ON THE ACCURACY OF AN ALGORITHM FOR EVALUATING THE STATE OF A WAVEFRONT

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The potential accuracies of an algorithm for evaluating phase distortions of the wavefront as a function of the conditions of observation, the data acquisition rate, and the properties of the turbulent atmosphere are analyzed.

We shall analyze the accuracy of the algorithm presented in Ref. 1 for evaluating the state of a wavefront for an adaptive optical system, in which separate corrections for the general tilt of the wavefront, defocusing, and local high-frequency distortions are made.

If the dynamics of wavefront distortions are adequately described in the model employed for synthesizing the estimation algorithm and in particular, if the statistical characteristics of the aberrations — the general tilts of the wavefront b_1 and b_2, defocusing b_3, and residual local tilts ψ_L and θ_L — are mutually independent, the accuracy with which the quantities b_1, b_2, and b_3 and the vectors ψ_L and θ_L are estimated is determined by the values of the diagonal matrix elements $D_z(kT)$ of the covariations of the estimation errors of the corresponding components of the vector $\bar{z}\,(kT) = (b_1;\ \bar{\psi}_L^T;\ b_2;\ \theta_L^T;\ b_3)$ of the state of the wavefront, calculated directly from the second and third recurrence relations of the algorithm (10) presented in Ref. 1.

For constant values of the variances $d_{fi}(kT) = d_{fi}$ ($i = 1, \ldots, N^2$) of the errors in the measurements of the local tilts of the wavefront in a Hartmann sensor, a comparatively short time after the algorithm for estimating the state of the wavefront starts up the covariation matrix of the estimation errors $D_z(kT)$ approaches some constant matrix D_z, and the values of the elements of the matrix D_z do not depend on the initial conditions. This indicates that the steady-state regime of the algorithm for estimating the state of the wavefront is stable.

To determine the effect of the statistical characteristics of the aberrations and the quality of the operation of the wavefront sensor on the accuracy of the estimation algorithm in the steady-state regime the values of the diagonal elements of the matrix D_z were found. To simplify the calculations, it was assumed that the quality of the measurements performed by the wavefront sensor is the some over the entire aperture, i.e., $d_{fi} = d_f$. As a result it was found that the steady-state variances of the estimation errors d and the extrapolation errors d_e for the general tilts b_1 and b_2 are determined by the relations

$$d = \frac{d_e d_\xi}{d_e + d_\xi}, \qquad d_e = \beta^2 d + d_\eta\ , \tag{1}$$

where $d_\xi = \dfrac{d_f}{N^2 k_D^2}$ is the variance of the errors in measuring the systematic component of the wavefront tilt based on the combined information from the outputs of all N^2 sensors; K_s is the slope of the characteristic of the wavefront sensor relative to the displacement of the interferogram; the coefficient $\beta = \exp(-T/\tau_0)$ depends on the data acquisition period T and the correlation time constant τ_0 of the wavefront tilts; d_η is the variance of the noise in the model of disturbances of the general wavefront tilt ($d_\eta = d_b(1 - \beta^2)$, d_b is the variance of the fluctuations of the general wavefront tilt).

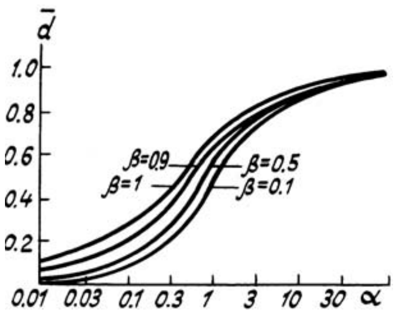

Fig. 1

Figure 1 shows a family of curves of the normalized variances of the estimation errors $\bar{d} = d/d_\xi$ as a function of the quantity $a = d_\xi/d_\eta$ for a series of values of the parameter β.

One can see that the algorithm forms an estimate of the general wavefront tilt for which the obtained variance of the error is more than two times smaller than the variance obtained based on information directly from the wavefront sensor, if $a \le 1$. It should be noted that for $T \ll \tau_0$ and constant variance of fluctuations of the general wavefront tilt d_b = const the value of d_η can be very small. This condition makes it possible to choose a sampling period Γ so that the estimation algorithm produces an estimate and an extrapolated value of the general waveferont tilt with the required accuracy (with known accuracy of single measurements of d_f).

The accuracy of the algorithm for estimating the coefficient b_3, characterizing the defocusing, is determined by analogous relations, if in them

$$d_\eta = d_{\eta 3}, \quad d_\xi = \frac{d_f}{k_D^2}\left[\sum_{i=1}^{N^2}\left[16u_i^2 + 16v_i^2\right]^{-1}\right]^{-1}, \tag{2}$$

where $d_{\eta 3} = d_{b3}(1 - \beta_3^2)$ is the variance of the noise in the model of disturbances of defocusing; u_i and v_i are the normalized coordinates of the i-th elementary cell relative to the center of the receiving aperture along two mutually orthogonal axes.

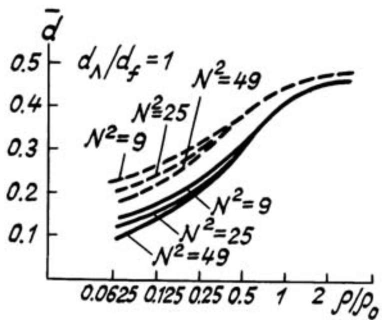

Fig. 2

The accuracy of the estimation of the residual local wavefront tilts $\vec{\psi}_L = \{\psi_{Li}\}$, $\vec{\theta}_L = \{\theta_{Li}\}$ in the steady-state regime depends not only on the intensity of fluctuations of the atmosphereand the quality of operation of the wavefront sensor, but also on the ratio of the distance ρ between neighboring cells of the sensor and the spatial correlation radius ρ_0 as well as on the total number of sensors N^2 and their location. The characteristic dependences of the relative variances $\bar{d} = d / d_f \cdot k_d^{-2}$ of the errors in estimating the residual local wavefront slopes at the center (solid lines) and at the edges (dashed lines) of the region of correction as a function of the ratio ρ/ρ_0 are shown in Fig. 2.

It should be noted that as ρ/ρ_0 0, i.e., when the fluctuations of the phase are completely correlated in the plane of the corrector, we are essentially evaluating the general wavefront tilt. Conversely, when $\rho/\rho_0 \gg 1$ and there is no correlation, the estimation of the local distortions of the wavefront becomes independent for each elementary section of the corrector (photosensor). In the last case the quality of estimation of the residual local tilts can be determined using Eq. (1), if it is assumed that $d_\xi = d_{fi} / k_D^2$, $d_\eta = d_{Li}(1 - \beta_{\alpha ii}^2)$, ($d_{Li}$ is the variance of the fluctuations of the local distortions). In this case, naturally, the estimation accuracy does not depend on the number of sensors.

We can draw the following conclusion based on the foregoing analysis of the accuracy of the algorithm for estimating the state of a wavefront. The algorithm permits estimation of the state of the wavefront with an error whose variance is less than that of the error obtained based on information directly from the wavefront sensor; this, in its turn, can significantly improve the accuracy of correction for wavefront distortions.

REFERENCE

1. S.V. Butsev and V.Sh. Khismatulin, Opt. Atmos., **2**, No. 2, 176 (1989).