Light scattering by cylindrical particles in the Rayleigh–Gans–Debye approximation. Part 2. Randomly oriented particles

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Approximate formulae for light scattering efficiency factors of randomly oriented circular and hexagonal cylinders in Rayleigh–Gans–Debye approximation at small diffraction parameters $ka < 1$, $kH < 1$ (where k, H, and a are the wavenumber, height, and radius of cylinder, respectively) are obtained. Some numerical results are presented.

Introduction

Considerable evidence on the light scattering by natural and artificial aerosols, ice crystals,[1] marine hydrosols, suspensions of biological particles[2] has been accumulated. As a rule, such particles are non-spherical and chaotically oriented. Studying in detail of characteristics of light scattering by chaotically oriented particles is of great theoretical interest.

It is expedient to use the Rayleigh–Gans–Debye (RGD) and anomalous diffraction (AD) approximations for optically "soft" ($|n - 1| << 1$, where n is the relative refractive index of a particle) light scattering particles of non-spherical shape. The formulae for cylindrical particles of round and hexagonal cross sections in the AD approximation have been obtained earlier.[3,4]

Analysis of the efficiency factor for light scattering by chaotically oriented cylinders with round and hexagonal cross sections in the RGD approximation is carried out in this paper.

1. Light scattering cross section

Let us assume that the natural light (polarization is chaotic) falls on a cylinder with the height H and the radius a. Then the light-scattering cross section[5] σ_s of a round RGD cylinder at small diffraction parameters $ka < 1$ and $kH < 1$ is:

$$\sigma_s = \frac{(ka)^4 H^2 (n^2 - 1)^2}{16} UC(H, a, \theta_i, \phi_i), \qquad (1)$$

where

$$UC(H, a, \theta_i, \phi_i) = UC_1 + UC_2;$$

$$UC_1 = \frac{2\pi}{3}\left\{4 - (kH)^2\left[\frac{2}{15} + \frac{\cos^2(\theta_i)}{3}\right] - (ka)^2\left[\frac{3}{5} + \sin^2(\theta_i)\right]\right\},$$

$$UC_2 = \frac{2\pi}{3}\left\{(kH)^2 (ka)^2 \times \left[\frac{1}{84} + \frac{\sin^2(\theta_i)\cos^2(\theta_i)}{12} + \frac{\sin^2(\theta_i)}{30} + \frac{\cos^2(\theta_i)}{20}\right]\right\};$$

θ_i, ϕ_i are the angles in the spherical coordinate system showing the direction of the incident wave.

Analogously, for a hexagonal RGD cylinder at small $ka < 1$ and $kH < 1$

$$\sigma_s = \frac{27(ka)^4 H^2 (n^2 - 1)^2}{64\pi^2} UH(H, a, \theta_i, \phi_i), \qquad (2)$$

where

$$UH(H, a, \theta_i, \phi_i) = UH_1 + UH_2;$$

$$UH_1 = \frac{2\pi}{3}\left\{4 - (kH)^2\left[\frac{2}{15} + \frac{\cos^2(\theta_i)}{3}\right] - (ka)^2\left[\frac{2}{5} + \frac{\sin^2(\theta_i)(3\sin^2(\phi_i) + \cos^2(\phi_i))}{3}\right]\right\};$$

$$UH_2 = \frac{2\pi}{3}\left\{(kH)^2 (ka)^2\left(\frac{1}{126} + \frac{\sin^2(\theta_i)}{18} \times \left([3\sin^2(\phi_i) + \cos^2(\phi_i)]\left[\frac{1}{5} + \frac{\cos^2(\theta_i)}{2}\right]\right) + \frac{\cos^2(\theta_i)}{30}\right)\right\}.$$

2. The efficiency factor of light scattering

According to Ref. 6, the light scattering cross section σ_s normalized to the area S of the particle projection to the plane, perpendicular to the beam axis (or the efficiency factor of the light scattering Q_S), is:

$$\frac{\sigma_s}{S} = Q_S = \frac{\int_{4\pi} |\mathbf{f}(\mathbf{s}, \mathbf{i})|^2 d\omega}{S}, \qquad (3)$$

where $d\omega$ is the element of the solid angle (in the spherical coordinate system $\sin(\theta_s)d\theta_s d\phi_s$).

Then, for the round cylinder, the area S of the particle projection to the plane, perpendicular to the beam axis, is:

$$S(\theta_i,\phi_i) = 2a\left[H\sin(\theta_i) + \frac{\pi}{2}a\cos(\theta_i)\right], \quad (4)$$

and for the hexagonal cylinder:

$$S(\theta_i,\phi_i) = 2a\cos\left(\phi_i - \frac{\pi}{6}(2p-1)\right)\left[H\sin(\theta_i) + \frac{3}{2}a\cos(\theta_i)\right], \quad (5)$$

where p is the number from 1 to 6 depending on the range of changing the angle ϕ_i ($p = 1$ at $0 \le \phi_i < \frac{\pi}{3}$, $p = 2$ at $\frac{\pi}{3} \le \phi_i < \frac{2\pi}{3}$, etc.).

If the particle is chaotically oriented, the factor of light scattering efficiency Q_S^R (without taking into account the symmetry by the azimuth angle ϕ) can be defined as

$$Q_S^R = \frac{\int_0^{2\pi}\int_0^{\pi} Q_S(\theta_i,\phi_i)S(\theta_i,\phi_i)\sin(\theta_i)d\theta_i d\phi_i}{\int_0^{2\pi}\int_0^{\pi} S(\theta_i,\phi_i)\sin(\theta_i)d\theta_i d\phi_i}. \quad (6)$$

Substituting the areas (4) and (5) into Eq. (6), we obtain the denominator of the efficiency factor of the light scattering, that is $2\pi^2 aH$ for the round cylinder and, respectively, $6\pi^2 aH$ for the hexagonal cylinder.

So, using Eq. (1) and integrating the nominator in Eq. (6), we have for the chaotically oriented round RGD cylinder:

$$Q_S^R = \frac{(ka)^3(kH)(n^2-1)^2}{32\pi^2}QC(H,a), \quad (7)$$

where

$$QC(H,a) = \frac{4\pi^2}{3}\times$$
$$\times\left\{8 - \frac{22}{45}(kH)^2 - \frac{38}{15}(ka)^2 + \frac{13}{105}(ka)^2(kH)^2\right\}.$$

Also, from Eqs. (6) and (2) we finally obtain for the hexagonal RGD cylinder

$$Q_S^R = \frac{9(ka)^3(kH)(n^2-1)^2}{128\pi^3}QH(H,a), \quad (8)$$

where

$$QH(H,a) = \frac{4\pi^2}{3}\times$$
$$\times\left\{8 - \frac{22}{45}(kH)^2 - \frac{76}{45}(ka)^2 + \frac{26}{315}(ka)^2(kH)^2\right\}.$$

To estimate the errors of the approximate formulae (7) and (8), we have calculated directly the integrals in Eqs. (3) and (6) in the RGD approximation disregarding the expansions in the series of light-scattering cross sections (1) and (2) at $ka < 1$ and $kH < 1$. The relative error was calculated as $(F_{\text{approx.}} / F_{\text{precise}} - 1) \cdot 100\%$. The results of comparison are shown in the Table; the relative error of the formulae for the round cylinder is independent of the refractive index. At $ka < 1$, the absolute value of the relative error of the formulae does not exceed 15% for the round cylinder and 9.5% for the hexagonal cylinder.

Then the results of numerical calculations of the values of the efficiency factor of scattering by the approximate formulae and by the Parcell-Pennipacker method (or the method of discrete dipoles, or the method of joined dipoles)[7] were compared. Results of calculation of the efficiency factors of scattering by chaotically oriented round and hexagonal cylinders with the relative refractive index $n = 1.31 + i \cdot 10^{-4}$ by the formulae (7) and (8) and by the method of discrete dipoles are shown in Fig. 1.

The absolute value of the relative error calculated by the approximate RGD formulae (7) and (8) in comparison with the method of discrete dipoles at

Table. Relative errors for the efficiency factor of light scattering calculated by the approximate formulae (7) and (8) for chaotically oriented round and hexagonal RGD cylinders

ka	Cylinder					
	Round			Hexagonal		
	H/a = 0.5	H/a = 1	H/a = 2	H/a = 0.5	H/a = 1	H/a = 2
0.1	−0.001	−0.001	−0.001	−0.072	−0.031	0.135
0.2	−0.011	−0.011	−0.020	−0.292	−0.127	0.528
0.3	−0.056	−0.058	−0.102	−0.667	−0.298	1.141
0.4	−0.178	−0.187	−0.327	−1.209	−0.563	1.905
0.5	−0.442	−0.465	−0.815	−1.938	−0.947	2.716
0.6	−0.933	−0.983	−1.730	−2.876	−1.484	3.423
0.7	−1.766	−1.864	−3.290	−4.057	−2.220	3.820
0.8	−3.087	−3.265	−5.777	−5.517	−3.209	3.632
0.9	−5.079	−5.384	−9.540	−7.303	−4.522	2.510
1.0	−7.969	−8.468	−15.000	−9.472	−6.244	0.027

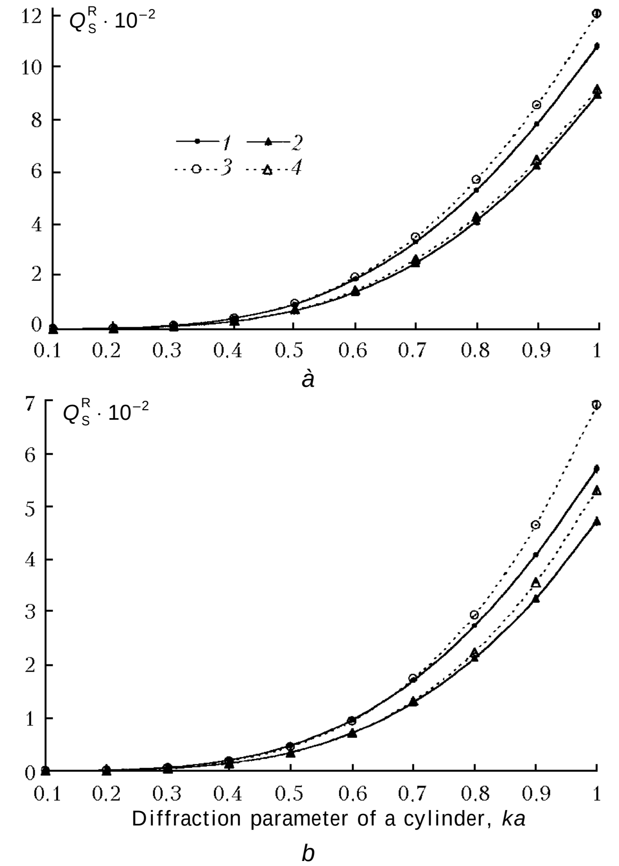

Fig. 1. The efficiency factor of the light scattering Q_S^R as a function of the diffraction parameter *ka* in the RGD approximation (*1*, *2*) and in the discrete dipole approximation (*3*, *4*) for the chaotically oriented cylinders of round (*1*, *3*) and hexagonal (*2*, *4*) cross sections with the related refractive index $n = 1.31 + i \cdot 10^{-4}$ and the constant ratio $H/a = 1$ (*à*), $H/a = 0.5$ (*b*).

$ka < 1$ and $kH < 1$ and at the ratio of the height to the radius from 0.5 to 2 does not exceed 17% for the round cylinder and 13% for the hexagonal cylinder. The higher (than in the Table) values of the relative error are explained by the fact that the direct calculation of the scattering efficiency factors in the RGD approximation without taking into account the expansion in series of the light scattering cross sections (1) and (2) at $ka < 1$ and $kH < 1$ in comparison with the method of discrete dipoles gives the absolute relative error no more than 3%.

Conclusion

The approximate formulae for the light scattering efficiency factors of the chaotically oriented round and hexagonal RGD cylinders are obtained at $ka < 1$ and $kH < 1$. The absolute value of the relative error calculated by approximate formulae as compared to the method of discrete dipoles at the ratio of the height to the radius from 0.5 to 2 does not exceed 17% for the round cylinder and 13% for the hexagonal cylinder.

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