OPTICAL BEAM TRANSFER THROUGH A BOUNDED SCATTERING VOLUME

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A solution of the problem of the optical beam transfer through a bounded medium is presented. Analytical formulas are derived for different experimental configurations with the light source and receiver being located at the boundaries of the medium, with the source being contained within the scattering volume while the receiver being located at its boundary or conversly, and with both the source and receiver being contained within the scattering volume. Theoretical results obtained in the paper agree well with experimental data thereby demonstrating the applicability of the exponential dependence of the radiation attenuation in the disperse media.

INTRODUCTION

In solving the scientific and applied problems of optical radiation transfer through the scattering media, we generally deal with the bounded scattering volumes and the radiation fluxes. The first run of theoretical and experimental studies made in this field in the 1960's (with the advent of laser radiation sources) has brought to light new regularities in the transfer of the narrow optical beams and has greatly contributed to the development of the approximate methods of calculation and to the qualitative interpretation of these regularities. However, up to date these and subsequent studies[1,2] did not provide a comprehensive quantitative account of the finite dimensions of the optical beams and the scattering volumes. Therefore, it might be of interest to solve the problems of a narrow optical beam transfer through a bounded scattering volume using a new analytical method which we presented in Ref. 3 for calculating the radiation fluxes with an acceptable level of accuracy.

A critical factor motivated us to employ a new method for solving the problem on propagation of a narrow beam through a bounded scattering medium is the transfer of a coherent fraction of radiation (direct radiation) of weakly–divergent laser beams at unexpectedly–larger optical thicknesses than with wide beams.[4,5] Unfortunately, the use of the empirical formula[1,6] for estimating the optical thickness for which the contrast between the brightness of direct and scattered radiation vanishes for narrow laser beams is restricted by experimental conditions. The physical interpretation[1,2,6] of this effect resulting from different brightnesses of multiply scattered radiation for narrow and wide optical beams propagating through the scattering media cannot be considered complete since the question of the role of different layers and shapes of the scattering volume remains unsolved. A new approach to the approximate solution of the radiative transfer equations with an account of boundary conditions for a medium and a beam seems to be promising for overcoming early existing difficulties in calculating and interpreting the results.

In this paper the solution of the above–formulated problem is based on the fact that the transfer of optical radiation through the scattering media can be treated independently for the direct radiation and for the incoherent background of the scattered radiation. In this case the attenuation of the direct radiation is described by the Bouguer law while the intensity of the background scattered radiation at small optical depths can be calculated with sufficient accuracy within the framework of the theory of single scattering of radiation.

Only at large optical depths the background multiscattered radiation can predominate and its level, in contrast to the level of the background singly scattered radiation, depends not only on the angular pattern of the receiving–transmiting system[1] but also on the optical dimensions of the beam and the scattering volume.[3]

It is expedient to separate out the single–scattered radiation intensity from a total one because of different ways of accounting for experimental conditions when calculations are made in single or multiple scattering approximations. Therefore the total intensity of the transmitted radiation I at any optical depth τ can be given in the form

$$I = I_0\exp(-\tau) + I_0\gamma\tau\exp(-\tau) + I_m , \qquad (1)$$

where the first component $I_0\exp(-\tau)$ describes the intensity of the direct radiation attenuated according the Bouguer law. The second component $I_0\gamma\tau\exp(-\tau)$ describes the singly–scattered radiation intensity, where the parameter γ is uniquely determined by the angular pattern of the receiving––transmitted system and by the scattering phase function in the forward direction. The third component I_m describes the transmitted radiation intensity associated with the multiple scattering effects. We will try to calculate the third component for different experimental conditions using the approximate analytical method described in Ref. 3.

CALCULATION OF MULTIPLY SCATTERED RADIATION INTENSITY TAKING THE POSITION OF THE SOURCE AND RECEIVER WITHIN THE SCATTERING VOLUME INTO ACCOUNT

In calculating the fluxes of multiply scattered radiation in the bounded scattering volume it is necessary to take into account not only its transverse dimensions but also longitudinal ones which are outside the region lying between the plane of the optical beam entering the medium and the plane of radiation reception if the scattering volume is not bounded by the planes in which the radiation source or the receiver are located. The longitudinal dimensions of the scattering volume must be taken into account depending on the experimental configuration due to the fact that in the process of multiple scattering the fraction of radiation scattered outside the region lying between the source and the

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receiver returns and contributes to the radiation flux recorded by the receiver. Thus, the intensity of multiply scattered radiation at the point of reception can be different depending on the experimental configuration.

When solving the problem of the optical beam propagation through the bounded volume we will consider the following four experimental configurations: the source and the receiver are at the boundaries (longitudinal) of the scattering volume; the radiation source is contained within the scattering volume and the receiver is at its boundary; the radiation receiver is contained within the scattering volume and the source is at its boundary; and, the radiation source and the receiver are contained within the scattering volume. We will also assume that the illumination and receiving planes are perpendicular to the source–receiver axis and the scattering volume is taken as a parallelepiped with dimensions τ_y and τ_z.

1. The source and the receiver are at the boundaries of the scattering volume. For this experimental configuration we can use the formulas for a one–dimensional case[7] with the generalized (for a three–dimensional case) parameters $K(\tau_y, \tau_z)$ and $R(\tau_y, \tau_z)$ in the form[3]:

$$A_1 = \frac{(1 - R^2)e^{-K\tau_0}}{1 - R^2e^{-2K\tau_0}}; \quad B_1 = \frac{R(1 - e^{-2K\tau_0})}{1 - R^2e^{-2K\tau_0}};$$

$$C_1 = \frac{(1 - R)(1 - e^{-K\tau_0})}{1 + Re^{-K\tau_0}}, \tag{2}$$

where $I_{m1} = I_0A_1$ is the transmitted radiation intensity, I_0B_1 is the reflected radiation intensity, I_0C_1 is the absorbed radiation intensity (comprising the radiation leaving the scattering volume), I_0 is the intensity of radiation incident on the medium, and τ_0 is the optical depth of the scattering layer lying between the source and the receiver (along the x axis).

2. The radiation source is contained within the scattering volume. An additional layer of the scattering medium lying behind the radiation source results in increase of the background of the multiply scattered light entering the receiver. Let us denote the optical depth of the layer lying behind the source as τ_1 and make use of the method of multiple reflections for calculating the value I_{m2} at the receiver.

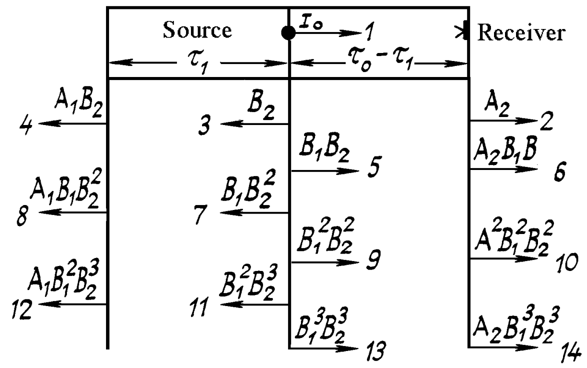

FIG. 1. A diagram for calculating the transmitted radiation intensity (in the plane of reception) for the experimental configuration with the source contained within a scattering volume.

In the upper part of Fig. 1 a schematic configuration of the experiment for this case is shown; in the lower part of this figure — a diagram which can be deciphered as step–by–step accounting for the intensities of the radiation transmitted through and reflected from the boundaries of the individual scattering layers.

After the radiation from the source with the intensity $I_0 = 1$ (step 1) has entered the scattering volume of the optical depth $(\tau_0 - \tau_1)$ the component of the intensity of radiation transmitted through the reception plane is A_2 (step 2) while the component of the intensity of radiation reflected from the plane of the radiation source is B_2 (step 3). But when the layer of the depth τ_1 is irradiated by the intensity B_2, the radiation with the intensity A_1B_2 (step 4) emanates the scattering layer of the depth τ_1 and the reflected radiation with the intensity B_1B_2 (step 5) enters the scattering layer of the depth $(\tau_0 - \tau_1)$. This reflected radiation after propagation through the layer $(\tau_0 - \tau_1)$ makes an additional contribution to the intensity of the radiation transmitted through the reception plane being equal to $A_2B_1B_2$ (step 6), and so on. Figure 1 also shows two more components of the intensity of radiation transmitted through the reception plane (steps 10 and 14) after propagation through the layer and reflection from the opposite planes of the scattering volume (steps 7–13).

Summing over all of the possible components of the intensity of radiation transmitted through the receiving plane, following the diagram of Fig. 1, we can write

$$I_{m2} = A_2\sum_{n=0}^{\infty}B_1^nB_2^n = \frac{A_2}{1 - B_1B_2}, \tag{3}$$

where the subscripts 1 and 2 stand for the components of the intensity of the transmitted A and reflected B radiation for the scattering layers of the depths τ_1 and $(\tau_0 - \tau_1)$, respectively. It follows from formula (2) that

$$B_1 = \frac{R(1 - e^{-2K\tau_1})}{1 - R^2e^{-2K\tau_1}}, \quad A_2 = \frac{(1 - R^2)\, e^{-K(\tau_0-\tau_1)}}{1 - R^2\, e^{-2K(\tau_0-\tau_1)}},$$

$$B_2 = \frac{R\,(1 - e^{-2K(\tau_0-\tau_1)})}{1 - R^2\, e^{-2K(\tau_0-\tau_1)}}. \tag{4}$$

Substituting the values B_1, A_2, and B_2 from Eq. (4) into Eq. (3) gives

$$I_{m2} = I_0\frac{(1 - R^2\, e^{-2K\tau_1})\, e^{-K(\tau_0-\tau_1)}}{1 - R^2\, e^{-2K\tau_0}}, \tag{5}$$

where the parameters $K(\tau_y, \tau_z)$ and $R(\tau_y, \tau_z)$ take into account the dependences of the total intensity of the radiation I_{m2} transmitted through the receiving plane on the transverse dimensions of the scattering volume.

3. The receiver is contained within the scattering volume. As for the previous configuration, we will consider one after another the intensity components of the transmitted and reflected radiation at the boundaries of the scattering layers bounded by the planes of the radiation source (a front wall of the scattering volume) and of the receiver and by the back wall of the scattering volume. The optical depth of the two layers identified in such a way is shown in the upper part of Fig. 2. The radiation with the intensity $I_0 = 1$ (step 1) incident on the first layer of the optical depth $(\tau_0 - \tau_2)$ results in the appearance of one of the intensity components of the reflected B_1 (step 2) and of the transmitted A_1 (step 3)

radiation in the receiving plane. After propagation of the radiation intensity A_1 through the layer τ_2, the radiation intensity A_2A_1 (step 4) leaves the scattering volume and the radiation intensity B_2A_1 (step 5), reflected from this layer, leaves the receiving plane.

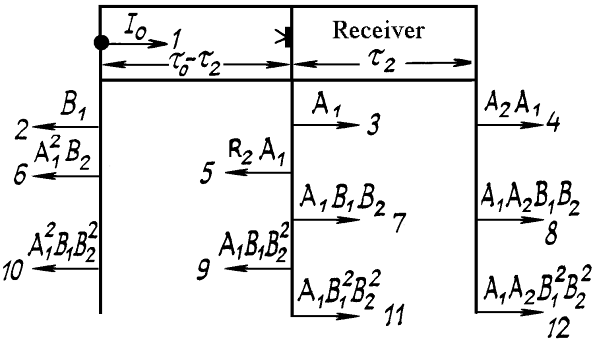

FIG. 2. A diagram for calculating the transmitted radiation intensity (in the plane of reception) for the experimental configuration with the source contained within a scattering volume.

After propagation of the radiation intensity B_2A_1 through the layer $(\tau_0 - \tau_2)$, the radiation $A_1^2B_2$ (step 6) leaves the scattering volume and the radiation intensity $A_1B_1B_2$, reflected from this layer, leaves the receiving plane, and so on. Summing over all possible components of the intensity of the radiation transmitted through the receiving plane, according to the diagram of Fig. 2, we can write

$$I_{m3} = A_1 \sum_{n=0}^{\infty} B_1^n B_2^n = \frac{A_1}{1 - B_1B_2}, \quad (6)$$

and after substituting A_1, B_1, and B_2 determined from formulas (2) we obtain for the total intensity

$$I_{m3} = I_0 \frac{e^{-K(\tau_0-\tau_2)} (1 - R^2 e^{-2K\tau_2})}{1 - R^2 e^{-2K\tau_0}}, \quad (7)$$

where, in analogy with the previous case, the parameters $K(\tau_y, \tau_z)$ and $R(\tau_y, \tau_z)$ depend on the transverse dimensions of the scattering volume.

4. The source and the receiver are contained within the scattering volume. To calculate the intensity of radiation transmitted from the source to the receiver with an account of multiple scattering for the given experimental configuration we may use one of the two above–described configurations. To this end, it is sufficient to separate out two scattering layers bounded by the front or the back walls of the scattering volume and by the plane of the receiver or the source, respectively. The subsequent discussions are similar to those given above, and the total intensity of the transmitted radiation in the receiving plane takes the form

$$I_{m4} = I_0 \frac{(1 - R^2e^{-2K\tau_1})(1 - R^2e^{-2K\tau_2})e^{-K(\tau_0-\tau_1-\tau_2)}}{(1 - R^2)(1 - R^2 e^{-2K\tau_0})}, \quad (8)$$

where $K(\tau_y, \tau_z)$ and $R(\tau_y, \tau_z)$ depend, as previously, on the transverse optical dimensions of the scattering volume.

Formula (8) is general, all previous relations follows from it as particular cases. Actually, if for the experimental configuration with the source or the receiver being contained within the scattering volume we assume $\tau_2 = 0$ or $\tau_1 = 0$, then formulas (5) and (7) follow from Eq. (8). If the source and the receiver are at the boundaries of the scattering volume, then $\tau_1 = \tau_2 = 0$ and from Eq. (2) the formula for A_1 can be obtained from Eq. (8).

CALCULATION OF THE INTENSITY OF MULTIPLY SCATTERED RADIATION TAKING THE DIMENSIONS OF THE OPTICAL BEAM INTO ACCOUNT

The formulas in the above section were obtained for the case in which the transverse dimensions of the optical beam coincide with those of the scattering volume. Here we deal with a more general case in which the dimensions of the beam with the optical cross section Δ_1 are smaller than transverse dimensions of the scattering volume. It follows from Eq. (1) that only the third component of the transmitted radiation intensity associated with the effects of multiple scattering depends on the relationship between the cross sections Δ_1 and Δ_2.

In the first approximation we can assume that the intensity component I_m associated with multiple scattering effects has a uniform distribution over the solid angle ω. It is the case at large optical depths[6] while at small optical depths the contribution of multiply scattered radiation to the total intensity of the transmitted radiation is negligible. In such an approximation I_m can be considered to comprise two components.

One of the components determines the fraction of the intensity of multiply scattered radiation which is formed within the region occupied by the beam and, according to formula (2) for the first experimental configuration is equal to

$$I'_{m1} = I_0 \frac{\omega}{2\pi} d_1,$$

$$d_1 = \frac{(1 - R_1^2) e^{-K_1\tau_0}}{1 - R_1^2 e^{-2K_1\tau_0}} - (1 + \gamma\tau_0) e^{-\tau_0}. \quad (9)$$

Here the subscript adjacent to K_1 and the subscript adjacent to R_1 indicate the calculation to be made for a medium whose optical transverse dimensions are equal to the diameter of the beam.

The second component describes the multiply scattered background radiation formed outside the region occupied by the beam and represents the response of those part of the medium which is not illuminated by the direct beam. It is given by the formula

$$I_{ms} = I_0 \frac{\omega}{2\pi} \nu(d'_1 - d_1),$$

$$d'_1 = \frac{(1 - R_2^2) e^{-K_2\tau_0}}{1 - R_2^2 e^{-2K_2\tau_0}} - (1 + \gamma\tau_0) e^{-\tau_0}, \quad (10)$$

where the subscript adjacent to K_2 and the subscript adjacent to R_2 indicate the calculation to be made for the entire medium and the factor ν depending on the optical parameters of the medium and the experimental configuration takes into account different density of radiation flux outside the region occupied by the beam.

Thus, in the first approximation the total value I_{m1} is given by the formula

$$I_{m1} = I'_{m1} + I_{ms} = I_0 \frac{\omega}{2\pi} [d_1 + \nu (d'_1 - d_1)] \ . \qquad (11)$$

Likewise, the values of I_{mi} are calculated for all experimental confugurations including the most typical configuration in which the source and the receiver are contained within the medium. For the fourth experimental configuration the calculation of I_{m4}, after taking the introduced factor ν and Eq. (8) into account, gives the formula

$$I_{m4} = I_0 \frac{\omega}{2\pi} [d_1 + \nu (d'_4 - d_4)] \ ,$$

$$d_4 = \frac{(1 - R_1^2 \, e^{-2K_1\tau_1}) \, (1 - R_1^2 \, e^{-2K_1\tau_2}) \, e^{-K_1(\tau_0-\tau_1-\tau_2)}}{(1 - R_1^2) \, (1 - R_1^2 \, e^{-2K_1\tau_0})} -$$

$$- [1 + \gamma(\tau_0 - \tau_1 - \tau_2)] \, e^{-(\tau_0-\tau_1-\tau_2)} \ , \qquad (12)$$

$$d'_4 = \frac{(1 - R_2^2 e^{-2K_2\tau_1}) \, (1 - R_2^2 e^{-2K_2\tau_2}) \, e^{-K_2(\tau_0-\tau_1-\tau_2)}}{(1 - R_2^2) \, (1 - R_2^2 \, e^{-2K_2\tau_0})} -$$

$$- [1 + \gamma(\tau_0 - \tau_1 - \tau_2)] \, e^{-(\tau_0-\tau_1-\tau_2)} \ .$$

Figure 3 shows the calculated results for the two components of multiply scattered radiation intensity I_{mi}, i.e., I'_{mi} and I_{ms} based on the above–derived formulas. It was assumed in calculations that the optical cross section of the beam $\Delta_1 = 4$.

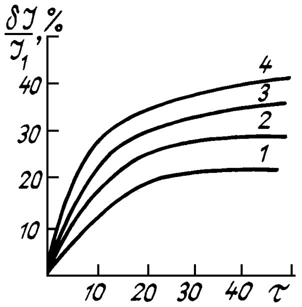

FIG. 3. Relative increase in the fraction of multiply scattered light as a function of the optical depth of the layer behind the source (the source is contained within the scattering volume): 1) $\tau_0 = 5$, *2)* $\tau_0 = 10$, *3)* $\tau_0 = 20$, *and 4)* $\tau_0 = 50$.

It can be seen from Fig. 3 that with increase of the optical depth τ the effect of the intensity component associated with the response of the medium becomes stronger and comparable with the effect of the component associated with multiple scattering inside the region occupied by the beam.

CONCLUSION

Our results provide a more comprehensive interpretation of the effect of multiply scattered background radiation on the brightness contrast of laser sources observed through the scattering medium. In particular, it is evident from the above–calculated data (Fig. 3) that the level of multiply scattered background radiation formed by the scattering volume lying outside the volume directly illuminated by the optical beam plays an important role at large optical depths. Therefore, the threshold optical depth τ_{th} at which the brightness contrast of the observed radiation source (the difference between the intensities of direct and scattered radiations) vanishes depends not only on geometric dimensions of the optical beam and scattering properties of the medium but also on the overall dimensions of the scattering volume. This is a new and principal aspect in interpreting the dependences of τ_{th} on the medium properties and the geometry of the experiment.

Figure 4 shows the results of estimating the applicability limits of the exponential dependence of attenuation on the parameter of a scattering medium ρ for different angular apertures of the radiation receiver ω. The angular divergence of the beam was taken to be 6′.

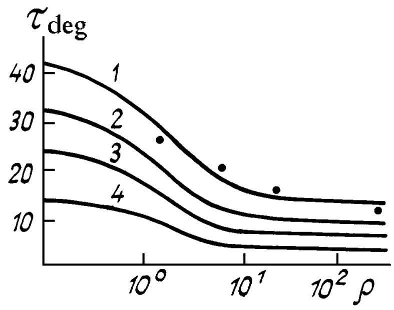

FIG. 4. Applicability limits of the exponential dependence of attenuation vs the parameter ρ *and the receiving aperture: 1)* $\omega = 0.1$, *2)* $\omega = 0.5$, *3)* $\omega = 2$, and *4)* $\omega = 10°$.

It can be seen from Fig. 4 that the strongest dependence on the scattering phase function takes place for $\rho < 2$. When $\rho > 2$ the ρ–dependence of τ_{th} is close to the linear one. Such a dependence is supported by a number of experimental data based on which an empirical formula for τ_{th} in the scattering medium has been written[1]

$$\tau_{th} = - 5 \lg(\sigma d) + b \ ,$$

where d is the diameter of the beam and b is the empirical parameter. The filled circles in Fig. 4 denote the experimental data obtained in Refs. 1–6. Satisfactory agreement between the curves calculated from our formulas and experimental data is indicative of the fact that in many cases the nonuniformity in the distribution of multiply scattered radiation intensity I_m can be neglected. We hope that in future the more detailed studies will allow us to identify the cases in which the adjustment factor ν must be introduced.

So, the simplest case of normal illumination of the scattering volume with a narrow beam has been treated in the present paper. The problems of inclined illumination have not yet been considered because of the cumbersome formulas being derived in this case. But it is the inclined illumination that provides the solution of the problem of optical radiation transfer through the spherical atmosphere and therefore it is of definite interest. The calculated results for this case and some other problems must be considered separately.

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