3(x_3, y_3) \otimes P(x_3, y_3) +$$
$$+ \exp[i\psi_3(x_3, y_3; a)]\left\{F\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes\right.$$
$$\otimes \Phi_1'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes \exp[-ikaR\,(M/l_1l_2)^2\,x_3] \times$$

$$\times \Phi_2'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes \Phi_3'\left[\frac{kMx_3}{l_1\,l_2}, \frac{kMy_3}{l_1\,l_2}\right] \otimes$$

$$\otimes \exp(-\,ikaMx_3/l_2^2)\;\Phi_1(x_3,\,y_3) \otimes$$

$$\otimes \exp[-\,ika\,(M-l_2)\,x_3/l_2^2]\;\Phi_2(x_3,\,y_3) \otimes$$

$$\otimes \exp[-\,ika\,(M-l_2)\,x_3/l_2^2]\;\Phi_3(x_3,\,y_3) \otimes$$

$$\left.\otimes \exp(-\,ikaMx_3/l_2^2)\;P(x_3,\,y_3)\right\}, \quad (8)$$

where

$$\psi_3(x_3,\,y_3;\,a) = \frac{k}{8r^2}\,(6x_3^2\,a^2 + 2y_3^2\,a^2 - 4x_3^3\,a - 4x_3a^3 -$$

$$-\,4x_3\,y_3^2\,a + 12x_3a^2\,c - 12x_3^2\,ac - 12x_3\,ac^2 - 4y_3^2\,ac)$$

is the phase function due to aberrations in the reference channel.

It follows from Eq. (8) that because the identical subjective speckles in both of the exposures are matched in the hologram plane, a lateral shear interferogram in infinitely wide fringes is located in it. As in Refs. 6 and 7, this interferogram largely characterizes the aberration of coma type, and the frequency of its interference fringes increases with the increase of the angle θ and photographic plane shift a, as well as with the decrease of the radius of curvature r. Therefore, at spatial filtering of the diffraction field in the hologram plane, the diameter of the filtering aperture of the spatial filter p located in this plane should be decreased. The decrease of this diameter, in its turn, leads to an increase in the size of the subjective speckle in the image plane $(x_4,\,y_4)$ of the ground glass screen, where the interferogram characterizing the controlled object is located. As the speckle size increases, the visibility of the interference pattern decreases down to zero, when the speckle size becomes comparable with the period of interference fringes. Consequently, aberrations in the reference channel restrict the interferometer sensitivity range, and they are absent when $r = \infty$ $(l_1 = f_1)$.

Assume, for brevity, that at the stage of hologram reconstruction the lens L_2 with the focal length f_2 is in its plane. Besides, the allowance for the third order of approximation in determination of the complex field amplitude in the plane $(x_4,\,y_4)$ leads only to the change in the distribution of the subjective speckle structure in the recording plane *3*, which is modulated by the interference fringes. Then, restricting our consideration to the parabolic approximation, we can write the distribution of the complex field amplitude in the plane $(x_4,\,y_4)$ in the following form:

$$u(x_4,\,y_4) \sim \iint\limits_{-\infty}^{\infty} p(x_3 + x_{03},\,y_3 + y_{03})\;u(x_3,\,y_3) \times$$

$$\times\,u_{01}(x_3,\,y_3)\,\exp\left[-\,\frac{ik}{2f_2}\,(x_3^2 + y_3^2)\right] \times$$

$$\times \exp\left\{\frac{ik}{2l_3}\,[(x_3 - x_4)^2 + (y_3 - y_4)^2]\right\}\,dx_3\,dy_3, \quad (9)$$

where $p(x_3 + x_{03},\,y_3 + y_{03})$ is the transmission function of the spatial filter[8] centered at the point with the coordinates x_{03} and y_{03}; l_3 is the distance between the planes $(x_3,\,y_3)$, $(x_4,\,y_4)$.

Let for brevity $l_3 = l_2$ and $1/f_2 = 1/r + 1/l_2$. Then, substituting Eqs. (4) and (8) in Eq. (9) and assuming that the diameter of the filtering aperture does not exceed the width of an interference fringe for the interferogram located in the hologram plane, and neglecting an inessential factor characterizing the phase distribution of a spherical wave, we have

$$u(x_4,\,y_4) \sim \Big\{ t(-\,\mu_1\,x_4,\,-\,\mu_1\,y_4)\;p(x_4,\,y_4) \times$$

$$\times \exp\Big\{ i\Big[\varphi(-\,x_4,\,-\,y_4) +$$

$$+\,\psi_1(-\,\mu_1\,x_4,\,-\,\mu_1\,y_4;\,\mu_2\,x_{03},\,\mu_2\,y_{03}) +$$

$$+\,\psi_2(-\,x_4,\,-\,y_4;\,x_{03},\,y_{03}) -$$

$$\left.\left.-\,\frac{k}{8}\left(\frac{\mu_1^4 + 1}{l_1^3} + \frac{\mu_1^4}{R^3} + \frac{1}{l_2^3}\right)(x_4^2 + y_4^2)^2\right]\right\} +$$

$$+\,t(-\,\mu_1\,x_4,\,-\,\mu_1\,y_4)\;p(x_4 + \mu_2\,a,\,y_4) \times$$

$$\times \exp\Big\{ i\Big\{\varphi(-\,x_4 - \mu_2\,a,\,-\,y_4) +$$

$$+\,\psi_1(-\,\mu_1\,x_4,\,-\,\mu_1\,y_4;\,\mu_2\,x_{03},\,\mu_2\,y_{03}) +$$

$$+\,\psi_2[-\,x_4 - (\mu_2 - 1)a,\,-\,y_4;\,x_{03},\,y_{03}] -$$

$$\left.\left.\left.-\,\frac{k\,\mu_1^4}{8l_1^3}\,(x_4^2 + y_4^2)^2\right\}\right\}\right\} \times$$

$$\times \exp\left\{-\,\frac{ik\mu_1^4}{8R^3}\left[\left(x_4 + \frac{a\mu_2}{1-\mu_1}\right)^2 + y_4^2\right]^2\right\} \times$$

$$\times \exp\left\{-\,\frac{ik}{8l_1^3}\,[(x_4 + \mu_2\,a)^2 + y_4^2\,]^2\right\} \times$$

$$\times \exp\left\{-\,\frac{ik}{8l_2^3}\{[x_4 + (\mu_2 - 1)a]^2 + y_4^2\}^2\right\}\Bigg\} \otimes P_0(x_4,y_4), \quad (10)$$

where $\mu_1 = l_1/M$ and $\mu_2 = M/l_2$ are the scaling coefficients;

$$P_0(x_4,\,y_4) = \iint\limits_{-\infty}^{\infty} p(x_3 + x_{03},\,y_3 + y_{03}) \times$$

$$\times \exp\left[-\,\frac{ik}{8l_2^3}\,(x_3^2 + y_3^2)^2\right] \times$$

$$\times \exp[-\,ik\,(x_3\,x_4 + y_3\,y_4)/l_2]\;dx_3\,dy_3$$

is the Fourier transform of the corresponding function.

Based on Eq. (10) and some statements from Refs. 1–3, we can determine the illumination distribution $I(x_4, y_4)$ in the interferogram plane, where identical subjective speckles of both exposures coincide. If the diameter D_0 of the illuminated area of the ground glass screen obeys the condition $D_0 \geq df_1 l_2/(f_1 l_1 + f_1 l_2 - l_1 l_2)$, then within the overlap of two images of the controlled lens pupil

$$I(x_4, y_4) \sim \{1 + \cos[\varphi(-x_4 - \mu_2 a, -y_4) - \varphi(-x_4, -y_4) + \psi_4(x_4, y_4; \mu_2 a) + \psi_5(x_4, y_4; x_{03}, y_{03}; \mu_2 a)]\} \times \Bigg| t(-\mu_1 x_4, -\mu_1 y_4) \times \exp\Bigg\{ i \Bigg[\psi_1(-\mu_1 x_4, -\mu_1 y_4; \mu_2 x_{03}, \mu_2 y_{03}) + \psi_2(-x_4, -y_4; x_{03}, y_{03}) - \frac{k}{8}\left(\frac{\mu_1^4 + 1}{l_1^3} + \frac{\mu_1^4}{R^3} + \frac{1}{l_2^3}\right)(x_4^2 + y_4^2)^2 \Bigg]\Bigg\} \otimes P_0(x_4, y_4) \Bigg|^2, \quad (11)$$

where

$$\psi_4(x_4, y_4; \mu_2 a) = -\frac{k}{8}\Bigg\{4x_4^3\left[\frac{1}{l_1^3} + \frac{(\mu_2 - 1)}{\mu_2 l_2^3} + \frac{\mu_1^3}{R^2 l_1}\right] \times (\mu_2 a) + (6x_4^2 + 2y_4^2)\left[\frac{1}{l_1^3} + \frac{(\mu_2 - 1)^2}{\mu_2^2 l_2^3} + \frac{\mu_1^2}{R\, l_1^2}\right](\mu_2 a)^2 + 4x_4\left[\frac{1}{l_1^3} + \frac{(\mu_2 - 1)^3}{\mu_2^3 l_2^3} + \frac{\mu_1}{l_1^3}\right](\mu_2 a)^3 + 4x_4 y_4^2\left[\frac{1}{l_1^3} + \frac{(\mu_2 - 1)}{\mu_2 l_2^3} + \frac{\mu_1^3}{R^2 l_1}\right](\mu_2 a)\Bigg\};$$

$$\psi_5(x_4, y_4; x_{03}, y_{03}; \mu_2 a) = -\frac{k}{8 l_2^3} \times [(12 x_4 x_{03}^2 + 12 x_4^2 x_{03} + 8 x_4 y_4 y_{03} + 4 x_4 y_{03}^2 + 4 y_4^2 x_{03} + 8 y_4 x_{03} y_{03})(\mu_2 - 1) a + (12 x_4 x_{03} + 4 y_4 y_{03})(\mu_2 - 1)^2 a^2]$$

are the phase functions caused by the third-order aberrations in the object channel.

It follows from Eq. (11) that a lateral shear interferogram in infinitely wide bands is formed in the image plane of the ground glass screen. This interferogram characterizes wave aberrations of the controlled lens. Interference fringes modulate the subjective speckle structure with the speckle size determined by the width of the function $P_0(x_4, y_4)$. The interferogram in this case can be distorted, if the functions $\psi_4(x_4, y_4; \mu_2 a)$ and $\psi_5(x_4, y_4; x_{03}, y_{03}; \mu_2 a)$ are nonzero.

It should be noted that when spatial filtering of the diffraction field is conducted in the hologram plane then to reconstruct the hologram by a small-aperture laser beam and in order to increase the image brightness, the intensity distribution in the Fourier plane of the lens L_2 for $f_2 = l_2$ takes the form similar to Eq. (11), but with different field distribution in the subjective speckle structure modulated by the interference fringes.

Let spatial filtering of the diffraction field be conducted in the hologram plane on the optical axis, then $\psi_5(x_4, y_4; 0, 0; \mu_2 a) = 0$, and possible distortions of the interferogram may be caused by the spherical aberration of the hologram in the object channel.

Since $l_1 < l_2$, for known d, f_1, l_1, and l_2 meeting the condition of the used approximation

$$[2\mu_1/(1 + \mu_1)]\sqrt[4]{0.8\lambda l_1^3} \leq d \leq [2\mu_1/(1 + \mu_1)]\sqrt[6]{1.6\lambda l_1^5}$$

we can find the maximum value $(\mu_2 a) \leq d/2$ of the lateral shift, at which we can still believe that $\psi_4(x_4, y_4; \mu_2 a) = 0$. For this purpose, we can use the well-known (see, for example, Ref. 9) criterion of the accuracy of determination of the phase equal to $0.1 \cdot 2\pi$. Since the phase change in differential interferometry is maximum at the shear axis, based on the equation for the function $\psi_4(x_4, y_4; \mu_2 a)$, the maximum permissible value of the lateral shear is the result of solution of the cubic equation:

$$4\left[\frac{1}{l_1^3} + \frac{(\mu_2 - 1)^3}{\mu_2^3 l_2^3} + \frac{\mu_1}{l_1^3}\right](d/2)(\mu_2 a)^3 + 6\left[\frac{1}{l_1^3} + \frac{(\mu_2 - 1)^2}{\mu_2^2 l_2^3} + \frac{\mu_1^2}{R l_1^2}\right](d/2)^2 (\mu_2 a)^2 + 4\left[\frac{1}{l_1^3} + \frac{(\mu_2 - 1)}{\mu_2 l_2^3} + \frac{\mu_1^3}{R^2 l_1}\right](d/2)^3 (\mu_2 a) - 0.8\lambda = 0.$$

In the case that spatial filtering of the diffraction field is conducted in an off-axis area then to control the off-axis wave aberrations of a positive lens or objective,[1–3] we can determine the field control range, within which

$$\psi_5(x_4, y_4; x_{03}, y_{03}; \mu_2 a) = 0.$$

From the form of the function $\psi_5(x_4, y_4; x_{03}, y_{03}; \mu_2 a)$, it follows that neglecting the terms of higher order of smallness, the maximum permissible diameter $D_{max} \leq d(1 + l_2/l_1 - l_2/f_1)$ in the hologram plane for the determined value of $(\mu_2 a)$ is a solution to the square equation:

$$12\,(d/2)\,(D_{max}/2)^2 + 12\,(d/2)^2\,(D_{max}/2) - 0.8\lambda\, l_2^3\, \mu_2/(\mu_2 - 1)\,(\mu_2 a) = 0,$$

and the restriction on the field control range is caused by the off-axis hologram aberration of the coma type in the object channel.

Some features of the considered method of recording double-exposure hologram follow from analysis of Eq. (8). Thus, the inclination angle is absent for the component $\Phi_1'\left[\frac{kMx_3}{l_1 l_2}, \frac{kMy_3}{l_1 l_2}\right]$ of the subjective speckle of the second exposure in the object channel. As a result, the control error due to spherical aberration of the hologram decreases. The components $\Phi_3'\left[\frac{kMx_3}{l_1\, l_2}, \frac{kMy_3}{l_1\, l_2}\right]$ of the subjective speckle of both exposures also turn out to coincide if the tilt angle between them is zero. This circumstance provides the decrease of the effect of off-axis hologram aberrations in the object channel on the control error. Besides, at $l_1 = f_1$, when the field control range is determined by the pupil diameter of the controlled lens, there is no control error caused by the off-axis hologram aberrations, and the error due to spherical hologram aberration decreases. In this case, the tilt angle between the components $\Phi_2(x_3, y_3)$ and $\Phi_3(x_3, y_3)$ of the subjective speckle in the hologram plane is zero.

Thus, analysis of formation of the lateral shear interferogram in coherent diffusely scattered light fields at double-exposure recording of the Fourier hologram of a ground glass screen for control of a positive lens or objective showed the following.

The increase of the pupil diameter of the controlled object is indicative of the need to take into account hologram aberrations in the object and reference channels. The third-order hologram aberrations of the coma type in the reference channel lead to a decrease in the interferometer sensitivity range. Axial and off-axis wave aberrations of the hologram in the object channel distort filtered interferograms, thus causing control errors. To exclude them, the interferometer sensitivity should be decreased. Within the preset boundaries of the sensitivity range, which depend on the hologram recording geometry in the object channel and the wavelength of the coherent radiation used, the filtered interferograms characterize the controlled object.

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