Effect of the refractive index variations within the intermediate layer on scattering characteristics

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A modified Mie model corresponding to spherical scatterers with the radially varying refractive index is proposed. For this model, the piecewise continuous hyperbolic approximation is developed and its accuracy is estimated. It is shown that the extinction and backscattering efficiency factors only slightly depend on the monotonically decreasing profile of the refractive index chosen within the intermediate layer and on its degree of smoothness.

In many cases, the results of measurement of optical scattering characteristics, for example, efficiency factors, cannot be successfully described within the framework of the Mie theory.[1,2] The generalized Mie theory based on solution of the problem of plane wave diffraction on a sphere with the radially varying continuous refractive index was proposed in Ref. 3. In Ref. 4 this theory was extended to the piecewise continuous approximation model. The algorithms for problem solution with the use of the piecewise hyperbolic approximation (PHA) was developed in Ref. 5.

Other authors (Refs. 6–7) have studied some particular cases of the problem. The solution proposed in Ref. 5 allows considering arbitrary profiles of the refractive index, and this solution uses, in addition to the Mie theory functions (their number is the same in both cases), only exponential functions with a complex exponent.

In Ref. 5, we considered spherical particles (R is the parameter of the particle size) with an intermediate layer $\rho_0 \le \rho \le R$ (ρ is the diffraction distance from the center of the sphere). The profile of the refractive index was written in the following form:

$$\mathbf{M}(\rho) = \begin{cases} m_0, & 0 \le \rho < \rho_0, \\ M(\rho), & \rho_0 < \rho < R, \\ M, & \rho > R, \end{cases} \quad (1)$$

where the nucleus radius ρ_0 is arbitrary, and m_0 and M are, respectively, the complex and real constants. The chosen model had the form

$$M(\rho) = \frac{\alpha_1}{\rho} + \frac{\alpha_2}{\rho^2} \ (\rho_0 < \rho < R), \quad (2)$$

where the values of α_1 and α_2 were determined by the equations

$$M(\rho_0) = m_0, \quad M(R) = M \quad (3)$$

(condition of the $\mathbf{M}(\rho)$ continuity). The extinction Q_{ext} and backscattering Q_{b} efficiency factors were calculated for the refractive index profile (1)–(3) by the PHA of the kth order, where k is the number of the approximations used (this scheme is shown below). It turned out that with the increasing width $\Delta = R - \rho_0$ of the intermediate layer, the factor Q_{ext} changes much slower than the factor Q_{b} does. On the other hand, it was shown that at $\Delta \to 0$ the limiting values of the factors Q_{ext} and Q_{b} coincide with the values determined from the Mie theory (to which $\Delta = 0$ corresponds). Because of the proper choice of the width of the intermediate layer Δ, we succeeded in achievement of the agreement between the theoretical and measured values[1] of the factors for space fluffy particles. It is significant that this agreement cannot be achieved within the framework of the Mie theory.[1,2]

In Ref. 4, it was shown that the equations for the components of scattered field and various efficiency factors have the same form in the problem of diffraction on a sphere with an intermediate layer and in the Mie problem. These equations depend on the scattering coefficients a_n and b_n (Ref. 8), which are referred to as the Mie coefficients in the problem of diffraction on a sphere. When the scattering sphere is replaced by a scattering sphere with an intermediate layer, the Mie coefficients a_n and b_n are to be replaced with the corresponding generalized Mie coefficients, which are denoted a_n and b_n as before. The PHA described below consists in determining the generalized Mie coefficients for the refractive index profile given by Eq. (1).

Let $\rho_1, \ldots, \rho_{k-1}$ and $x_1, \ldots, x_k$ be arbitrary sets of values meeting the condition

$$\rho_0 < x_1 < \rho_1 < x_2 < \rho_2 < \ldots < \rho_{k-1} < x_k < \rho_k = R. \quad (4)$$

Thus, we have ($j = 1$ corresponds to the electric wave, and $j = 2$ – to the magnetic wave)[5]

$$A_{jn} = -B_{jn}(\psi_n)/B_{jn}(\zeta_n), \quad (5)$$

where $A_{1n} = a_n$ and $A_{2n} = b_n$. In addition, (below the summation index $n = 1, 2, \ldots$ is omitted, k is the order of approximation)

$$B_j(g) = (a_{jk}\Delta_{j+} + c_{jk}\Delta_{j-})\, \delta_{j-}(g) - - (b_{jk}\Delta_{j+} + d_{jk}\Delta_{j-})\, \delta_{j+}(g), \quad (6)$$

where

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$$\Delta_{j\pm} = \begin{vmatrix} \rho_0^{-1.5+j\pm\nu_1} & \psi(m_0\rho_0) \\ (-1.5 + j \pm \nu_1)\rho_0^{-2.5+j\pm\nu_1} & \tau_{j0}^{-1}\, m_0\psi'(m_0\rho_0) \end{vmatrix}, \quad (7)$$

$$\delta_{j\pm}(g) = \begin{vmatrix} \rho_k^{-1.5+j\pm\nu_k} & g(\mu_k\rho_k) \\ (-1.5 + j \pm \nu_k)\rho_k^{-2.5+j\pm\nu_k} & \tau_{jk}\, \mu_k g'(\mu_k\rho_k) \end{vmatrix}; \quad (8)$$

$$\nu_i = [(n + 0.5)^2 - \gamma_i^2]^{0.5};\ \gamma_i = x_i M(x_i),\ i = 1 \dots k; \quad (9)$$

$$m_i = \lim_{\substack{\rho\to\rho_i \\ \rho<\rho_i}} m(\rho),\ \ \mu_i = \lim_{\substack{\rho\to\rho_i \\ \rho<\rho_i}} m(\rho),\ \ i = 1 \dots k; \quad (10)$$

$$\tau_{1i} = m_i^2\, \mu_i^{-2},\ \ \tau_{2i} = 1,\ i = 1 \dots k. \quad (11)$$

In their turn, the coefficients a_{jk}, b_{jk}, c_{jk}, and d_{jk} are determined by the following scheme presented in the matrix form. We have at $k = 1$

$$\begin{bmatrix} a_{j1} & b_{j1} \\ c_{j1} & d_{j1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (12)$$

and at $k \geq 2$

$$\begin{bmatrix} a_{jk} & b_{jk} \\ c_{jk} & d_{jk} \end{bmatrix} = \prod_{i=2}^{k} \begin{bmatrix} \alpha_{ji} & \beta_{ji} \\ \gamma_{ji} & \delta_{ji} \end{bmatrix}. \quad (13)$$

Here

$$\begin{cases} \alpha_{ji+1} = \omega_{ji}(\nu_i, \nu_{i+1}), & \beta_{ji+1} = \omega_{ji}(\nu_i, -\nu_{i+1}), \\ \gamma_{ji+1} = \omega_{ji}(-\nu_i, \nu_{i+1}), & \delta_{ji+1} = \omega_{ji}(-\nu_i, -\nu_{i+1}), \end{cases} \quad (14)$$

where

$$\omega_{ji}(u, v) = 0.5\rho_i^{-u+v}\, \{1 + u^{-1}[0.5 + \tau_{ji}(-0.5 + v)]\}. \quad (15)$$

For typical models, let us study the effect of the refractive index profile $M(\rho)$ described by Eq. (1) on the factors Q_{ext} and Q_{b} calculated by use of the PHA scheme provided that the width of the intermediate layer Δ is fixed. Consider the models

$$M(\rho) = \sum_{i=1}^{4} \frac{\alpha_i}{\rho^i} \quad (16)$$

$$M(\rho) = \beta + \alpha(1 - e^{\rho-\rho_0}). \quad (17)$$

Figures 1 and 2 show, respectively, the factors Q_{ext} and Q_{b} for the size parameters R ($0 \leq R \leq 10$) and $m_0 = 1.5$ that are calculated by the Mie theory (curves *1*) and the generalized Mie theory (curves *2–4*) at different width Δ of the intermediate layer. Curves *2* describe the two-parameter model (2) being a particular case of the model (16) (at $\alpha_3 = \alpha_4 = 0$). Conditions (3) provide for continuity of the refractive index $M(\rho)$ at any ρ. In their turn, curves *3* describe the four-parameter model (16), whose parameters α_1, α_2, α_3, and α_4 are determined from the conditions

$$M(\rho_0) = m_0,\ M(R) = M,\ M'(\rho_0) = M'(R) = 0, \quad (18)$$

providing for continuity of the refractive index $M(\rho)$ and its first derivative.

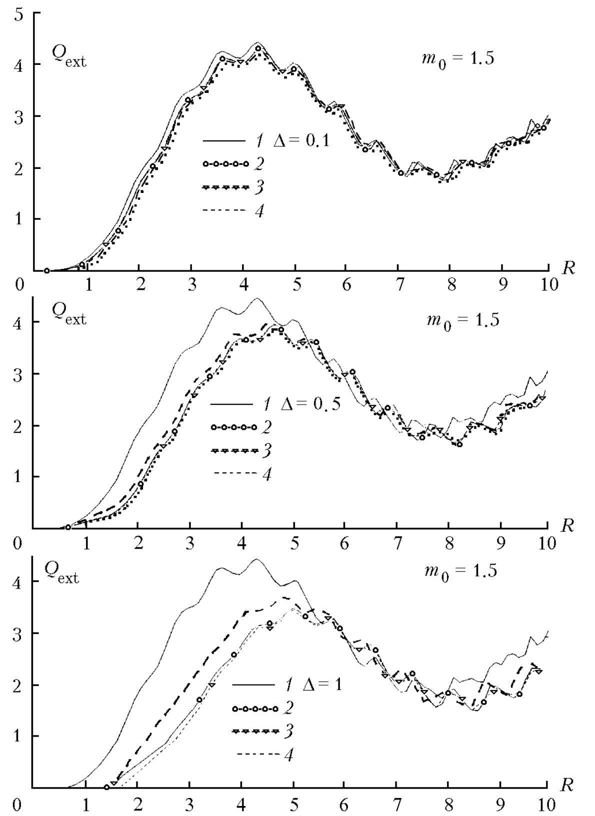

Fig. 1. Dependence of the factor of extinction efficiency on the refractive index profile.

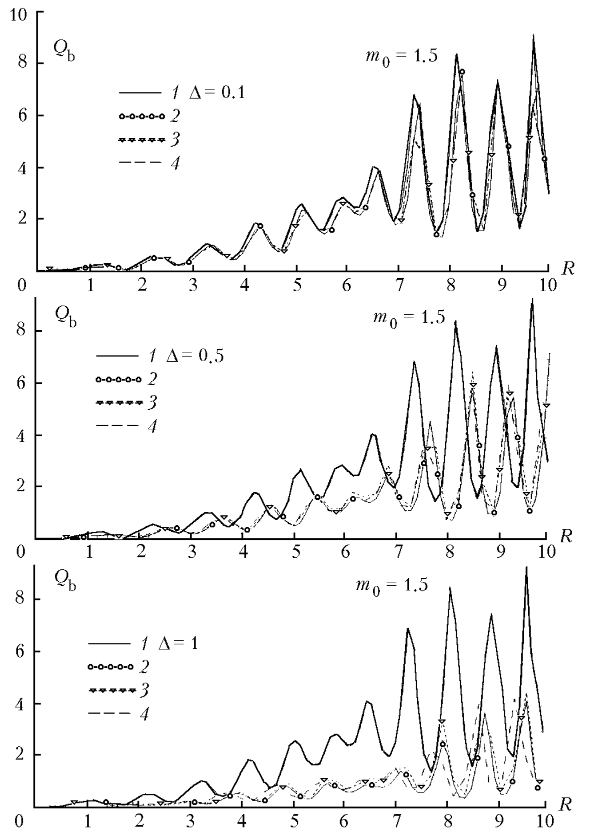

Fig. 2. Dependence of the factor of backscattering efficiency on the refractive index profile.

The result is similar for an arbitrary set of positive values of the parameters α_1, α_2, α_3, and α_4 if the conditions (3) are satisfied. Curves *4* correspond to the two-parameter model (17) with the parameters α and β (here $\beta = m_0$).

The calculated results describe variations of the factors Q_{ext} and Q_{b} depending on different models of the refractive index profile in the intermediate layer of the fixed width Δ. It is shown that Q_{ext} is almost independent of the specific profile, and Q_{b} depends on it very weakly.

Thus, if some data of optical measurements cannot be described within the framework of the Mie theory, they can be described, in the first approximation, with the PHA of any continuous profile of the refractive index by varying the width Δ of the intermediate layer. This trick was illustrated in Ref. 5 for the case of space fluffy particles with the use of the model (2) and (3) of the refractive index.

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