FORMATION OF LATERAL SHEAR INTERFEROGRAMS IN DIFFUSIVELY SCATTERED LIGHT AT A DOUBLE-EXPOSURE RECORDING OF HOLOGRAM OF AN AMPLITUDE SCATTERER FOCUSED IMAGE BY THE GABOR METHOD

V.G. Gusev

Tomsk State University
Received December 9, 1996

Analysis of lateral shear interferometer based on hologram recording of an amplitude scatterer focused image is presented. It is shown that spatial filtering of the diffraction field provides the interferogram which determines spherical aberration of a lens with doubled sensitivity.

In Ref. 1 I considered the case of double-exposure recording by the Gabor by matching speckle-fields of two exposures of an amplitude scatterer focused image when it is irradiated with a coherent light of a converging quasi-spherical waveform with radius of curvature R which does not exceed the distance l_1 from the scatterer to the principal plane of the lens forming the image. It is shown that at the stage of hologram reconstruction with a small-aperture laser beam interference pattern characterizing spherical aberration of the lens and resulting from a combination of the lateral shear interferogram in bands of infinite width and the interference pattern in bands of equal thickness arising from the interference of waves in (−1) and (+1) diffraction orders appears in the Fourier plane. Besides, at $R < l_1$ when regular component of the amplitude scatterer transmittance at the stage of hologram recording overlaps the pupil of a controllable lens, in the course of spatial filtering of the diffraction field out of the hologram the lateral shear interferogram is recorded in the Fourier plane with a doubled sensitivity.

The present paper analyzes some peculiarities in formation of interference patterns in diffusively scattered fields when at the stage of double-exposure recording based on the matching of speckle-fields of the two exposures of an amplitude scatterer focused image hologram by the Gabor method the scatterer is irradiated with a coherent light of a diverging quasi-spherical or converging with a radius of curvature $R < l_1$.

As shown in Fig.1*a*, the amplitude scatterer located in the plane (x_1, y_1) is irradiated by a coherent light with a diverging quasi-spherical wave with the radius of curvature R. Its image is constructed in the photoplate plane (x_3, y_3) by lens L_1 and the focused image hologram is recorded by the Gabor method in a time of first exposure. Before the second exposure the amplitude scatterer is displaced in its plane, for instance, in the direction of x axis at a distance a, while the photoplate is displaced in the opposite direction at a distance $b = a/\mu_1$, where μ_1 is the scale transformation coefficient, l_2 is the distance from the principal plane of the lens L_2 to the photoplate.

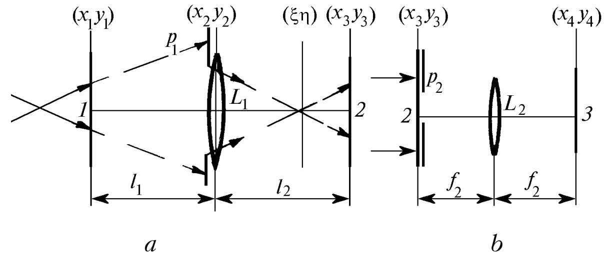


FIG. 1. Optical arrangement of recording (a) and reconstructing (b) of a double-exposure hologram of an amplitude scatterer image: 1, 2, and 3 are amplitude scatterer, photoplate-hologram, plane of interference pattern recording, respectively; L_1, L_2 *are lenses;* p_1 *is the objective aperture,* p_2 *is a spatial filter.*

After photographic treatment the double-exposure Gabor hologram recorded by the above procedure is exposed to a plane wave from a light source used at the stage of recording and interference pattern is recorded in the second focal plane of the lens L_2 with the focal length f_2 (see Fig. 1*b*). The spatial filtering of the diffraction field is performed using an opaque screen p_2 with a round hole placed in the hologram plane *2*.

According to Ref. 2, if the diameter D_0 of the illuminated area in the subject plane satisfies the condition $D_0 \geq dR/(l_1 + R)$, where d is the lens L_1 pupil diameter of the distribution of complex amplitude of the field corresponding to first exposure at a distance $l = f_1 (l_1 + R)/(l_1 + R - f_1)$ from the lens principal plane takes the following form:

$$u_1(\xi,\eta) \sim \exp\left[\frac{ik\,(\xi^2+\eta^2)}{2l}\right]\left\{\exp\left[-\frac{ik\,M\,(\xi^2+\eta^2)}{2l^2}\right] F\left[\frac{kM\,\xi}{l_1\,l},\frac{kM\,\eta}{l_1\,l}\right] \otimes P_1(\xi,\eta)\right\}, \quad (1)$$

where $\otimes$ is the convolution symbol; k is the wave number; $M = l_1(l_1 + R)/R$;

$$F\left[\frac{kM\,\xi}{l_1\,l},\frac{kM\,\eta}{l_1\,l}\right] = \iint\limits_{-\infty}^{\infty} [1 - t(x_1, y_1)] \exp i\,\varphi_0(x_1, y_1) \exp\left[\frac{-\,ik\,(x_1\,\xi + y_1\,\eta)\,M}{l_1\,l}\right] \mathrm{d}x_1\,\mathrm{d}y_1$$

is the Fourier-transform of the input function $[1 - t(x_1, y_1)] \exp i\,\varphi_0\,(x_1, y_1)$; $1 - t(x_1, y_1)$ is the scattering screen transmittance amplitude being a random function of coordinates; $\varphi_0\,(x_1, y_1)$ is the determinate function describing possible phase aberrations of the radiation wave front illuminating the amplitude scatterer due to optical forming system;

$$P_1(\xi,\eta) = \iint\limits_{-\infty}^{\infty} p_1(x_2, y_2) \exp i\varphi_1(x_2, y_2)\ \exp\left[\frac{-\,ik\,(x_2\,\xi + y_2\,\eta)}{l}\right] \mathrm{d}x_2\,\mathrm{d}y_2$$

is the Fourier-transform of the generalized function of $p_1(x_2, y_2) \exp i\,\varphi_1(x_2, y_2)$ of the lens L_1 pupil (see Ref. 3) considering its axial wave aberration.

As is evident from Eq. (1), due to spatial limitation of the field by the aperture p_1 of the lens L_1 (see Fig.1*a*) each point of the Fourier-transform of the input function is broadened in the plane (ξ, η) up to the size of a subjective speckle. The latter is determined by the width of the function $P_1\,(\xi, \eta)$.

Complex field amplitude distribution in the photoplate plane is a result of the Fresnel transformation of the field distribution in the plane of the Fourier-transform formation

$$u_1(x_3, y_3) \sim \iint\limits_{-\infty}^{\infty} u_1(\xi,\eta) \exp\left\{\frac{ik}{2\,(l_2 - l)}\,[(\xi - x_3)^2 + (\eta - y_3)^2]\right\} \mathrm{d}\xi\,\mathrm{d}\eta. \quad (2)$$

By substituting Eq. (1) into Eq. (2) we obtain

$$u_1'(x_3, y_3) \sim \exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2\,(l_2 - l)}\right]\left\{\exp\left[-\frac{ikl\,(x_3^2 + y_3^2)}{2\,l_2\,(l_2 - l)}\right] \otimes\right.$$

$$\otimes\left\{\exp\left[\frac{ikl^2\,(x_3^2 + y_3^2)}{2\,M\,(l_2 - l)^2}\right] \otimes [1 - t(-\,\mu_1 x_3, -\,\mu_1 y_3)] \exp i\varphi_0(-\,\mu_1 x_3, -\,\mu_1 y_3)\right\}\ p_1\left(-\frac{l}{l_2 - l}\,x_3, -\frac{l}{l_2 - l}\,y_3\right) \times$$

$$\left.\times \exp i\varphi_1\left(-\frac{l}{l_2 - l}\,x_3, -\frac{l}{l_2 - l}\,y_3\right)\right\}. \quad (3)$$

Based on the known integral representation of convolution the Eq. (3) takes the following form:

$$u_1(x_3, y_3) \sim \exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2\,l_2}\right]\left\{[1 - t(-\,\mu_1 x_3, -\,\mu_1 y_3)] \exp i\varphi_0(-\,\mu_1 x_3, -\,\mu_1 y_3) \exp\left[ik\,\frac{\mu_1\,(l_1 + R)}{2\,Rl_2}\,(x_3^2 + y_3^2)\right] \otimes P_1(x_3, y_3)\right\}, \quad (4)$$

where

$$P_1(x_3, y_3) = \iint\limits_{-\infty}^{\infty} p_1(x_2, y_2) \exp i\,\varphi_1(x_2, y_2) \exp\left[\frac{-\,ik\,(x_2\,x_3 + y_2\,y_3)}{l_2}\right] \mathrm{d}x_2\,\mathrm{d}y_2$$

is the Fourier-transform of the generalized function of pupil of the lens L_1.

Since the width of the function $P_1\,(x_3, y_3)$ is of the order of $\lambda l_2/d$ (see Ref. 4), where λ is the wavelength of a coherent light used for the hologram recording and reconstruction, let us assume that within the hologram area the change in the phase of a spherical wave with the radius $Rl_2/\mu_1(l_1 + R)$ does not exceed π. Then in the photoplate plane (x_3, y_3) at the diameter $D \le d\left(\frac{l}{l_2 - l}\right)^{-1}$, the quadratic phase factor $\exp\left[ik\,\frac{\mu_1\,(l_1 + R)}{2\,Rl_2}\,(x_3^2 + y_3^2)\right]$ in Eq. (4) can be removed from the integral of convolution with the function $P_1(x_3, y_3)$ and the following expression can be derived:

$$u_1(x_3, y_3) \sim \exp\left\{\frac{ik\,[R + \mu_1\,(l_1 + R)]}{2\,Rl_2}\,(x_3^2 + y_3^2)\right\}\{[1 - t(-\mu_1\,x_3,\,-\mu_1\,y_3)]\,\exp\,i\varphi_0(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3) \otimes P_1(x_3, y_3)\}. \quad (5)$$

From Eq. (5) it follows that each point of the amplitude scattering screen image in the photoplate plane is broadened to the size of a subjective speckle determined by the width of function $P_1(x_3, y_3)$ resulted from the diffraction of a plane wave on the pupil of the lens L_1. In this situation the field of subjective speckles is superimposed by the field of phase distortions of the wave illuminating the scattering screen (if one assumes that the period of function $\varphi_0(-\mu_1 x_3, -\mu_1 y_3)$ exceeds the size of a speckle, T_3^2,) and by the phase distribution of a diverging spherical wave having the curvature of radius $r = Rl_2/[R + \mu_1(l_1 + R)]$.

Let us write expression describing distribution of the complex field amplitude in the photoplate plane before recording the second exposure

$$u_2'(x_3, y_3) \sim \exp\left\{\frac{ik\,[(x_3 + b)^2 + y_3^2]}{2\,(l_2 - l)}\right\}\left\{\exp - \frac{ikl\,[(x_3 + b)^2 + y_3^2]}{2l_2\,(l_2 - l)} \otimes \left\{\exp\frac{ikl^2\,[(x_3 + b)^2 + y_3^2]}{2M\,(l_2 - l)^2} \otimes [1 - t(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3)]\,\times\right.\right.$$

$$\left.\left.\times\,\exp\,i\varphi_0(-\,\mu_1\,x_3 - \mu_1\,b,\,-\,\mu_1\,y_3)\right\}\,p_1\left[-\frac{l}{l_2 - l}\,(x_3 + b),\,-\frac{l}{l_2 - l}\,y_3\right]\exp\,i\varphi_1\left[-\frac{l}{l_2 - l}\,(x_3 + b),\,-\frac{l}{l_2 - l}\,y_3\right]\right\}, \quad (6)$$

which, according to the abovesaid is reduced to the form

$$u_2(x_3, y_3) \sim \exp\left\{\frac{ik\,[R + \mu_1\,(l_1 + R)]}{2\,Rl_2}\,[(x_3 + b)^2 + y_3^2]\right\}\left\{[1 - t(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3)]\,\exp\,i\varphi_0(-\,\mu_1\,x_3 - \mu_1\,b,\,-\,\mu_1\,y_3) \otimes\right.$$

$$\left.\otimes\,\exp\left[\frac{ik\,x_3\,b\,\mu_1\,(l_1 + R)}{Rl_2}\right]P_1(x_3, y_3)\right\}. \quad (7)$$

Let us assume that the photolayer exposed to light with the intensity $I(x_3, y_3) = u_1(x_3, y_3)u_1^*(x_3, y_3) + u_2(x_3, y_3)u_2^*(x_3, y_3)$, is developed and the negative is made on the linear part of the characteristic blackening curve. Then, as in Ref. 1, at $t(x_1, y_1) \ll 1$ the hologram transmittance in Fig.1*b* $\tau(x_3, y_3)$ for diffusively scattered light component is described by the following expression:

$$\tau(x_3, y_3) \sim [\exp\,i\varphi_0(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3) \otimes P_1(x_3, y_3)]\,[t(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3)\,\exp - i\varphi_0(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3) \otimes$$

$$\otimes\,P_1^*\,(x_3, y_3)] + [\exp - i\varphi_0(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3) \otimes P_1(x_3, y_3)]\,[t(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3)\,\exp\,i\varphi_0(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3) \otimes$$

$$\otimes\,P_1(x_3, y_3)] + \left\{\exp\,i\varphi_0(-\,\mu_1\,x_3 - \mu_1\,b,\,-\,\mu_1\,y_3) \otimes \exp\left[\frac{ik\,x_3\,b\,\mu_1\,(l_1 + R)}{R\,l_2}\right]P_1(x_3, y_3)\right\}\times$$

$$\times\left\{t(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3)\,\exp - i\varphi_0(-\,\mu_1\,x_3 - \mu_1\,b,\,-\,\mu_1\,y_3) \otimes \exp\left[\frac{-\,ik\,x_3\,b\,\mu_1\,(l_1 + R)}{R\,l_2}\right]P_1^*(x_3, y_3)\right\} +$$

$$+\left\{\exp - i\varphi_0(-\,\mu_1\,x_3 - \mu_1\,b,\,-\,\mu_1\,y_3) \otimes \exp\left[\frac{-\,ik\,x_3\,b\,\mu_1\,(l_1 + R)}{R\,l_2}\right]P_1^*(x_3, y_3)\right\}\times$$

$$\times\left\{t(-\,\mu_1\,x_3,\,-\,\mu_1\,y_3)\,\exp\,i\varphi_0(-\,\mu_1\,x_3 - \mu_1\,b,\,-\,\mu_1\,y_3) \otimes \exp\left[\frac{ik\,x_3\,b\,\mu_1\,(l_1 + R)}{R\,l_2}\right]P_1(x_3, y_3)\right\}. \quad (8)$$

If the size of a subjective speckle in the plane (x_3, y_3) is small as compared to the period of phase function $\varphi_0\,(-\mu_1 x_3, -\mu_1 y_3)$ and when performing spatial filtering of the diffraction field on the optical axis in the hologram plane, one can show that the complex field amplitude distribution in the second focal plane of the lens L_2 (see Fig. 1*b*) within the overlaping area of the pupil images P_1 (see Fig. 1*a*) for two exposures takes the following form:

$$u(x_4, y_4) \sim \left\{\exp - i\varphi_1(-\mu_2 x_4, -\mu_2 y_4) + \exp i\varphi_1(-\mu_2 x_4, -\mu_2 y_4) + \exp - i\varphi_1\left[\mu_2 x_4 + \frac{\mu_1 (l_1 + R)}{R} b, \mu_2 y_4\right] + \right.$$

$$\left. + \exp i\varphi_1\left[-\mu_2 x_4 + \frac{\mu_1 (l_1 + R)}{R} b, \mu_2 y_4\right]\right\} F(x_4, y_4) \otimes P_2(x_4, y_4), \quad (9)$$

where $\mu_2 = l_2/f_2$ is the coefficient of scaling transformation; $F(x_4, y_4) = \iint\limits_{-\infty}^{\infty} t(-\mu_1 x_3, -\mu_1 y_3) \times \times \exp[-ik(x_3 x_4 + y_3 y_4)/f_2]\, dx_3\, dy_3$ is the Fourier-transform of the absorption function of the amplitude scatterer in its image plane; $P_2(x_4, y_4) = = \iint\limits_{-\infty}^{\infty} p_2(x_3, y_3) \exp[-ik(x_3 x_4 + y_3 y_4)/f_2]\, dx_3\, dy_3$ is the Fourier-transform of the transmission function of an opaque screen p_2 with a round hole (see Ref. 5).

Assuming that the size of a subjective speckle determined by the width of the function $P_2(x_4, y_4)$ in the observation plane *3* (see Fig.1*b*) is small as compared to the period of the phase function in Eq. (9) modulating speckle-field superposition of correlating speckle-fields of the two exposures causes the following illumination distribution for the even phase function $\varphi_1(x_2, y_2)$

$$I(x_4, y_4) \sim [1 + \cos 2\varphi_1(\mu_2 x_4, \mu_2 y_4)]\left\{1 + \cos\left[\frac{\partial\varphi_1(\mu_2 x_4, \mu_2 y_4)}{\partial \mu_2 x_4} \frac{b\mu_1 (l_1 + R)}{R}\right]\right\} |F(x_4, y_4) \otimes P_2(x_4, y_4)|^2, \quad (10)$$

where

$$\frac{\partial\varphi_1(\mu_2 x_4, \mu_2 y_4)}{\partial \mu_2 x_4} \frac{b\mu_1 (l_1 + R)}{R} = \varphi_1\left[\mu_2 x_4 + \frac{b\mu_1 (l_1 + R)}{R}, \mu_2 x_4\right] - \varphi_1(\mu_2 x_4, \mu_2 y_4).$$

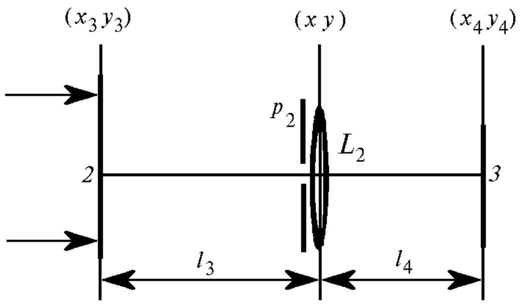


FIG. 2. Optical arrangement of recording interference pattern localized in the hologram plane.

From Eq. (10) it follows that in the observation plane the subjective speckle-structure is modulated by interference fringes. As in Ref. 1, the interference pattern consists of a combination of a lateral shear interferogram in the bands of infinite width and interference pattern in bands of equal thickness characterizing aberration of the controllable lens. If the pattern has doubled sensitivity, then that of a lateral shear interferometer is determined by the shear value b and the geometric factor $G = \mu_1(l_1 + R)/R$. As R decreases, the interferometer sensitivity increases at a given shear value at the expense of the geometric factor because the relative inclination angle $\beta = b\mu_1(l_1 + R)/Rl_2$ between the speckle-fields of the two exposures at the stage of the hologram recording increases. At $R = \mu_1 l_1/(1 - \mu_1)$ the sensitivity depends only on the lateral shear value b.

Let now the two-exposure Gabor hologram of an amplitude scatterer focused image be reconstructed according to Fig. 2 when the lens L_2 constructs the hologram image in the observation plane *3*. Then for the hologram transmittance amplitude $\tau'(x_4, y_4)$ to be determined we can use Eqs. (3) and (6). Based on the above assumptions that the period of the phase function $\varphi_0(-\mu_1 x_3, -\mu_1 y_3)$ exceeds the size of the subjective speckle in the hologram plane $t(x_1, y_1) \ll 1$, expression for diffusively scattered component of the transmittance amplitude takes the following form:

$$\tau'(x_3, y_3) \sim \left\{\exp\left[\frac{-ik(x_3^2 + y_3^2)}{2L}\right] \otimes \left\{\exp\left[\frac{ik(x_3^2 + y_3^2)}{2L}\right] p_1(-\mu_1' x_3, -\mu_1' y_3) \exp i\varphi_1(-\mu_1' x_3, -\mu_1' y_3\right\}\right\} \times$$

$$\times \left\{\exp\left[\frac{ik(x_3^2 + y_3^2)}{2L}\right] \otimes \left\{\exp\left[\frac{-ik(x_3^2 + y_3^2)}{2L}\right] \otimes t(-\mu_1 x_3, -\mu_1 y_3)\right\} \times p_1(-\mu_1' x_3, -\mu_1' y_3) \times\right.$$

$$\times \exp - i\varphi_1(-\mu_1' x_3, -\mu_1' y_3)\Bigg\} + \Bigg\{\exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes \Bigg\{\exp\left[\frac{-ik\,(x_3^2 + y_3^2)}{2L}\right] p_1(-\mu_1' x_3, -\mu_1' y_3) \times$$

$$\times \exp - i\varphi_1 - (-\mu_1' x_3, -\mu_1' y_3)\Bigg\}\Bigg\}\Bigg\} \Bigg\{\exp\left[\frac{-ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes \Bigg\{\exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes t(-\mu_1 x_3, -\mu_1 y_3)\Bigg\} \times$$

$$\times p_1(-\mu_1' x_3, -\mu_1' y_3) \exp i\varphi_1(-\mu_1' x_3, -\mu_1' y_3)\Bigg\} + \Bigg\{\exp\left[\frac{-ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes \Bigg\{\exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2L}\right] \times$$

$$\times p_1[-\mu_1'(x_3 + b), -\mu_1' y_3] \exp i\varphi_1[-\mu_1'(x_3 + b), -\mu_1' y_3]\Bigg\}\Bigg\}\Bigg\} \Bigg\{\exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes$$

$$\otimes \Bigg\{\exp\left[\frac{-ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes t(-\mu_1 x_3, -\mu_1 y_3)\Bigg\}\; p_1[-\mu_1'(x_3 + b), -\mu_1' y_3] \exp - i\varphi_1[-\mu_1'(x_3 + b), -\mu_1' y_3]\Bigg\} +$$

$$+ \Bigg\{\exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes \Bigg\{\exp\left[\frac{-ik\,(x_3^2 + y_3^2)}{2L}\right] p_1[-\mu_1'(x_3 + b), -\mu_1' y_3] \exp - i\varphi_1[-\mu_1'(x_3 + b), -\mu_1' y_3]\Bigg\}\Bigg\} \times$$

$$\times \Bigg\{\exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2L}\right] \otimes \Bigg\{\exp\left[\frac{ik\,(x_3^2 + y_3^2)}{2L}\right] t(-\mu_1 x_3, -\mu_1 y_3)\Bigg\}\, p_1[-\mu_1'(x_3 + b), -\mu_1' y_3] \times$$

$$\times \exp i\varphi_1[-\mu_1'(x_3 + b), -\mu_1' y_3]\Bigg\}, \qquad (11)$$

where $L = l_2R/\mu_1(l_1 + R)$; $\mu_1' = \mu_1(l_1 + R)/R$ are used to shorten the expression.

Similarly for the sake of brevity let us assume that the lens L_2 in Fig. 2 constructs the image with the unit magnification, i.e., $l_3 = l_4 = l_0$. Then the complex field amplitude distribution in the observation plane (x_4, y_4) is determined accurate to the quadratic phase factor insufficient for further consideration by the following form:

$$u'(x_4, y_4) \sim \tau'(x_4, y_4) \otimes P_2'(x_4, y_4), \qquad (12)$$

where

$$P_2'(x_4, y_4) = \iint\limits_{-\infty}^{\infty} p_2(x, y) \exp\left[\frac{-ik\,(x\,x_4 + y\,y_4)}{l_0}\right] \mathrm{d}x\,\mathrm{d}y$$

is the Fourier-transform of the transmittance function of the objective aperture p_2 of the lens L_2.

By substituting Eq. (11) into Eq. (12) and considering the change of the phase function describing spherical aberration of the controllable lens to be slow one can write the following the complex field amplitude distribution within the overlaping zone of functions as

$$p_1\left[\frac{\mu_1\,(l_1 + R)}{R}\,x_4, \frac{\mu_1\,(l_1 + R)}{R}\,y_4\right], p_1\left[\frac{\mu_1\,(l_1 + R)}{R}\,(x_4 - b), \frac{\mu_1\,(l_1 + R)}{R}\,y_4\right]$$

$$u'(x_4, y_4) \sim \left\{1 + \exp\left[-\frac{i\,\partial\varphi_1\,(\mu_1' x_4, \mu_1' y_4)}{\partial\,\mu_1' x_4}\,2\,\mu_1' b\right]\right\} t\,(\mu_1 x_4, \mu_1 y_4) \otimes P_2'(x_4, y_4). \qquad (13)$$

Hence, based on Eq. (13) and on the condition that the size of a subjective speckle in the observation plane *3* (see Fig. 2) determined by the width of the function $P_2'(x_4, y_4)$, is much less than the period of the phase function modulating the speckle-field gives rise to the following illumination distribution:

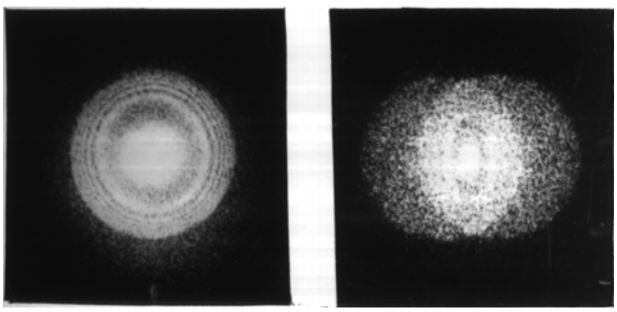
$$I'(x_4, y_4) \sim \left\{1 + \cos\left\{\frac{\partial\varphi_1\left[\frac{\mu_1\,(l_1 + R)}{R}\,x_4, \frac{\mu_1\,(l_1 + R)}{R}\,y_4\right]}{\partial\mu_1\,\frac{(l_1 + R)}{R}\,x_4}\,2b\,\frac{\mu_1\,(l_1 + R)}{R}\right\}\right\} \left|t\,(\mu_1 x_4, \mu_1 y_4) \otimes P_2'(x_4, y_4)\right|^2, \qquad (14)$$

which characterizes the subjective speckle-structure modulated by the interference fringes. The interference pattern represents lateral shear interferogram in the bands of infinite width, caused by spherical aberration of the lens controlled. The lateral shear interferogram sensitivity is in this case doubled at a given shear value.

In the case of illuminating the amplitude diffuse screen by a coherent light with converging quasi-spherical wave with the radius of curvature $R > l_1$ and if the diameter of the diffuse screen illuminated $D_0 \geq dR/(R - l_1)$, from Ref. 6 it follows that the complex field amplitude distribution at a distance $l = f_1(R - l_1)/(f_1 + R - l_1)$ from the principal plane of the lens L_1 (see Fig.1*a*) is described by Eq. (1) with $M = l_1(R - l_1)/R$. Now the Fourier-transform of the input function appears between the lens and its second focal plane in contrast to the previous case when it is formed in the gap between the second focal plane of the lens L_1 and the photoplate. For this reason all the above expressions are valid if $(l_1 + R)$ is replaced by $(R - l_1)$. As a result, the value of lateral shear interferometer sensitivity at a given b increases with decreasing radius of curvature R because of the geometric factor $G = \mu_1(R - l_1)/R$ and at $R = \dfrac{\mu_1 l_1}{\mu_1 - 1}$ the factor is equal to unity.

In our experiments, single- and double-exposure holograms of the amplitude scatterer focused image were recorded by the Gabor method on photoplates of Micrat VRL type using a He-Ne laser emitting at the wavelength $\lambda = 0.63$ μm. As in Ref. 1, the lens with the focal length $f_1 = 160$ mm and pupil diameter $d = 27$ mm forming the paraxial image of the scattering screen in the photoplate plane with the unity magnification was used as a controllable object.

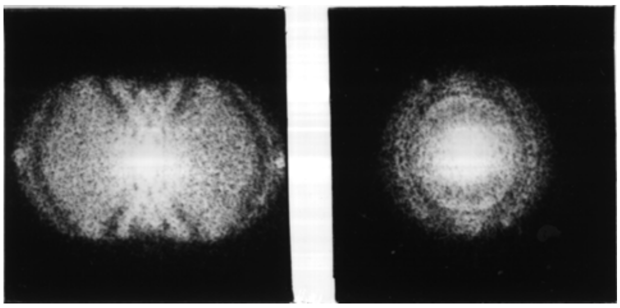
Figure 3 shows interference pattern in the bands of equal thickness characterizing spherical aberration of the lens controlled. In accordance with Fig. 1*b* it was recorded in the focal plane of the camera with the focal length $f_2 = 50$ mm when performing spatial filtering of the diffraction field on the optical axis by reconstructing single-exposure hologram using a small aperture (≈ 2 mm) laser beam.



a *b*

FIG. 3. Interference patterns in the bands of equal thickness recorded when performing spatial filtering in the hologram plane on (a) and off (b) of the optical axis.

The hologram was recorded when illuminating the amplitude scattering screen by a coherent quasi-spherical wave. As in Ref. 1, for the case of illuminating the screen with a converging quasi-spherical wave with the radius of curvature $R < l_1$, the hologram displacement about the reconstructing laser beam ($x_3 = 8$ mm, $y_3 = 0$) leads to a partial spatial separation of the diffracted quasi-plane waves in (−1) and (+1) diffraction orders (see Fig. 3*b*) and gives rise to an angle between the directions of their propagation. Spatial separation of the wave fronts in (−1) and (+1) diffraction orders increasing with the increasing hologram displacement about the laser beam is caused by vignetting which is not evident when the image is formed with a telescope optical system of the Keppler type (for instance, see Ref. 7). In this case the results of single-exposure Gabor hologram recording (see Ref. 8) indicates that only the angle between directions of propagation of the waves diffracted in (−1) and (+1) diffraction orders appears and this angle increases with the increasing hologram displacement with respect to the reconstructing laser beam. Hence, for a given hologram point located out of the optical axis the form of the frequency transfer function of the optical system forming the image (see Ref. 9) determines the angle between the directions of the waves diffracted in (−1) and (+1) diffraction orders. As a result, certain peculiarities appear when recording the interference pattern in the bands of equal thickness (see Fig. 3*a*). The less is the radius of diverging quasi-spherical wave of a coherent radiation or a converging one in Ref. 1 used to illuminate the amplitude scattering screen *1* (see Fig. 1*a*) , the less is the diameter of filtering hole (see Fig. 1*b*) wherein the waves diffracted in (−1) and (+1) diffraction orders propagate in the same direction. The diameter increases as the radius of converging quasi-spherical wave decreases (or increases as in Ref. 1). At $R = l_1$ in the absence of vignetting when the frequency transfer function of the optical system forming the image is uniform and the diameter of the filtering hole reaches image size.

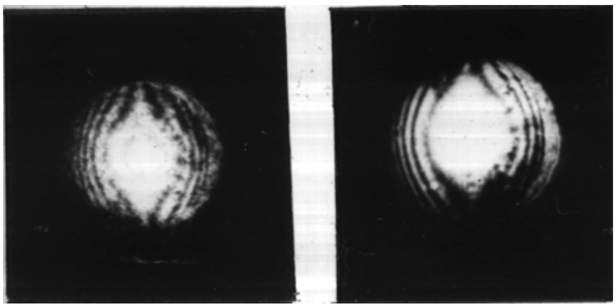


a *b*

FIG. 4. Interference patterns recorded when performing spatial filtering of the diffraction field in the plane of a double-exposure hologram off (a) and on (b) the optical axis.

In Fig.4*a* is shown an interference pattern recorded when double-exposure Gabor hologram is reconstructed with a small-aperture laser beam at the point located on the shear axis at a distance of 10 mm from the optical axis. Interference fringes in (−1) and (+1) diffraction orders are caused by a doubled exposure of photoplate when at the stage of the hologram recording at lateral shear value $b = 2.1$ mm the amplitude scattering screen is illuminated by quasi-plane wave. As in Ref. 1 we have additional interference pattern resulted from the superposition of inversed quasi-plane wave fronts in the zone of overlaping diffraction orders, whereas at the stage of the hologram reconstruction on the optical axis the illumination distribution in the observation plane (see Fig. 4*b*) is expressed by Eq. (10). The interference pattern resulting from a combination of a lateral shear interferogram and interference pattern in the bands of infinite thickness characterizes spherical aberration of the lens under control.

According to Fig. 2, in the case of that double-exposure hologram reconstruction the lateral shear interferogram shown in Fig. 5*a* appears in the observation plane when performing spatial filtration of the diffraction field using the objective aperture p_2. This interferogram describes spherical aberration of the lens with a doubled sensitivity at a given shear value (see Fig. 4). Spatial filtering is necessary to reduce the area, in the observation plane, where the light from regular component of the hologram transmittance is concentrated. The area diameter decreases with the increasing objective aperture. However in this case the size of a subjective speckle increases in the observation plane. If the period of interference fringes is comparable to the speckle size visibility of the interference pattern vanishes (see Ref. 10).



a *b*

FIG.5. Lateral shear interferograms localized in the hologram plane reconstructed using a coherent (a) and polychromatic (b) light.

The possibility of reconstructing focused image hologram recorded using polychromatic (white) light using an off-axis reference wave is well known (see Ref. 11). This is due to a constant size of the image localized in the hologram plane independent of wavelength. As a result, a spectral colored image is observed in the hologram plane. If in the case of a double-exposure hologram of a focused image interference pattern is localized in the hologram plane interference fringes observed are also spectral colored .

The lateral shear interferogram observed in a white light (Fig. 5*b*) is a system of achromatic interference fringes, because no dispersion occurs in this case, the dispersion being caused by the use of an off-axis reference wave when recording a hologram of a focused image. The speckle-structure in the observation plane is not observed. Hence, when a double-exposure Gabor hologram of an amplitude screen focused image is reconstructed with a collimated polychromatic (white) light (see Fig. 2), diameter of the objective aperture p_2 can be decreased.

As our experimental investigation have demonstrated, the interference pattern of the lateral shear interferogram type in the bands of infinite width with doubled sensitivity at a given shear value localizes only in the hologram plane if the double-exposure recording of the hologram is made at the amplitude screen is illuminated by a coherent quasi-spherical wave with the radius of curvature $R > l_1$. Then as in the case of illuminating by a coherent converging quasi-spherical wave with $R > l_1$ when spatial filtering of the diffraction field is made in the near zone (see Ref. 10) the interference pattern is localized in the Fourier plane. This may be explained as follows.

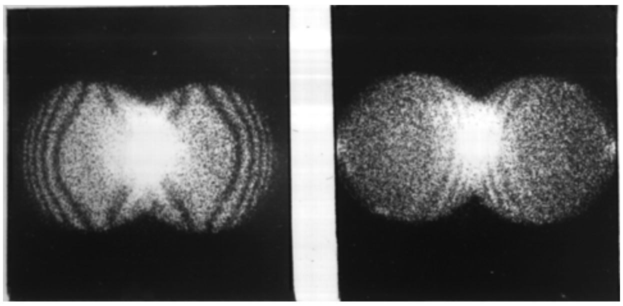


FIG.6. Lateral shear nterferograms in (−1) and (+1) diffraction orders. The amplitude scattering screen is illuminated by a converging, with radius of curvature $R > l_1$ (a) and diverging with $R > l_2$, spherical waves.

In all cases of reconstructing the double-exposure hologram with a small-aperture laser beam and if at the stage of the hologram recording the amplitude scattering screen is illuminated by a coherent quasi-spherical wave with the radius of curvature $R > l_1$ typical interference pattern presented in Fig. 6*a* is recorded in the hologram extreme points located on the shear axis. This pattern is typical in the following aspect. The lateral shear interferograms obtained in (−1) and (+1) orders of diffraction that characterize the combination of axial and off-axis wave aberrations of a lens under control always have more dense interference fringes in the pattern periphery that is in the area farther from the point where regular component of the hologram transmittance is focused. In the case when the amplitude scatterer is illuminated by a coherent converging or diverging quasi-spherical wave of curvature radius $R > l_1$ the lateral shear interferograms in (−1) and (+1) diffraction orders shown in Fig. 6 in the vicinity of the point where the hologram transmittance regular component is focused always have interference fringes spaced more closely. This clearly demonstrates that in the former case normals to the wave fronts of (−1) and (+1) diffraction orders diverge, whereas in second case they converge thus providing the interference pattern localization in the hologram plane with an enhanced sensitivity.

It should be pointed out that when a double-exposure Gabor hologram of the amplitude scattering screen focused image is recorded using a telescope system of Keppler type the lateral shear interferogram formed at the stage of the hologram reconstruction has doubled sensitivity at a given shear value (see Ref. 12). In the case when spatial filtering of diffraction field is performed in the near zone the pattern is localized in the Fourier plane. This is typical for the cases when the field distribution in the image plane is formed as a result of two successive Fourier transformations of the input distribution in the subject plane.

Thus, the results of the experiments performed showed that, as in Ref. 1, when the amplitude scattering screen is irradiated by a coherent converging or diverging quasi-spherical wave with radius of curvature $R > l_1$ single-exposure recording of its focused image by Gabor method is accompanied, at the stage of the hologram reconstruction, by appearance of the interference pattern in bands of equal thickness which characterizes spherical aberration of an object under control with a doubled sensitivity.

As in Ref. 1, when recording a double-exposure hologram at the stage of its reconstruction with a small-aperture laser beam at the point located on the optical axis in the Fourier plane the interference pattern describing spherical aberration of the object and resulting from a combination of the interference pattern in the bands of equal thickness and lateral shear interferogram. In contrast to Ref. 1 the lateral shear interference pattern with a doubled sensitivity at a given shear value is localized in the hologram plane what allow us to reconstruct it using white light and to record achromatic interference fringes.

REFERENCES

1. V.G. Gusev, Atmos. Oceanic Opt. **10**, No. 2, 89–96 (1997).
2. V.G. Gusev, *ibid.* **5**, No. 2, 73–78 (1992).
3. D. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968).
4. M.Franson, *Optics of Speckles* [Russian translation] (Mir, Moscow,1980).
5. M. Born, E. Wolf, *Principles of Optics* (Pergamon Press, Oxford, New York, Paris, Frankfurt, 1964).
6. V.G. Gusev, *Lateral Shear Interferogram Formation in Diffusely Scattered Field when Recording Double-Exposure Lens Fourier Hologram* (Moscow, dep. VINITI, Dec. 11, 1991, No. 4892–B91), 20 pp.
7. M.I. Apenko and A.S. Dubovic, *Applied Optics* (Nauka, Moscow, 1982), 348 pp.
8. V.G. Gusev, Atmos. Oceanic Opt. **9**, No. 7, 569–574 (1996).
9. R.J. Collier, C.B. Burckhardt, and L.H. Lin, *Optical Holography* (Academic Press, New York, 1971).
10. R. Jones and C. Wykes, *Holographic and Speckle Interferometry* (Cambridge University Press, 1986).
11. I.S. Klimenko, E.G. Matinian, and I.P. Nalimov, Opt. Spektrosk. **26**, No. 6, 1019–1026 (1969).
12. V.G. Gusev, Izv. Vyssh. Uchebn. Zaved, Ser. Fizika, No. 2, 89–96 (1996).