FLEXIBLE MEMBRANE MIRROR FOR ADAPTIVE OPTICAL INFORMATION-MEASURING SYSTEMS

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The construction of a flexible membrane mirror is described. Its basic statistical characteristics — the sensitivity and the response function of the mirror — are studied. The chosen arrangement of actuators is analyzed theoretically. The possibility of using such a mirror in an atmospheric adaptive optical system is examined.

The random distortions of the wavefront caused by atmospheric turbulence degrade the characteristics of optical systems used for transmitting and receiving information through the atmosphere, in particular, large ground-based telescopes. The resolution of a telescope can be substantially improved with the help of an adaptive optical system with a controllable mirror.[1] Membrane mirrors with electrostatic control are promising mirrors of this type.[2] They can operate with low voltages, they are simple to build, and they permit reproducing comparatively simply quite complicated phase aberrations of the distorted light wave.[3]

In this paper the basic questions pertaining to the calculation of controllable membrane mirrors for through-the-atmosphere viewing systems are studied. A model of a mirror which has been built is described its characteristics are studied.

The construction of an electrostatistically controllable membrane mirror is described in Refs. 2 and 3. Such a mirror consists of a mirror metallic film uniformly tensioned onto a ring. The shape of the mirror surface is controlled with the help of electrodes placed on the back side of the mirror. In order that the mirror be able to bend in both directions a transparent electrode is placed in front of it. Control of the mirror depends on the requirements which the adaptive system as a whole must meet. In problems in which the best correction of the distorted wavefront must be achieved the mirror is controlled with the help of the voltages of each electrode separately. The effectiveness of the correction in this case can be evaluated in accordance with the results of Ref. 4 under the assumption that the response function of the mirror is local.[3] In adaptive systems that implement modal correction of the wavefront the mirror is controlled with the help of specially computed electrode voltages.[5] The number of control channels in this case is usually chosen to be less than the number of electrodes. We shall study such systems. We shall describe the characteristic forms of the wavefront distortions with the help of the lowest-order Zernike polynomials z_j, widely employed in adaptive atmospheric systems.[1,6] Then we shall represent the distorted phase φ of the light wave in the form

$$\varphi(r) \approx \sum_{j=1}^{m} \beta_j Z_j(r),$$

where $Z_1 = 1$, $Z_{2,3} = 2\rho\exp^{i\theta}$, $Z_4 = \sqrt{3}(2\rho^2 - 1)$, 1

$$Z_{5,6} = \sqrt{6}\,\rho^2\exp^{i2\theta},$$

$$Z_{8,7} = \sqrt{8}\,(3\rho^3 - 2\rho)\exp^{i\theta},\quad Z_{9,10} = \sqrt{8}\,\rho^3\exp^{i3\theta}, \tag{1}$$

$$\rho = 2r/D,$$

(r and θ are the polar coordinates; D is the diameter of the receiving aperture Ω; β_j are the expansion coefficients whose variances are:

$\langle\beta_j^2\rangle = 0.499(D/r_0)^{5/3}$, $j = 2, 3$;

$\langle\beta_j^2\rangle = 0.023(D/r_0)^{5/3}$, $j = 4–6$;

$\langle\beta_j^2\rangle = 0.006(D/r_0)^{5/3}$, $j = 7–10$;

$\sum_{j=11}^{\infty}\langle\beta_j^2\rangle = 0.0401(D/r_0)^{5/3}$; r_0 is Fried's correlation radius; and, the brackets denote averaging over an ensemble of realizations).

The control of the mirror is based on compensation of each polynomial z_j separately. In this case the error J in the correction of the wavefront can be written approximately in the form

$$J = \left\langle \frac{1}{S}\int_\Omega \left[\sum_{j=2}^{m}\beta_j\left[Z_j - \sum_{k=1}^{N} P_k^j R_k\right] + \sum_{j=m+1}^{\infty}\beta_j Z_j\right]^2 d^2r\right\rangle \simeq$$

$$\simeq \sum_{j=2}^{m}\langle\beta_j^2\rangle J_j + \sum_{j=m+1}^{\infty}\langle\beta_j^2\rangle, \tag{2}$$

where

$$J_j = \frac{1}{S}\int_\Omega \left[Z_j - \sum_{k=1}^{N} P_k^j R_k\right]^2 d^2r \tag{3}$$

are the errors in the approximation of the polynomials Z_j; $S = \pi D^2/4$; N is the number of electrodes; P_k^j is the controlling force on the k-th electrode in correcting $Z_j(\bar{r})$; $R_k(r)$ is the response function of the mirror, determined from the solution of the membrane deflection equation[7]:

$$T\Delta W = -q; \quad W(r = R_0) = 0, \tag{4}$$

Δ is the Laplacian operator; W is the deflection of the membrane; q is the load acting on the membrane; and, T is the tension acting on the mirror in a frame with radius R_0.

The response function of the mirror for a stress created by a central circular electrode with radius r_{sa} is presented in Ref. 3 and Green's function for a membrane is given in Ref. 7. By comparing them it can be shown that the response function of an arbitrarily positioned actuator, producing a load q_s = const in its region, will be determined by the expression

$$R_s(\vec{r}) = \frac{1}{4\pi T}\begin{cases} \ln(B/r_p^2)+1-A/r_p^2, & A \le r_p^2, \\ \ln(B/A), & A > r_p^2, \end{cases} \tag{5}$$

where $A = r^2 + r_s^2 - 2rr_s\cos(\theta - \theta_s)$;

$$B = r_s^2 r^2/R_0^2 + R_0^2 - 2rr_s\cos(\vartheta - \vartheta_s), \quad s = 1, \dots, N; \tag{6}$$

and, (r_s, θ_s) are the polar coordinates of the center of the s-th actuator.

We call attention to the following. Superthin, free, metal films have a very low intrinsic bending stiffness, which is neglected in most cases. If, however, the tension T is sufficiently small, this stiffness can affect the character of the deformation of the mirror. In this case the shape of the mirror will be described by the equation for the deflection of thin plate under tension:[8]

$$(D_0\Delta\Delta - T\Delta)W = q; \tag{7}$$

$$W(r = R_0) = 0; \tag{8}$$

$$\frac{\partial}{\partial r} W(r = R_0) = 0, \tag{9}$$

where D_0 is the cylindric stiffness of the mirror.

In accordance with the general procedure for solving Eq. (7) for a circular region, it is not difficult to show[8] that the response function of the mirror for a concentrated force applied at a point with the coordinates $(\vec{r}_s, \theta_s)$ can be approximately calculated using the formula

$$R_s(\vec{r}) = \frac{1}{2\pi T}\left\{\frac{1}{2}\ln\frac{B}{A} + K_0(\omega B) - K_0(\omega A) + C_0\left[I_0(\omega B) - I_0(\omega A)\right]\right\},$$

$$C_0 = \left[K_1(\omega_1) - \frac{1}{\omega_1}\right]/I_1(\omega_1), \quad \omega_1 = (r_s + R_0)\omega, \tag{10}$$

where $\omega^2 = T/D_0$, and I_0, I_1, K_0, and K_1 are modified Bessel functions.

This response function satisfies Eqs. (7) and (8) for the approximate condition (9) in the form of $\frac{\partial W}{\partial r}\left(r = r_s + R_0, \theta = \frac{\pi}{2}\right) = 0$. At $r_s = 0$ it is the exact solution of the problem (7)–(9). Graphs of the response functions (5) and (10) in the case of the action of an actuator at the center ($r_s = 0$) are presented in Fig. 1.

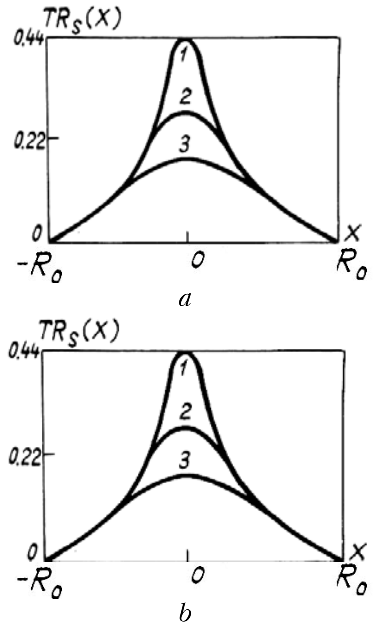

FIG. 1. The response of a membrane mirror to the action of an actuator at the center ($r_s = 0$): a) $D_3 = 0$: 1 – $r_a = 0.1\ R_0$, 2 – $r_a = 0.3\ R_0$, 3 – $r_a = 0.5\ R_0$, b) $D_0 \neq 0$, $r_a = 0$: 1 – $\omega = 2$, 2 – $\omega = 20$.

The controlling forces P_k^j, required for approximating the corresponding polynomial $Z_j(\bar{r})$, can be calculated with the help of the expressions(5) and (10) obtained above. Since the condition that the mirror be clamped on the frame does not permit giving the form of z_j in the entire region of the mirror the diameter D of the region of correction must be chosen to be less than $2R_0$. We shall study the dependence of the approximation errors (3) on geometry of the arrangement of the actuators. In what follows, for simplicity, we shall everywhere assume that the membrane mirror is made of an absolutely flexible metallic film ($D_0 = 0$). In this case the functions $r^n \exp[in\theta]$, $n = 0, 1, 2, \dots$ satisfy Eq. (4) identically.[7]

These functions include the first ten Zernike polynomials, with the exception of z_4 and $z_{8,7}$. For this reason, to correct $z_j(\bar{r})$, $j = 1, 2, 3, 5, 6, 9$, and 10, the electrodes must be placed on the boundary of the region Ω or outside it. The electrodes should be placed inside Ω only to obtain Z_4, $Z_{8,7}$ ($m \le 10$).

Five variants of the arrangement of circular actuators with radius r_a at the nodes of Cartesian and polar coordinate grids were calculated. The first two variants are shown in Fig. 2a, while the remaining three variants are shown in Fig. 2b (the solid lines show the arrangements of the actuators). We denote these variants by the numbers 1–5. They correspond to the following values of the parameters: N, r_a, a_i:
1 – $N = 21$, $a_0 = 2r_a = 0.25\ D$;
2 – $N = 37$, $a_0 = 0.165D$, $2r_a = 0.15\ D$;
3 – $N = 7$, $a_1 = 2r_s = 0.4\ D$;
4 – $N = 19$, $a_2 = 2a_1 = 4r_a = 0.5\ D$;
5 – $N = 43$, $a_3 = 1.5$, $a_2 = 3$, $a_1 = 0.495\ D$,
$r_a = 0.08\ D$. In all the variants the radius of the mirror was assumed to be $R_0 = 0.65\ D$. The controlling forces necessary to reproduce Z_j were calculated using the formula[1]

$$\sum_{k=1}^{N} P_k^j R_k(\vec{r}_s) = Z_j(\vec{r}_s); \quad s = 1, \ldots, N;$$
$$j = 1, \ldots, m, \tag{11}$$

where $\vec{r}_s$ are the coordinates of the centers of the electrodes.

TABLE I.

Variant No.	N_{zj}					
	1	2.3	4	5.6	7.8	9.10
1	0.12	0.22	0.53	0.33, 0.36		
2	0.05	0.08	0.19	0.13	0.36	0.19
3	0.14	0.27	0.56	0.39, 0.44		
4	0.09	0.17	0.44	0.23	0.85	0.28; 0.32
5	0.05	0.09	0.21	0.13	0.40	0.19

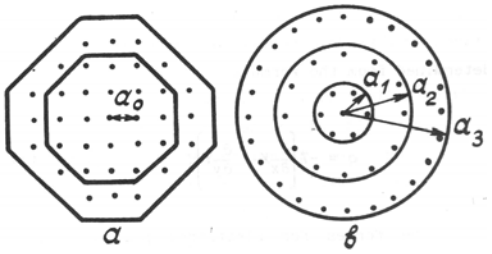

FIG. 2. Diagrams of the positions of the centers of the actuators.

The forces determined in this manner correspond to the experimental arrangement of the control system.[1] The errors $\sqrt{J_j}$ in the approximation of the Zernike polynomials Z_j by a membrane mirror with response functions of the form (5) are given in Table I. In this table N_{z_j} is the number of the approximated polynomial Z_j and N_{var} is the variant of the electrode arrangement. In the most general formulation of the problem of correcting an arbitrary wavefront $\varphi(r_s)$ the controlling forces exerted by the electrodes must also be calculated using the formula (11), in which Z_j must be replaced by tp. In so doing, it is implicitly assumed in the expression (11) that the phase of the wave φ at the points $\vec{r}_s$ can be measured directly (for example, with the help of a Zernike phase contrast sensor[1]). At the same time, to observe weak light sources (astronomical objects) sensors of local tilts, for example, a Hartman sensor, are widely employed.[1] In this case there arises the problem of the preliminary reconstruction of $\varphi(r_s)$. This can be avoided in the calculation of the control of membrane mirrors. It is well known[1,9] that algorithms for reconstructing the wavefront that minimize the errors in the measurement of the local tilts of the wavefront are, as a rule, a discrete analog of the following equation:

$$\Delta\varphi = \frac{\partial}{\partial x}X + \frac{\partial}{\partial y}Y = \frac{1}{r}\left[\frac{\partial}{\partial r}(r\varphi_r) + \frac{\partial}{\partial\vartheta}\varphi_\vartheta\right], \tag{12}$$

where X, Y, φ_r, and φ_θ are the measured (with noise) average values of $\frac{\partial\varphi}{\partial x}$, $\frac{\partial\varphi}{\partial y}$, $\frac{\partial\varphi}{\partial r}$, and $\frac{\partial\varphi}{\partial\theta}$ over the subapertures of the sensors.

Then, comparing Eq. (4) with Eq. (12), it is obvious that the controlling forces for electrodes placed inside the receiving aperture Ω can be determined from the formula

$$q = -T\left[\frac{\partial}{\partial x}X + \frac{\partial}{\partial y}Y\right], \tag{13}$$

while the forces for electrodes placed on the boundary of Ω can be calculated using the well-known procedure[9] of finding the best rms approximation of the tilts of the mirror to the tilts of the distorted wavefront. The expression (13) is especially simple in the case when the electrodes are placed at the nodes of a square grid (Fig. 2a). If the slope sensors are placed at the nodes of the covering grid, shifted by $0.5a_0$ along the x and y axes, the formula for calculating the actuator forces assumes the following form:

$$q_{ij} = -\frac{T}{2a_0}\left\{X_{i+\frac{1}{2},j+\frac{1}{2}} + X_{i+\frac{1}{2},j-\frac{1}{2}} - X_{i-\frac{1}{2},j-\frac{1}{2}} - X_{i-\frac{1}{2},j+\frac{1}{2}} + Y_{i+\frac{1}{2},j+\frac{1}{2}} + Y_{i-\frac{1}{2},j+\frac{1}{2}} - Y_{i-\frac{1}{2},j-\frac{1}{2}} - Y_{i+\frac{1}{2},j-\frac{1}{2}}\right\} \tag{14}$$

Here the indices (i, j) refer to the position of the node (x_j, y_j) of the grid.

In calculating the control system of a real mirror it is necessary to determine the effect of the clamping errors on the quality of the mirror surface. In accordance with Ref. 7 it is not difficult to show that for a clamping error of the form $W(r = R_0, \theta) = C \times \cos n\theta$ the rms error in the deviation of the mirror surface from a flat shape on the region Ω is $0.5C^2(D/2R_0)^{2n}/(n + 1)$.

A model of a controllable membrane mirror for systems of observation through the turbulent atmosphere was built based on the foregoing analysis. The mirror-smooth film, prepared by the method of vacuum deposition of layers of copper and aluminum and having a diameter of 110 mm, was stretched with uniform tension on a ring with an inner diameter of 100 mm. The mirror was grounded and placed between the input window, coated with a transparent conducting layer, and 37 electrostatic actuators. The electrostatic actuators consisted of contact pads 6.5 mm in diameter spaced by 7.8 mm on an insulated base. The actuator assembly was filled with epoxy resin, and the end surface of the actuators was ground and polished. The distance between the contact pads and the mirror film was equal to 50–100 mm. The errors in the approximation (3) of the lowest order Zernike polynomials computed for this mirror on the region of correction Ω 5 mm in diameter were as follows:

N_{z_j}	1	2.3	4	5	6	7.8	9.10
$\sqrt{J_j}$	0.019	0.052	0.13	0.078	0.012	0.25	0.17

The controlling forces were determined from the condition that the errors (3) be minimum, and the response functions of the mirror were calculated using formula (5). The deformations of the mirror so obtained were measured with the help of a modified shadow method on a Ten'-5 apparatus. Figure 3a shows the shadow pattern of the initial profile of the mirror; Fig. 3b shows the profile of the mirror (upper curve) and the profile of the tilt angle (bottom curve) in a section along the diameter of the mirror. The breaks at the edges of the curves correspond to the mirror frame. The distortion of a flat shape of the mirror near the frame is explained by the imperfection of the technology employed to fabricate the mirror with uniform tension. For a region of correction D=50 mm such distortions are of no consequence; in the worst case they can always be compensated with the help of the outer actuators. The profiles of the mirror surface and the tilt angle of the mirror in the case when a voltage of 200 V is applied to the central electrode are shown in Fig. 3c. The sensitivity of the deflection of the mirror to the voltage on the central electrode was quadratic and equaled 10 μm with a voltage of 200 V.

In conclusion we note that the obtained range of variations of the shape of the membrane mirror built indicates that such a mirror can be used in atmospheric adaptive systems. The theoretical analysis of the deformations of membrane mirrors performed above will make it possible to simplify the design of the control systems of such mirrors.

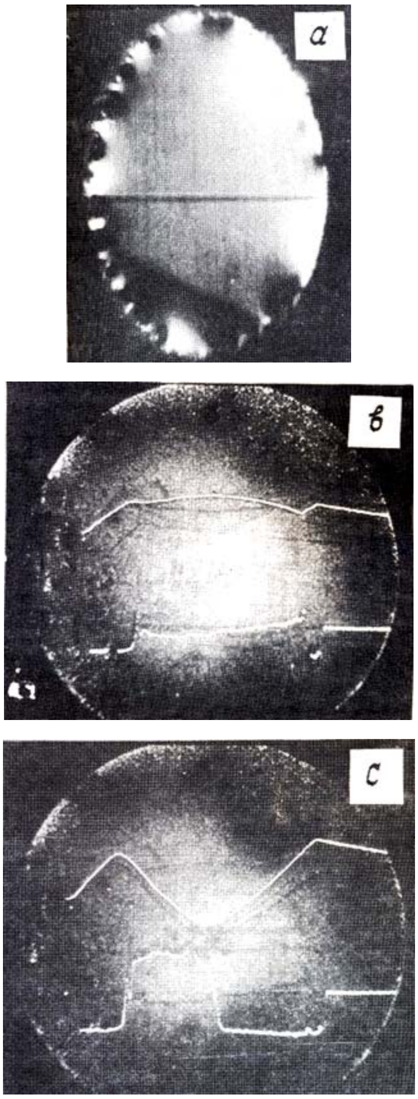

FIG. 3. The deformation of the surface of a membrane mirror: a) shadow picture of the starting surface of the mirror; b), c) profiles of the surface of the mirror (top) and tilt angle of the mirror (bottom) in the initial state (b) and with the voltage of 200 V on the central electrode (c).

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