PRELIMINARY RESULTS OF THE EXPERIMENTAL STUDY OF A MODEL FOR THE SPECTRAL TENSOR OF THE WIND VELOCITY FIELD IN THE GROUND ATMOSPHERE

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Attempt is described to use a model approach to description of a spatial structure of the turbulence in the region of large spatial scales. The idea of the approach is in the determination of the spectral tensor based on measurement data or a model setting of one–dimensional fluctuation spectra of the three wind velocity components. The spectral tensor obtained in such a manner is used to predict spatial coherence of the velocity components fluctuating in the plane perpendicular to the mean wind direction. Comparison of the predicted and experimental coherences is presented. The applicability and limitations of this model of spectral tensor of the anisotropic velocity field for homogeneous turbulence in the near–ground atmospheric layer is analyzed based on the data obtained.

Mathematical and physical modeling of the boundary atmospheric layer is being rapidly developed in recent years. But, in spite of certain successes, no common universal model of the boundary layer of the atmosphere has been developed so far. At the same time there exist a large number of particular models[1] every of which satisfactorily reproduces some or other properties of the boundary layer.

The choice of a model is determined by the problem to be solved. In particular, there are important practical problems requiring good understanding of the spatial structure of the wind velocity field in the boundary layer of the atmosphere. For example, the calculations of wind loads give rise to the problem of correlations between wind pulsations at different levels of the boundary layer when the spectral coherence is used to estimate spatial variability of the wind pressure on constructions.

Numerous observations of the one–dimensional component of the velocity fluctuation spectrum in the boundary atmospheric layer described in the literature are not sufficient to describe spatial structure of the turbulence when the distances between measuring points are of the turbulence scale.

Kinematic turbulence model based on the representation of the spatial spectrum of wind velocity fluctuations in the form of anisotropic tensor $\Phi_{ij}(\boldsymbol{\kappa})$ was proposed in Ref. 2 for investigating turbulence in the region of large spatial scales.

The spectrum $\Phi_{ij}(\boldsymbol{\kappa})$ is a generalization of the homogeneous and isotropic spatial spectrum $\Phi(\kappa)$ to the case of a homogeneous but anisotropic velocity field in the boundary layer.

Let us consider a homogeneous incompressible turbulent velocity field. For such a field the ensemble–averaged wind velocity V_0 is constant in space, and the covariant tensor

$$B_{ij}(\mathbf{r}) = \langle [V_i(\mathbf{r}_1) - V_{0i}]\, [V_j(\mathbf{r}_2) - V_{0j}] \rangle, \quad i, j = 1, 2, 3 \quad (1)$$

is a function of the spacing vector $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ only.

Let us introduce a spatial coordinate system in such a way that the longitudinal direction (along the average wind) will be characterized by the unit vector $\mathbf{i}_1$, the vertical direction – by the unit vector $\mathbf{i}_3$, and the transvere direction – by the vector $\mathbf{i}_2 = \mathbf{i}_3 \times \mathbf{i}_1$, respectively.

The following expression for the spectral tensor, $\Phi_{ij}(\boldsymbol{\kappa}) = \dfrac{1}{(2\,\pi)^3} \int B_{ij}(\mathbf{r}) \exp(-\, i\, \boldsymbol{\kappa}\, \mathbf{r})\, \mathrm{d}^3 r$, has been proposed in Ref. 2:

$$\Phi_{ij}(\boldsymbol{\kappa}) = \sum_{l=1}^{3} A_l(\kappa) \left\{\delta_{li} - \frac{\mathrm{k}_l\, \mathrm{k}_i}{\mathrm{k}^2}\right\}\left\{\delta_{lj} - \frac{\mathrm{k}_l\, \mathrm{k}_j}{\mathrm{k}^2}\right\}, \quad (2)$$

where $A_1(\kappa)$, $A_2(\kappa)$, and $A_3(\kappa)$ are real independent scalar functions of κ. Assuming that all the three functions in Eq. (2) are similar $A_1(\kappa) = A_2(\kappa) = A_3(\kappa) = E(\kappa)/(4\pi\kappa^2)$ and summing over l we obtain the well–known expression[3] for the isotropic spectral tensor

$$\Phi_{ij}(\boldsymbol{\kappa}) = \frac{E(\mathrm{k})}{4\,\pi\,\mathrm{k}^2} \left\{\delta_{ij} - \frac{\mathrm{k}_i\, \mathrm{k}_j}{\mathrm{k}^2}\right\}.$$

In other words, isotropy is included into the model as a particular case.

Here we investigate the velocity field in a plane perpendicular to the direction of the average wind $V_0\, \mathbf{i}_1$, i.e., we define the shift as

$$\mathbf{R} = R \cos\theta\, \mathbf{i}_2 + R \sin\theta\, \mathbf{i}_3\,, \quad (3)$$

where θ is the angle between the direction of the shift and the horizon.

One of the most important applications of the spectral tensor is studying of the spatial structure of turbulence. A useful instrument for these purposes is the coherence function

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$$\Gamma^2_{ij}(\mathbf{R}, \omega) = \frac{|W_{ij}(\mathbf{R}, \omega)|^2}{[W_{ij}(0, \omega)\, W_{ji}(0, \omega)]}, \quad (4)$$

where $W_{ij}(\mathbf{R}, \omega) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} B_{ij}(\mathbf{R}, \tau)\exp(-i\omega\tau)d\tau$ is the cross spectrum.

Transforming the spatiotemporal correlation function $B_{ij}(\mathbf{R}, \tau)$ into purely spatial, in accordance with Taylor's freezing hypothesis, expanding it into a three–dimensional spatial spectrum, and integrating over τ we obtain for the cross spectrum the following formula

$$W_{ij}(\mathbf{R}, \omega) = \frac{1}{V_0}\int\limits_{-\$}^{\$} \Phi_{ij}\left(-\frac{w}{V_0}\mathbf{i}_1 + k_2\,\mathbf{i}_2 + k_3\,\mathbf{i}_3\right) \times$$

$$\times \exp\,[i\,R\,(\kappa_2 \cos\theta + \kappa_3 \sin\theta)]\, d\kappa_2\, d\kappa_3\,, \quad (5)$$

where $\Phi_{ij}(\mathbf{\kappa})$ is defined by the expression (2).

Let us consider, for definiteness, the coherence spectrum for longitudinal velocity components ($i = j = 1$) when the observation points are spaced in a plane perpendicular to the direction of the average wind.

By transforming Eq. (5) to the form in polar coordinates and integrating over the angle variable we obtain

$$W_{11}(\mathbf{R}, \omega) = \frac{2\,\pi}{V_0}\int\limits_0^{\infty} \frac{k^3}{\kappa^4}\, d\,\kappa \left\{A_1(\kappa)\,\kappa^2\,J_0(R\,\kappa) + \right.$$

$$+ A_2(\kappa)\left(\frac{\omega}{V_0}\right)^2\left[J_0(R\,k)\cos^2\theta - \frac{J_1(R\,k)}{R\,k}\cos\,(2\,\theta)\right] +$$

$$+ A_3(\kappa)\left(\frac{\omega}{V_0}\right)^2\left[J_0(R\,k)\sin^2\theta + \frac{J_1(R\,k)}{R\,k}\cos\,(2\,\theta)\right], \quad (6)$$

where $\kappa = \sqrt{k^2 + \omega^2/V_0^2}$, $J_n(x)$ are Bessel functions of the nth order.

Thus, now the problem is to find unknown functions $A(\kappa)$. Kristensen and Lenschow[2] have created a model of the spectral tensor $\Phi_{ij}(\mathbf{\kappa})$ in which the functions $A_1(\kappa)$, $A_2(\kappa)$, and $A_3(\kappa)$ are expressed in terms of one–dimensional spatial spectra of fluctuations of wind velocity components $F_{ii}(\kappa_1\mathbf{i}_1) = \int\limits_{-\infty}^{\infty}\Phi_{ii}(\mathbf{\kappa})d\kappa_2 d\kappa_3$. The idea of such an approach is to define the spectral tensor by measurement data or by a model setting of one–dimensional spatial fluctuation spectra $F_u(\kappa)$, $F_v(\kappa)$, and $F_w(\kappa)$ of the longitudinal (u), transverse (v), and vertical (w) wind velocity components. One–dimensional model spectra have the property that they are close to locally isotropic when the wave numbers are large. But, they can have different scales and curvature when wave numbers in the energetic portion of the spectrum are small.

Thus, following the model from Ref. 2 we can calculate A–functions defining the spectral tensor and, therefore, predict the coherence.

A series of experiments has been performed to verify this model. Simultaneous measurements of the longitudinal, transverse, and vertical components, u, v, and w, of the wind velocity were performed using three acoustic meteorological stations[4] spaced in both horizontal and vertical directions.

Auto– and crosscorrelation functions, autospectra, phase spectra, and coherence spectra were calculated for all three velocity components when processing measurement data. The data analysis is not yet completed. Nevertheless, one can compare coherence spectra of fluctuations of the longitudinal wind velocity component for transverse horizontal spacings (what corresponds to the angle $\theta = 0$).

In Fig. 1 dashed lines show the coherence spectra predicted for different spacings based on the model representation[2] and related to the neutral boundary layer (curves *1'–4'*), while solid lines show coherence functions found directly in the experiment (curves *1–4*).

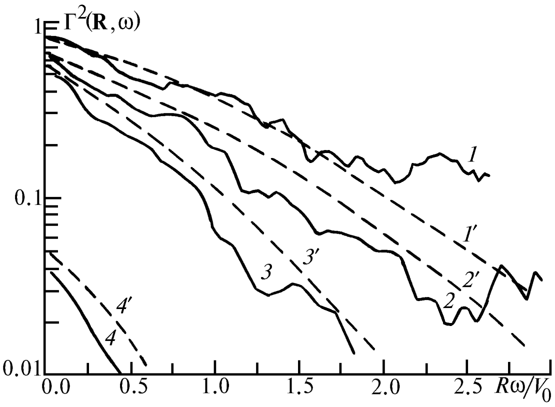

FIG. 1. Experimentally obtained (1–4) and calculated using a model (1'–4') coherence spectra for longitudinal wind velocity component for different spacings in the transvers horizontal direction: $R/L = 0.064$ *(1 and 1'),* 0.127 *(2 and 2'),* 0.509 *(3 and 3'), and* 1.018 *(4 and 4').*

In accordance with the data obtained the coherence for spacings which are small as compared with the outer turbulence scale L is well predictable. Moreover, the best coincidence is observed in the region of small wave numbers. For large distances the predicted coherence slightly exceeds the experimentally obtained one. This can probably be explained by the restrictions imposed in the model because of freezing hypothesis used when coming from frequency spectra to spatial ones and by the fact that the model description used is applicable to the homogeneous turbulence only.

Updating the model, taking into account "local freezing" and its extension to the case of inhomogeneous wind velocity field in the boundary layer of the atmosphere may be the next step in our study.

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