A METHOD OF RESTORATION OF THE IMAGE FROM ITS CONVOLUTION WITH AN UNKNOWN PULSE RESPONSE

D.V. Makarov and A.D. Ryakhin

Scientific–Production Union "Astrofizika", Moscow
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A new method of restoration of the image from its convolution with an unknown pulse response based on solving the phase problem has been proposed. Its efficiency has been shown. Results of numerical simulation have been presented.

The problem of restoration of the images of an object is important in different fields of applied physics. As a rule, the recorded image $J(\mathbf{x})$ is a convolution of the true image $O(\mathbf{x})$ with the pulse response $H(\mathbf{x})$ of the imaging system

$$J(\mathbf{x}) = \int O(\mathbf{x}') \, H(\mathbf{x} - \mathbf{x}')\mathrm{d}\mathbf{x}' . \quad (1)$$

Thus, the problem of restoration of the image is equivalent to the problem of solving the convolution equation.

The widespread methods of restoration are based on using the *a priori* data on the image $O(\mathbf{x})$ or on the pulse response $H(\mathbf{x})$ or on an averaging over a great number of copies of the recorded images when the initial image $O(\mathbf{x})$ is the same and the pulse response $H(\mathbf{x})$ varies randomly.[1] Along with the foregoing, an investigation of the possible ways of restoration of the true image directly from the recorded one is of particular interest. A number of such restoration methods are well known.[2,3] However, they are very sensitive to noise and require an accurate assignment of the image dimensions and the pulse response.

The proposed method of restoration of the image is based on solving the phase problem, which essentially reduces to the reconstruction of the spatially bounded function based on the modules of its spatial spectrum.

Performing a two–dimensional Fourier transform of the convolution equation (1), we obtain

$$\tilde{J}(\mathbf{f}) = \tilde{O}(\mathbf{f}) \, \tilde{H}(\mathbf{f}) , \quad (2)$$

where $\tilde{J}(\mathbf{f})$ is the spatial spectrum of the recorded image, $\tilde{O}(\mathbf{f})$ is the spatial spectrum of the true image, and $\tilde{H}(\mathbf{f})$ is the optical transfer function.

It is well known[4] that the two–dimensional problem of reconstruction of the phase of the finite function spectrum based on its modules can be solved, as a rule, uniquely. At the same time, in the given particular case, when the spatial spectrum of the recorded image $\tilde{J}(\mathbf{f})$ is the product of the two functions $\tilde{O}(\mathbf{f})$ and $\tilde{H}(\mathbf{f})$, one can reconstruct four possible spectra $\tilde{J}(\mathbf{f})$, $\tilde{J}^*(\mathbf{f})$, $\tilde{O}^*(\mathbf{f})\,\tilde{H}(\mathbf{f})$, and $\tilde{O}(\mathbf{f})\,\tilde{H}^*(\mathbf{f})$ based on the modules $|\tilde{J}(\mathbf{f})|$ of the recorded spectra. In addition, the images, which correspond to the first and second spectra, and likewise to the third and fourth spectra, will differ only in rotation by an angle of 180°.

Thus, using the well–known iterative algorithms for reconstructing the finite function spectrum based on its modules, we can reconstruct the function $\tilde{O}^*(\mathbf{f})\tilde{H}(\mathbf{f})$ or its complex conjugate based on the modules $|\tilde{J}(\mathbf{f})|$. Without limitation of the generality, we shall assume that the function $\tilde{O}^*(\mathbf{f})\tilde{H}(\mathbf{f})$ has been obtained. A ratio of the spatial spectrum of the recorded image to the reconstructed function can be written as

$$\frac{\tilde{J}(\mathbf{f})}{\tilde{O}^*(\mathbf{f})\tilde{H}(\mathbf{f})} = \exp[\, i \; 2\arg\tilde{O}(\mathbf{f})] . \quad (3)$$

Taking the logarithm of Eq. (3) we separate out the doubled phase of the spatial of the spectrum true image, from which it is easy to obtain the tangent of the phase and to determine the phase by itself of the spatial spectrum of the image to an accuracy of π.

Restoration of an image based on the phase or the tangent of the phase of its spatial spectrum is a simpler problem than the solution of the phase problem. The appropriate iterative algorithms of reconstruction converge rapidly and are stable enough with respect to the noise.[5,6]

It should be noted that in the case in which the pulse response of the imaging system is a centrally symmetric function, the data on the phase of the spatial spectrum of the true image can be retrieved directly from the spatial spectrum of the recorded image.

We have performed a computer simulation of the proposed method of restoration of the image. In so doing, in order to retrieve the function $\tilde{O}^*(\mathbf{f})\,\tilde{H}(\mathbf{f})$, we used a well–known iterative algorithm for reconstruction of the spatial spectrum of the finite function based on its modules, which employs shaking up, i.e., relaxation.[7] To restore the true image based on the tangent of the phase of its spatial spectrum, we made use of an iterative algorithm which was proposed by Oppenheim and Lim.[6] In the course of simulation, we realized a pulse response corresponding to a multiple image motion.

The results of simulation have shown that in order to retrieve the function $\tilde{O}^*(\mathbf{f})\,\tilde{H}(\mathbf{f})$ with an error of 10% it is sufficient, as a rule, 50–100 iterations; in this case, the optimum relaxation coefficient lies in the range 0.4–0.6. Restoration of the image itself does require 30–40 iterations. In so doing, the relative rms error of the image is about 15%. Figure 1 shows the results of numerical simulation for illustration. Our investigations have shown that this method of restoration is insufficiently noise–proof. Already when the signal–to–noise ratio is equal to 15–20, it does not yield a fair estimate of the image.

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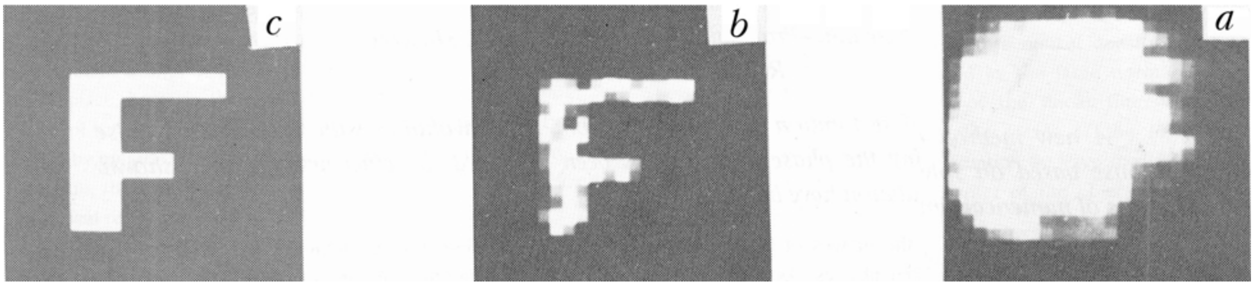

FIG. 1. The results of simulation: the distorted image (a), the image being restored after 30 iterations (b), and the true image (c).

Thus, a principal feasibility of the proposed method of restoration has been established. However, its applicability in practice is substantially hindered by unavoidable noise of the imaging system.

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