Influence of mesoscale vortices on vertical transport of impurities in the atmosphere

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A semiempirical model of a mesoscale atmospheric vortex with a vertical axis of rotation is proposed. The mass of liquid or solid particles entered the vortex from a water surface or sand-covered land is estimated.

Introduction

Mesoscale atmospheric vortices with a nearly vertical rotation axis often arise in the atmosphere. Depending on the size and intensity, these vortices are referred to differently: dust vortex, waterspout, sandstorm, and tornado. The diameter of the smallest dust vortex is several meters, and the largest tornados are about 1 km in diameter. These vortices are characterized by the presence of intense rotational motions and vertical streams: from 1 to 20 m/s in dust vortices and from 20 to 100 m/s in spouts and tornados.

Intense motions near the underlying surface favor saltation processes above sand-covered or dusty surfaces and liquid fraction exchange between the atmosphere and water bodies. Intense upward streams cause vertical transport of liquid and solid impurities, including very large ones, to high altitudes. The vertical dimension of mesoscale vortices, as a rule, far exceeds their horizontal dimension.

Impurity particles lifted to high altitudes are transported to long distances. So they can affect various processes even in remote regions. Thus, salt particles formed from evaporated sea drops, as well as sand and dust particles serve as the nuclei of vapor condensation and raindrop coagulation.

As a spout or tornado passes over industrial and chemical enterprises or atomic power stations, solid, liquid, and gaseous toxicants may enter the atmosphere and present a severe hazard to human health at large territories. Nevertheless, processes of mass exchange between the surface and the atmosphere in the case of mesoscale vortices are poorly studied. This paper is an attempt to fill, although partially, this gap. In this paper, the simplified model[1,5] of a mesoscale vortex with the vertical axis is used for theoretical study of the mass exchange between the atmosphere and a water body or sand-covered land.

Statement of the problem and its solution

Papers developing the theory of mesoscale vortices with the vertical axis are reviewed in Refs. 2 and 3. It was shown by the numerical model proposed in Ref. 4 that mesoscale atmospheric vortices are convective cells with significantly changed spatiotemporal structure. These changes occur under the effect of air twisting around the cell axis because of the angular momentum in the atmosphere. This numerical model is rather complicated, and it does not allow unambiguous treatment of the results. Therefore, two simplified analytical models were proposed in Ref. 5: linear one for explaining time changes in the vortex structure and nonlinear one for explaining the spatial structure of the vortex.

By comparing the simplified nonlinear model with the numerical model,[1,5] it was shown that the simplified model rather accurately reconstructs the structure of the mesoscale vortex at the stage of its maximum development. The initial set of equations of the simplified model was derived from the Boussinesq equations at neglect of time dependence, viscosity, and Archimedes force. It has the following form[1,5]:

$$u\,\frac{\partial w}{\partial r} + w\,\frac{\partial w}{\partial z} = -R\,\theta_0\,\frac{\partial}{\partial z}\left(\frac{p'}{p}\right); \tag{1}$$

$$u\,\frac{\partial v}{\partial r} + w\,\frac{\partial v}{\partial z} + \frac{uv}{r} = 0; \tag{2}$$

$$\frac{v^2}{r} = R\,\theta\,\frac{\partial}{\partial r}\left(\frac{p'}{p}\right); \tag{3}$$

$$\frac{\partial ur}{\partial r} + \frac{\partial wr}{\partial z} = 0, \tag{4}$$

where u and w are the radial and vertical velocity components in the cylindrical coordinates r and z; v is

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the rotational velocity component; p' is the deviation of the pressure p from its value typical for the unperturbed atmosphere; R is the universal gas constant; θ is the mean temperature of the convective layer.

The solution to the set (1)–(4) was obtained in Refs. 1 and 5. It has the following form:

$$w = (1 - (r/r_0)^2) \exp(-(r/r_0)^2) F;$$

$$v = \sqrt{2}\,(r/r_0) \exp[-(r/r_0)^2] F; \quad (5)$$

$$u = -(r/2) \exp[-(r/r_0)^2] \frac{\partial F}{\partial z};$$

$$p' = -\frac{p}{2R\theta_0} \exp[-2(r/r_0)^2] F^2. \quad (6)$$

Hereinafter we assume that the arbitrary function $F(z) > 0$. In this case, air moves upward in the tornado core ($w > 0$ at $r < r_0$) and downward at its periphery ($w < 0$ at $r > r_0$). Note that just the vortex core must be visible because of opaque particles and things lifted from the surface by intense air streams. The vertical component is maximum along the vortex axis $w = w_{max} = F(z)$ at $r = 0$. The mean vertical component in the vortex core is

$$\bar{w} = \frac{2\pi}{\pi r_0^2} \int\limits_0^{r_0} wr\,\mathrm{d}r\mathrm{d}z = F\mathrm{e}^{-1}. \quad (7)$$

It is easy to calculate that at the same altitude the maximum speed of upward streams is much higher than that of downward motions. Nevertheless, downward motions are the compensating ones, since $2\pi/(\pi r_0^2) \int\limits_0^{\infty} wr\mathrm{d}r\mathrm{d}z = 0$. The rotational component of the velocity is maximum in the vortex core: $v = v_{max} = \exp(-0.5)F$ at $r = r_0/\sqrt{2}$. At the vortex periphery, v decreases quickly with increasing r. The vertical structure of the vortex is determined by the function F. Since air particles do not penetrate the surface, $F = 0$ at $z = 0$. Then F increases ($\mathrm{d}F/\mathrm{d}z > 0$) up to the altitude about several meters. Air in this area moves toward the vortex axis.

According to observations at a 1-km long path, the visible part of the vortex only slightly depends on the altitude. We believe that in this area $F \approx$ const and $u \approx 0$. At high altitudes, the vortex begins to weaken nearby a cloud generating it ($\mathrm{d}F/\mathrm{d}z < 0$). In this part of the vortex, $u > 0$ and air particles moving from the center come to the vortex periphery in the area of downward motions. Reaching the near-surface part of the vortex, the air particles again enter the vortex core. Thus, air particles in this model do not leave some closed local volume, which, on average, does not exceed several cubic kilometers. Actually, observations show that air can be at relative rest in the vicinity of a tornado.

Theory and approximate estimates for an actual tornado[3] show that the vertical and rotational velocity components are close to each other. The analytical model gives the results very close to actual findings at the stage of relative stabilization of a vortex that lasts likely no more than one hour.

Influence of vortices on vertical transport of impurities

Let us complement Eqs. (1)–(4) with the following simplified equation for impurity transport:

$$u\frac{\partial s}{\partial r} + (w - w_0)\frac{\partial s}{\partial z} = 0, \quad (8)$$

where s is the volume concentration of some impurity; w_0 is the sedimentation rate of particles.

The solution of Eq. (7) has the following form:

$$s = ar(v - \psi) \text{ at } v \geq \psi,\ s = 0 \text{ at } v < \psi, \quad (9)$$

where v is determined from Eq. (5); $\psi = \sqrt{2} r w_0/r_0$; a is a constant determined from the conditions for s at the lower boundary.

Let us set these conditions. To do this, we consider the scenario in which impurities look like small droplets coming to air from rough sea due to passage of a vortex. The salt dissolved in the seawater also comes into the atmosphere in this case. Thus formed aerosol plays an important role in formation of precipitations. Following Ref. 1, we assume that for seawater droplets at $z = z_0$

$$\log s = -94.6V^2/(V^3 + 172) + 6, \quad (10)$$

where V is the wind speed near the water surface; s is the specific content of liquid droplets (liquid water content), in g/m^3.

Let us now turn our attention to empirical data. According to Ref. 1, the main characteristic of tornado danger is the class of its intensity K. In this case, the mean speed of rotational motion v_k, the mean rotational speed at the surface level v_0, vortex width L_k, and vortex height H_k are determined from the following equations:

$$v_k = 6.3(K + 2.5)^{1.5} \text{ m/s}; \quad (11)$$

$$v_0 = 1.6(K + 2.5)^{1.5} \text{ m/s}; \quad (12)$$

$$L_k = 1.609 \cdot 10^{0.5(K + 1.5)} \text{ m}; \quad (13)$$

$$H_k = 6.6 \cdot 10^{0.5(K+1.5)} \text{ m},\ 0 \leq K \leq 5. \quad (14)$$

Let us now attempt to join the empirical equations (11)–(14) with the theoretical solution (5)–(9). It is obvious that

$$r_0 = \frac{L_k}{2},\ v_k = \bar{v} = \frac{2\pi}{\pi r_0^2} \int\limits_0^{r_0} vr\mathrm{d}r\mathrm{d}z \approx 0.79 v_{max},$$

$$v_0 = \approx 0.79 v^0_{max}; \quad (15)$$

at $z = z_0$

$$\log s_0 = -94.6 v_0^2 / (v_0^3 + 172) + 6 \quad (16)$$

and

$$v_{\max}^0 = v_{\max},\ s_0 = \frac{2\pi}{\pi r_0^2} \int_0^{r_0} s r \mathrm{d}r \approx 0.61 a r_0 v_0. \quad (17)$$

Thus, we have found the relation between the liquid water content and vortex intensity near the surface. Taking into account that in vortices $v \gg \psi$ and neglecting the small term in Eq. (9), for the total amount of the solution M_s contained in the vortex core we have the following approximation:

$$M_\mathrm{s} = \pi r_0^2 H_k s_0. \quad (18)$$

In derivation of Eq. (18), it was assumed that the impurities with the mean concentration s_0 fill the vortex core fast enough under the effect of intense upward streams. At the vortex periphery, under the effect of less intense compensating downward streams and the force of gravity, the impurities slowly deposit on the surface. Note that the volume of the periphery part of the vortex exceeds the volume of its core, and the vertical speeds at the periphery are much lower. Therefore, the mass of the impurities contained in the periphery part of the vortex (as it exists for sufficiently long time) can exceed the mass of the impurities contained in the core. Unfortunately, the empirical model does not give values for the vertical velocity component. The theoretical model gives the relation between the intensity of vertical and rotational velocity components in the vortex core:

$$\bar{w} = \mathrm{e}^{0.5} / 0.79 \bar{v} = 0.95 v_k. \quad (19)$$

Now let us consider the case that the source of an impurity is a sand-covered surface. In this case, we distinguish the diffusion and saltation processes assuming that at $w \gg w_0$ all particles having vertical momentum are captured by the vortex and then transported in the form of suspension. The near-surface concentration is determined by solving the inverse problem on motion of particles with simultaneous calculation of dynamic characteristics of the surface layer. A limited volume of this paper does not allow us to present all the used equations; they can be found in Ref. 6, which also gives the statement of the problem and detailed analysis of the solution.

Table. Vortex parameters and mass of impurity in vortex core

K	0	1	2	3	4	5
$\bar{v}$, m/s	25	38	56	82	104	128
$\bar{w}$, m/s	24	36	54	78	99	122
v_0, m/s	6.1	9.1	14	20.5	26	32
L, m	9	29	90	290	900	2900
H, m	35	110	350	1100	3500	11000
M_s, t	$2{\cdot}10^{-6}$	$7{\cdot}10^{-5}$	2	660	$2{\cdot}10^{5}$	$6.7{\cdot}10^{7}$
M_sand, t	0.1	4.3	181	8282	$3.1{\cdot}10^{5}$	$1.3{\cdot}10^{7}$

The main parameters for vortices of different classes of intensity are given in the Table. This Table also presents the mass of solution in the vortex core as calculated by Eq. (18) and the mass M_sand of sand with the mean diameter of 125 μm in the vortex core.

Conclusion

It is seen from the Table that vortices of the first and second classes of intensity practically do not lift water drops, because waves do not collapse at low speed. However, intense vortices of the fourth and, especially, fifth classes lift a great amount of the solution. Thus, according to the Table, the intense vortex can lift water in the amount equal to a small lake, what is actually confirmed by observations.

Passage of such vortices over cooling pools of atomic power stations[1] and settlers of industrial and chemical enterprises is of particular danger. Apparently, as intense vortices pass over seas and oceans, a great amount of salt comes into the atmosphere. After evaporation of sea drops, this salt can remain in the atmosphere for a long time as fine aerosol. However, to estimate this mechanism of delivery of salt particles to the atmosphere, we need statistical data on mesoscale vortices over seas and oceans. As we know, such data are not available now. Some information can be apparently obtained from the processing of space photographs.

As for the uplift of dust particles and sand by mesoscale vortices, it occurs, according to the Table, even at relatively weak vortices. Actually, in deserts, semi-deserts, and dry steppes, dust storms of low and moderate intensity often take place. They can be seen from the distance of many kilometers because of the impurities they contain. However, intense vortices like tornado are always accompanied by intense cyclonic activity causing heavy showers, thunderstorms, hail, which abundantly water the land. These conditions hamper the uplift of fine particles from the surface, and this case is not described by our model.

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