OPTICAL RECONSTRUCTION OF REFRACTION-CHANNEL PROFILES

V.V. Nosov

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
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An optical phase algorithm for reconstructing the characteristics of refraction channels is proposed. The algorithm is based on transverse optical sounding (transillumination) of the channel in different directions. Diffraction gratings are used in standard photodetectors.

Refraction channels (in air or other media) can be created by different methods, for example, photometrically.[1–3] The characteristics of such channels are analyzed by different methods,[1–4] in particular, longitudinal sounding of the channel with a narrow beam.

The dielectric constant (index of refraction) in the refraction channel is usually a real quantity (no absorption) and varies continuously across the channel. In the scheme of transverse optical sounding of the refraction channel the phase of the received wave depends on the characteristics of the channel. The phase increments in the optical wave can be described quite accurately by the method of geometric optics.[1,2] However square-law photodetectors (which are usually used in practice) do not respond to the phase of the wave. For this reason, more complicated receiving systems (for example, optical heterodyning systems) must be used to analyze the phase of the signal. It is obvious that in the process the methods used to process the received signal also become more complicated.[5]

In this paper we propose an optical phase algorithm for reconstructing the characteristics of refraction channels. These characteristics usually include the effective radii of the channel in different directions, the dielectric constant on the axis of the channel, and other parameters of the distribution of the dielectric constant across the channel. The algorithm is based on transverse optical sounding of the refraction channel, under the condition that the position of the axis of the channel is known with some accuracy. Since the channel characteristics can be constructed best by transilluminating the channel in different directions, the proposed method is a tomographic method.

Reconstructive tomography is now being actively developed for performing diagnostics of phase objects (for example, in fluid, gas, and plasma mechanics, etc.).[6–8] The mathematical basis for tomographic reconstruction is the Radon transformation, in which to functions defined in a volume there are associated integrals of the functions over hyperplanes. *A priori* constraints, connected with the symmetry properties of the object under study, are often introduced into the reconstruction problem. In the one-dimensional case this results in an Abel integral equation.

Until recently, any progress made in the methods of reconstruction of objects with high temporal and spatial resolution involved the improvement of instrumentation.[7] The phase algorithm proposed here requires simpler instrumentation than the traditional interferometric methods for reconstructing the properties of phase objects. The purpose of this paper is to illustrate the effectiveness of this method (the possibility of three-dimensional reconstruction) for the example of the tomography of refraction channels.

Let the refraction channel lie between the source and the detector and let it be perpendicular to the source-detector line. We assume that the source is a point source with wavelength λ. The source can move in a plane parallel to the input aperture of the detector.

The receiver consists of the standard receiving, telescope, in which the received flux is divided into three channels. Standard square-law photodetectors are placed in each receiving channel (in the focal plane of the telescope). Matrices (transparencies) with fixed intensity transmittances τ_n (n = 0, 1, and 2 is the channel number) are placed in front of the photodetectors:

$$\tau_0 = 1, \quad \tau_1(y) = (1 + \cos\xi y)/2,$$
$$\tau_2(y) = (1 + \sin\xi y)/2, \quad \xi > 0. \qquad (1)$$

Here y is one of the transverse coordinates in the focal plane of the detector (the y axis is perpendicular and the z axis is parallel to the channel axis; $y \perp z$). A simple model of the matrix τ_1 is a diffraction grating whose lines are separated by a distance $d = 2\pi/\zeta$, where ζ is the spatial frequency of the grating (in m^{-1}). The matrix is the same diffraction grating, but shifted by $d/4$ relative to τ_1 along the y axis.

Let an optical wave $u(x, \rho)$, $\rho = (y, z)$, which has traversed a distance x (across the refraction channel), be incident on the receiving aperture of a telescope of radius a_t. The intensity distribution in the image plane (at a distance F from the receiving lens) has the form[9,10]

$$I(F, \rho) = \left(\frac{k}{2\pi F}\right)^2 \int d^2R \int d^2t \times$$
$$\times \exp\left\{-\frac{R^2}{a_t^2} - \frac{t^2}{4a_t^2} + \frac{ik}{F}\left(1 - \frac{F}{F_0}\right)Rt - \frac{ik}{F}t\rho\right\} \times$$

$$\times \gamma(x, R, t);$$

$$\gamma(x, R, t) = u(x, R + t/2)u^*(x, R - t/2), \quad (2)$$

where F_0 is the focal length of the telescope and $k = 2\pi/\lambda$.

We denote the electric signals (photocurrent) at the output of the photodetector in the nth channel as E_n. Then

$$E_n = \int d^2\rho \tau_n(y) I(F, \rho), \quad \rho = (y, z).$$

Using in this relation the representation (2) we obtain

$$E_0 = \int d^2R \, e^{-R^2/a_t^2} \gamma(x, R, 0), \quad R = (R_1, R_2),$$

$$E_1 - E_0/2 = \frac{1}{4} \exp\left\{- [F\xi/(2ka_t)]^2\right\} \times$$

$$\times \int d^2R e^{-R^2/a_t^2}\left[\exp\left\{i(1 - F/F_0)R_1\xi\right\}\gamma\left(x, R, \frac{F\xi}{k}, 0\right) - \right.$$

$$\left. - \exp\left\{-i(1 - F/F_0)R_1\xi\right\}\gamma\left(x, R, -\frac{F\xi}{k}, 0\right)\right];$$

$$E_2 - E_0/2 = \frac{1}{4i} \exp\left\{- [F\xi/(2ka_t)]^2\right\} \times$$

$$\times \int d^2R \, e^{-R^2/a_t^2}\left[\exp\left\{i(1 - F/F_0)R_1\xi\right\}\gamma\left(x, R, \frac{F\xi}{k}, 0\right) - \right.$$

$$\left. - \exp\left\{-i(1 - F/F_0)R_1\xi\right\}\gamma\left(x, R, -\frac{F\xi}{k}, 0\right)\right]; \quad (3)$$

In the approximation of geometric optics the field of the spherical wave $u(x, \rho)$ is given as follows:[9,10]

$$u(x, \rho) = \frac{ku_0}{2\pi i x} \exp\left\{ikx - \frac{ik}{2x} (\rho - \rho_0)^2 + \right.$$

$$\left. + \frac{ik}{2} \int_0^x d\xi \varepsilon_1\left[\xi, \frac{\xi}{x} \rho + \left(1 - \frac{\xi}{x}\right) \rho_0\right]\right\}. \quad (4)$$

Here $\rho_0 = (\rho_{01}, \rho_{02})$ is the radius vector of the center of the source; $\varepsilon_1 = \varepsilon - 1$, where $\varepsilon(x, \rho)$ is the real dielectric constant of the medium; and, the constant u_0 is determined from the intensity $I_s(x)$ of the spherical wave at a distance x: $u_0^2 = I_s(x)(2\pi x/k)^2$, $I_s(x) = \gamma(x, R, 0)$. The function γ can be written, with the help of the expression (4), in the form

$$\gamma(x, R, \rho) = \left[\frac{ku_0}{2\pi x}\right]^2 \exp\left\{\frac{ik}{x} \rho(R - \rho_0) + \right.$$

$$\left. + \frac{ikx}{2} \left[J(R, \rho) - J(R, -\rho)\right]\right\},$$

$$J(R, \rho) = \int_0^1 ds \, \varepsilon_1\left[sx, sR + (1 - s)\rho_0 + \frac{s\rho}{2}\right]. \quad (5)$$

We shall assume further that within the field of view of the detector the characteristics of the channel remain virtually constant along the channel (along the z axis). This means that the dielectric constant $\varepsilon_1(x, y, z)$ in Eq. (5) does not depend on z. For this reason, the profile of ε_1 can be given by the following model function:

$$\varepsilon_1(x, y, z) = \varepsilon_{10}\exp\left[-\frac{(x - x_0)^2}{a_x^2} - \frac{(y - y_0)^2}{a_y^2}\right], \quad (6)$$

where ε_{10} = const is the dielectric constant on the axis of the channel; x_0 and y_0 are the coordinates of the channel axis; and, a_x and a_y are the radii of the channel along the x and y axes, respectively. [The origin of the coordinate system lies in the plane of the radiator on the optical axis of the receiver. For this reason, x_0 is the distance from the origin of the coordinate system up to the channel axis in the direction of propagation of the sounding wave, i.e., along the x axis; y_0 is the distance from the optical axis of the receiver to the channel axis.]

For the profile (6) the function $J(R, \rho)$ in Eq. (5) has the form

$$J(R, \rho) = \frac{\varepsilon_{10} \sqrt{\pi} \, a_x}{x \sqrt{1 + \eta^2 a_x^2/a_y^2}} \exp\left[-\frac{(m + \eta x_0/a_y)^2}{1 + \eta^2 a_x^2/a_y^2}\right],$$

$$\eta = \frac{R_1 + \rho_1/2 - \rho_{01}}{x}, \quad m = \frac{\rho_{01} - y_0}{a_y},$$

$$R = (R_1, R_2), \quad \rho = (\rho_1, \rho_2). \quad (7)$$

Here the limits of integration were replaced by infinite limits, i.e., the function ε_1 given by Eq. (6), which defines the channel, is concentrated on a finite section of the path ($2a_x \ll x$).

For the further analytical analysis we shall simplify the expression (7). For this we require that the conditions

$$a_t \ll x, \ \rho_{01} \ll x, \ F\zeta/k \ll 2x$$

be satisfied (the path length is greater than the transverse displacement of the source and radius of the detector). It then follows from the relations (3), (5), and (7) that in the region $R_1 \lesssim a_t$, which is significant for the integration in Eq. (3), $\eta \ll 1$. We shall also assume that in the x direction the source lies near the channel, for example, at a distance of several radii a_x from the channel axis. In this case, for bounded values of the ratio a_x/a_y (channels which are not strongly oblate along the y axis), from the condition $\eta \ll 1$ there follow the conditions $\eta a_x/a_y \ll 1$, $\eta x_0/a_y \ll 1$. Expanding' Eq. (7) for finite values of m in a Taylor series in η and retaining the first three terms in the expansion, we obtain

$$\frac{ikx}{2}\left[J(R,\ \rho) - J(R,\ -\rho)\right] = i\omega_1 R_1 - i\omega_0,$$

$$\omega_1 = \varepsilon_{10}\frac{a_x(a_x^2 + 2x_0^2)\rho_1}{a^2\ x}\sqrt{\pi}\ e^{-m^2}\left[m^2 - \frac{1}{2}\right], \quad (8)$$

Substituting Eq. (8) into Eq. (5) and Eq. (5) into Eq. (3), we find

$$\frac{E_1 - E_0/2}{E_0/2} = \mu\cos\Omega_0,\quad \frac{E_2 - E_0/2}{E_0/2} = -\mu\sin\Omega_0;$$

$$\mu = \exp\left\{-\left[\frac{F\xi}{2ka_t}\right]^2 - \left[\frac{\xi a_t}{2}\left[1 - \frac{F}{F_0} + \frac{F}{x}\right] - \frac{\Omega_1 a_t}{2}\right]^2\right\};$$

$$\Omega_0 = \frac{F\xi a_x}{x}\left\{\frac{a_y}{a_x}\left[m + \frac{y_0}{a_y}\right] + \varepsilon_{10}\frac{x_0}{a_y}\sqrt{\pi}\ e^{-m^2}\times\right.$$

$$\left.\times\left[m + \frac{(a_x^2 + 2x_0^2)}{xx_0}\left[m^2 - \frac{1}{2}\right]\left[m + \frac{y_0}{a_y}\right]\right]\right\};$$

$$\Omega_1 = \omega_1\Big|_{\rho_1 = \frac{F\xi}{x}}. \quad (9)$$

The expression (9) is the basis for finding the unknown parameters ε_{01}, a_x, a_y of the dielectric constant distribution in the channel. In a chosen (fixed) region where $\tan\Omega 0_Q$ is a monotonic function it follows from Eq. (9) that

$$\Omega_0 = -\arctan\frac{E_2 - E_0/2}{E_1 - E_0/2}. \quad (9a)$$

Choosing the position of the source so that $m = 0$ ($\rho_{01} = y_0$) we obtain, for example,

$$\varepsilon_{10} = \frac{\sqrt{\pi}a_x}{2}\frac{(a_x^2 + 2x_0^2)F\xi y_0}{a_y^2\ x^2} = \frac{F\xi y_0}{x} + \arctan\frac{E_2 - E_0/2}{E_1 - E_0/2} \equiv A_0 \quad (10)$$

Changing the path length ($x \to x_1$) and (or) the longitudinal position of the source ($x_0 \to x_{01}$), we obtain the second equation

$$\frac{\varepsilon_{10}\sqrt{\pi}}{2}\cdot\frac{a_x(a_x^2 + 2x_{01}^2)F\xi y_0}{a_y^2 x_1^2} \equiv A_1. \quad (11)$$

From these equations, dividing one by the other, we find the longitudinal (along the x axis) radius of the channel

$$a_x = \sqrt{\frac{2(x_0^2 - \nu x_{01}^2)}{\nu - 1}},\quad \nu = \frac{A_0 x^2}{A_1 x_1^2}.$$

We shall now transilluminate the channel in a direction perpendicular to the preceding direction. For the new direction the radii a_x and a_y are interchanged. For this reason, applying the preceding procedure (Eqs. (10) and (11)), we find the radius a_y. Using the known values of a_x and a_y we find from Eqs. (10) or (11) the value of the dielectric constant on the channel axis ε_{10}.

Thus if the position of the channel axis known, then a_x, a_y, ε_{10} can be found from Eq. (9). The values found for a_x, a_y, ε_{10} can be used as the starting information for a more detailed description of the profile $\varepsilon_1(x,\ y,\ z)$ in the channel. For example, expressing the profile ε_1 as a finite sum of functions of the form Eq. (6)

$$\varepsilon_1(x,y,z) = \sum_{n=-N}^{N}\varepsilon_{10}^{(n)}\exp\left[-\frac{(x - x_0 - n\Delta_x)^2}{a_{xn}^2} - \frac{(y - y_0 - n\Delta_y)^2}{a_{yn}^2}\right]$$

where $\varepsilon_{10}^{(n)}$, a_{xn}, a_{yn} are unknown parameters, N, Δ_x, Δ_y are fixed parameters, $\Delta_x \ll a_x$; $\Delta_y \ll a_y$; and x_0, y_0 are the coordinates of the channel axis, we obtain from Eq. (9) or (9a) the system of equations for finding $\varepsilon_{10}^{(n)}$, a_{xn}, a_{yn} for $-N \le n \le N$. This system of equations is obtained either by moving the source (along the x and y axes) or by moving the detector (also along the x and y axes) or by changing the direction of transillumination of the channel.

When the position of the channel axis is unknown, ε_1 given by the formula (6) will be a function of z. Expressing the profile ε_1 by an appropriate expression with a collection of unknown parameters, a system of equations for these unknown parameters can also be derived from Eq. (9) or (9a).

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