Identification of spiral dislocation of wave front and compensation for its influence on formation of optical structures in ring interferometer

I.V. Izmailov, A.L. Magazinnikov, and B.N. Poizner

Tomsk State University

Received March 24, 2000

A modification of the Akhmanov–Vorontsov model describing formation of optical structures in a nonlinear ring interferometer (NRI) is proposed. The case of optical field turning through some angle in the laser beam cross section (in NRI feedback) is studied. It is found that both the order of spiral dislocation of the wave front at the entrance to NRI and phase delay of the field in the NRI feedback have an identical effect on formation of optical structures. Theoretical analysis and computer experiments show new applications of the NRI: for identifying the order of spiral dislocation of an optical vortex (by the shape of optical structures in the laser beam cross section in the NRI), for compensating for the influence of an optical vortex on the process and result of structure formation in the NRI, and as a basis of an arithmetic and logic unit executing the operation of addition.

Introduction

In recent years, light fields with spiral dislocations of the wave front attracted attention of researches in atmospheric optics and laser physics. Perturbations of such a kind cause the spiral character of propagation of the radiant energy. It is believed that optical vortices arise due to isolated points with zero intensity in the laser beam cross section, for example, because of random inhomogeneities in the optical parameters of a propagation channel (see Refs. 1–4 and references therein). The methods of correcting such perturbations are developed in adaptive optics.

The traditionally considered types of distortions of equiphase surfaces do not change topology of phase fronts. However, appearance of optical vortices in a light beam is necessarily accompanied by appearance of singular points, at which the phase is uncertain. By analogy with two-dimensional lattice defects, these points are called spiral dislocations (SDs). Appearance of SDs changes cardinally the topology of phase fronts. Upon passing around a SD, the phase changes by $2\pi V_d$ (V_d is an integer non-negative number equal to the SD order). Depending on the sign of the phase change, left and right SDs are distinguished. Let the origin of the coordinate system coincide with a singular point. Then the complex amplitude of the field in the vicinity of the singular point is

$$A = Cr^{V_d} \exp (\pm j\ V_d\ az), \qquad (1)$$

where C is an arbitrary constant; $r \equiv |\mathbf{r}|$ is the distance from the SD; $\mathbf{r} \equiv (x, y)$ is the radius vector of a point in the cross section; j is imaginary unit; az is the azimuth angle. The current methods for theoretical determination of the conditions for appearance of SD and its position use mostly the apparatus of wave optics. Progress in this field is considered in Ref. 2.

Formation of SDs on the wave front of laser beams is a purely phase effect. Therefore, the only mean providing reliable identification of SDs is the method based on the use of interferometric information. Interferograms of the beam cross section can be obtained in different ways.

In the opinion of Korolenko,[1] the most convenient way is the method based on recording of the interference structure of the studied field with a plane or spherical homogeneous wave acting as a reference one. This allows the SD order and number to be determined from the structure of the interference pattern. This method can be implemented using, for example, the Michelson interferometer.[3] Nevertheless, the necessity of formation of a reference wave being coherent with the wave having passed through the turbulent atmosphere or other inhomogeneous medium poses well-known difficulties. Besides, the reliability of SD determination and identification of SD order assumes the availability of technical means for analysis of the structure of an interference pattern.

Therefore, it is worth considering existence of an alternative method of SD detection. It is desirable for this method, first, to open up the possibility of simpler detection of SD and identification of its order and, second, to provide interference of the studied light wave with the same wave subjected to spatial transformations with a linear element. Third, this method should provide the possibility of application of nonlinear-optics effects. Fourth, for further extension of the problem, it is worth considering the capability of the new device of compensating for the influence of spiral dislocations. One of such ways is, in our opinion, analysis of the type of dynamics in a nonlinear ring interferometer (NRI) used in atmospheric adaptive optics.[5–7]

In this paper, we consider the mathematical model of formation of an optical structure in the cross section

0235-6880/00/09 747-07 $02.00

of a laser beam in the NRI for the case that radiation with spiral dislocations of the wave front comes at the NRI entrance. The results of mathematical simulation are discussed as well.

1. Model of processes in nonlinear ring interferometer

Let us consider an optical system shown in Fig. 1. Here NM is nonlinear medium of length l, G is a linear element responsible for large-scale transformation of the field in the feedback unit; it deflects a ray from the point $\mathbf{r}' \equiv (x', y')$ of the beam cross section to the point $\mathbf{r} \equiv (x, y)$; the reflectance of the mirrors M_1 and M_2 is R, and that of M_3 and M_4 is 100%.

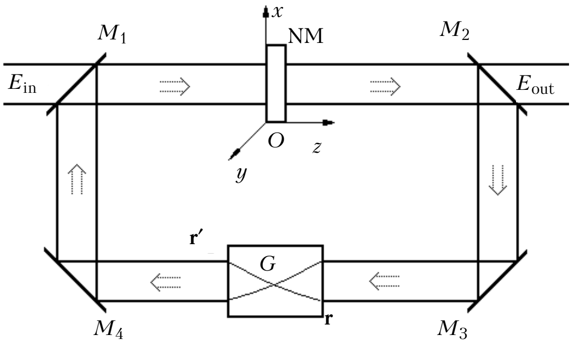


Fig. 1. Optical layout of nonlinear ring interferometer.

The mathematical model of the dynamics of transverse distribution of the nonlinear phase incursion U in the case of a ring interferometer with a thin Kerr medium (i.e., a medium with square dependence of its refractive index on the electric field strength of radiation) is considered in detail in Ref. 8 in the approximation of high loss (or in the approximation of a single pass) and dispersion-less diffraction-less propagation of the field. Reference 8 considers a rather general case. The light beam at the NRI entrance is characterized by the angular rate of rotation of the polarization plane Ω, circular optical frequency ω, and slowly varying (for the time $2\pi/\omega$) parameters: position of the polarization plane ψ, phase φ, and amplitude A.

The Kerr medium under the exposure to the light electric field polarizes and becomes anisotropic acquiring the properties of a uniaxial crystal with an optical axis parallel to the vector of electric field. According to the definition, the main optical plane passes through the crystal optical axis and the light wave vector (Ref. 9, p. 542). Hereinafter we assume that $\Omega = 0$ and $\psi = \text{const}$. Then, since the main optical axis is parallel to the light field strength vector, the latter lies in the main optical plane and, by definition, corresponds to an extraordinary ray. An ordinary ray is not formed at such geometry.

Taking into account the above-said, the mathematical model proposed in Ref. 8 reduces to the form

$$A^2_{\text{in NM}}(\mathbf{r}, t) = (1 - R)\,\{A^2(\mathbf{r}, t - \tau) + \gamma\eta(\mathbf{r}', t - \tau)\, A(\mathbf{r}', t - \tau)\, A(\mathbf{r}, t) \times \cos[\omega\tau + \varphi(\mathbf{r}, t) - \varphi(\mathbf{r}', t - \tau)] + pA^2(\mathbf{r}', t - \tau)\};$$

$$\tau_n \mathrm{d}U(\mathbf{r}, t)/\mathrm{d}t = n_2\, lk\, A^2_{\text{in NM}}(\mathbf{r}, t) - U(\mathbf{r}, t) + D_e\, \Delta_{xy}\, U(\mathbf{r}, t), \qquad (2)$$

where $A_{\text{in NM}}$ is the amplitude of the light field at the entrance to the nonlinear medium; $\tau = \tau(\mathbf{r}', t) = t_e(\mathbf{r}') + U(\mathbf{r}', t)/\omega$;

$$p = \begin{cases} 0, & \text{in the approximation of high loss,} \\ [\gamma\eta(\mathbf{r}, t - \tau)]^{2/4}, & \text{in the single pass approximation;} \end{cases}$$

$\eta = \eta(\mathbf{r}, t) = \sigma^{-1}(1 + 1/\omega\, \mathrm{d}U(\mathbf{r}, t)/\mathrm{d}t)^{-1/2}$; τ is the total time needed for a light beam to pass over the interferometer; σ is the coefficient of beam extension in the element G; τ_n is the relaxation time of the nonlinear response of a medium (for example, liquid crystal); γ is the coefficient of loss in the field amplitude while passing over the NRI; D_e is the diffusion coefficient of polarized molecules of a liquid crystal normalized to the beam radius and τ_n; Δ_{xy} is the Laplace operator of cross coordinates.

The single-pass mode here is a mode, at which some component of the light field has passed the feedback unit, but at the next passage it was removed and thus did not come to the nonlinear medium. If the power loss in the interferometer is so high that we can neglect the term with $\{\gamma/\sigma/2\}^2$, then it is generally accepted to speak about the approximation of high loss.

If we take $\eta = 1$, $t_e = 0$, $A = \text{const}$, $I_0 = A^2$, $p = 0$, and nonlinearity parameter $K = n_2\, lk\, (1 - R)\, I_0$, then we obtain the result coinciding with the models in Refs. 5, 6, and 10 and differing from the model from Ref. 8 by the absence of the factor 0.5 in the expression including the product $n_2\, lk$ (since the ray is extraordinary).

2. Regularities in formation of optical structures in NRI for the case of optical field with spiral dislocations

Let an optical field with spiral dislocations of the wave front come to the entrance of the NRI. Assume that the center of this field (singular point at which the intensity is zero) coincides with the optical axis of the interferometer Oz.

According to Ref. 1, the complex amplitude of the field strength in the vicinity of the singular point of the field with spiral dislocation of the wave front is described by Eq. (1). Consequently, the model (2) describes this case, if we assume

$$A(r, t) = Cr^{V_{\text{d}}}; \qquad (3)$$
$$\varphi(\mathbf{r}, t) = \pm\, V_{\text{d}}\, az,$$

where az is the azimuth angle measured from the positive direction of the axis Ox counterclockwise up to the ray **r** plotted from the point O. Assume that in some region of the beam cross section we can neglect the dependence of the amplitude A on V_d, i.e., the presence of this dependence does not provoke appearance of bifurcations in the solution of Eq. (2) like, for example, in the situations described in Ref. 5 (Fig. 5) and Ref. 6 (Fig. 6.9). Our further considerations apply only to this region.

Case of field turning in the feedback unit. Let the feedback unit of the NRI (with the element G) turns the field through the angle $\Delta = 2\pi M/m$ in the beam cross section (Fig. 2), where M and m are coprime numbers. Because the angles $\Delta_1 = 2\pi(M + im)/m$ and $\Delta_2 = 2\pi M/m$ are equivalent, it is sufficient to consider the case that M belongs to the half-open interval $[1, m)$. It is obvious that the whole set of points of the beam cross section in the NRI is divided into an infinite number of subsets. These subsets are chains of points, at which light fields and nonlinear phase incursions U interact sequentially (see Fig. 2).

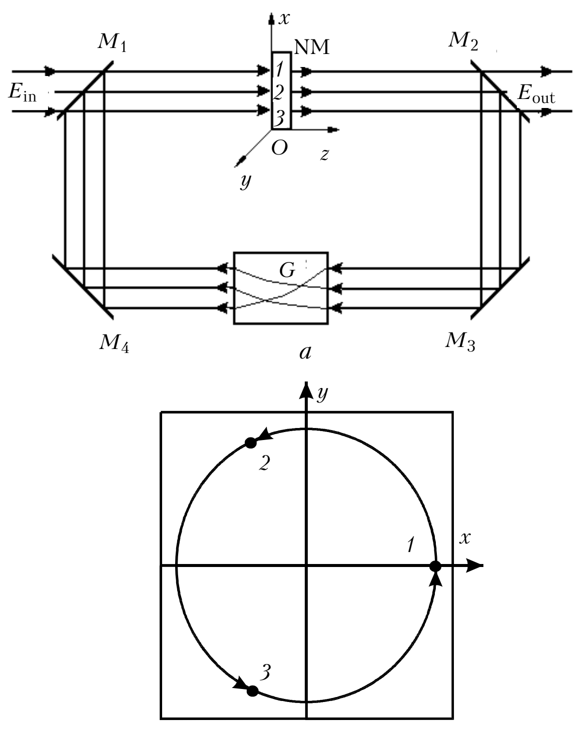


Fig. 2. Ray geometry as light field turns through the angle $\Delta = 120°$ in the cross plane in NRI feedback unit: trajectories of three rays *1*, *2*, and *3* closed after three passages in the NRI (*a*), projection of the closed trajectories of the rays *1*, *2*, and *3* onto the plane of the beam cross section.

In other words, upon passage through the nonlinear medium and NRI feedback at the point i (for example, in Fig. 2 $i = 1, 2, 3$), the ray acquires the phase incursion U_i and experiences the delay t_{ei}. Because of the presence of the element G, the ray comes to the point $i + 1$. Here, adding together with one of the input rays of the interferometer, it, according to the model (2), affects the rate of change of the nonlinear phase incursion U_{i+1}. (Note that because the trajectories of the rays in Fig. 2 are closed, the value of the index $i + 1 = 4$ should be taken equal to $i + 1 = 1$). The phase incursion U_i at the point i affects the incursion U_{i+1} at the point $i + 1$ just in this way. According to the terminology accepted in Ref. 10, the points of this type are called transposition points, and m is called the transposition order. At such an organization of the feedback, the ray trajectory is closed after the ray passes around the NRI m times. It can be easily seen that, according to the accepted method of numeration of the transposition points, $i + 1$ means the operation $((i+1) \bmod m) + 1$, where $(i+1) \bmod m$ is the remainder of division of $i + 1$ by m. Physically, this means that the ray from the mth point comes to the first one.

The transposition points are characterized by the azimuth angles $az_i = az_0 + (2\pi/m)\{(iM) \bmod m\}$, where az_0 belongs to the half-open interval $[0, 2\pi/m)$. Then the field phase at the ith transposition point is $\varphi_i = \pm V_d\, az_i$. In the model (2) the difference between phases at two transposition points $\varphi(\mathbf{r}, t) - \varphi(\mathbf{r}', t - \tau)$ can be taken equal to

$$\varphi(\mathbf{r}, t) - \varphi(\mathbf{r}', t - \tau) \equiv \varphi_{i+1} - \varphi_i = \pm V_d (2\pi/m) \times$$
$$\times \{[((i + 1) M) \bmod m] - [(iM) \bmod m]\}.$$

Since this difference is under the cosine sign in the model (2), it is sufficient to determine it accurate to 2π. It follows that $\varphi_{i+1} - \varphi_i = \pm V_d (2\pi M/m)$. Therefore, as the order of the spiral dislocation changes by an integer number N, the value of $\varphi_{i+1} - \varphi_i$ changes by $\Delta(\varphi_{i+1} - \varphi_i) = \pm N (2\pi M/m)$.

To reveal the periodicity in the dependence of dynamics of the nonlinear phase incursion (and intensity) of the field in the NRI on the value of V_d, let us take the change $\Delta(\varphi_{i+1} - \varphi_i) = 2\pi s$, where s is integer. Then

$$N = \pm sm/M. \quad (4)$$

Obviously, there always exists such s that Eq. (4) is fulfilled (recall that N is integer). Consequently, within the model (2) as the order of the spiral dislocation V_d changes by N, the dynamics of the nonlinear phase incursion $U(\mathbf{r}, t)$ (and intensity $A^2_{\text{in NM}}(\mathbf{r}, t)$), in the NRI is the same as before the change. That is, due to the NRI there exists a periodicity in mapping of the set of fields with spiral dislocations of the wave front into the set of processes $U(\mathbf{r}, t)$, $A^2_{\text{in NM}}(\mathbf{r}, t)$. The minimal period of mapping corresponding to the minimal N (or s) in Eq. (4) is m.

We can prove the statement that φ, ψ, and t_e have the equivalent influence on the dynamics of U and $A^2_{\text{in NM}}$. This statement, in view of the approximations (accepted in the beginning of Section 2) on the field parameters at the entrance to the NRI, transforms to the following: if the value of $U_i(t) - U_i(t + \delta_2)$ is negligible, then within the model (2) the influence of the modulation of phase $\varphi_i \neq 0$ on the dynamics of processes in the NRI can be reduced to the joint influence of modulation of the delay time in the NRI feedback unit $t_{e2} = t_{ei} + \delta_{2i}$ and phase $\varphi_{2i} = 0$. Here $\delta_{2i} = [\varphi_{i+1} - \varphi_i]/\omega$. We believe that the rate of transfer of the wave front with spiral dislocation along the axis Oz in the NRI is independent of V_d. Then the field with $V_\mathrm{d} \neq 0$ at the NRI entrance generates, with the delay t_{ei}, the same dynamics of the phase incursion and intensity as the field with $V_\mathrm{d} = 0$ at the entrance to the NRI with $t_{e2i} = t_{ei} + [\varphi_{i+1} - \varphi_i]/\omega = t_{ei} \pm V_\mathrm{d}(2\pi M/m)/\omega$.

Let us take into account the above periodicity in mapping of the set of vortex fields with different dislocation orders into the set of processes in the NRI given by Eq. (4), as well as the equivalence of influence of the delay t_e (phase delay φ_fb in the feedback unit) and the difference of phases $\varphi_{i+1} - \varphi_i$ at two points in space and time. Then we can formulate the following theses:

– within the NRI model (2) in the approximation of high loss or single pass of the monochromatic linearly polarized vortex field with the order of the regular spiral dislocation V_d, the set of fields with different V_d is mapped into the set of optical structures $U(\mathbf{r}, t)$ and $A^2_{\mathrm{in\ NM}}(\mathbf{r}, t)$ (static or rotating[5,6,10]) in the laser beam cross section;

– if the vortex optical field coming to the NRI is deflected by the angle $\Delta = 2\pi M/m$ in the cross section and delayed in phase by

$$\varphi_\mathrm{fb} = \varphi_{0\mathrm{fb}} - \{\pm 2\pi[(LM) \bmod m]\}, \qquad (5)$$

due to field delay, then for the cross section region, where the dependence of the field amplitude on V_d is negligible (see Section 2), the map as a function of V_d (and as a function of L) has the period m;

– then, knowing the value of V_d for the field at the NRI entrance and the value of L from Eq. (5), the structures from the set $U(\mathbf{r}, t)$, $A^2_{\mathrm{in\ NM}}(\mathbf{r}, t)$ can be assigned the number N_s by the following rule:

$$N_s = V_\mathrm{d} \oplus_m L, \qquad (6)$$

where $\oplus_m$ is the modulo sum of m; M and L are integer non-negative numbers (M and m are coprime numbers).

Thus, using the NRI, we can identify the order of spiral dislocation of an optical vortex (by the structures in the beam cross section), as well as compensate for its influence (by fitting the value of L meeting the condition (6) at given N_s and V_d) on the behavior and result of formation of optical structures in the interferometer. Such compensation is urgent in the context of problems of atmospheric adaptive optics.[7]

Case of specular reflection of the field in the feedback unit. Let now the beam be reflected specularly relative to the axis Ox in the feedback unit (see Fig. 1). As in the case of field turning by the angle $\Delta = 2\pi M/m$, the whole set of the points of the beam cross section in the NRI is divided into the infinite number of subsets. However, now the latter ones consist of pairs of transposition points equidistant from the axis Ox, and the rays coming from the origin of coordinates to such points form the angle, whose bisector is the axis Ox. Therefore, $\pi - az_1 = -(\pi - az_2)$.

Let us fix the square distance from the origin of coordinates to some point $x^2 + y^2$. When moving from the point of the cross section $(-x, 0)$ to the point $(x, 0)$ $(x > 0)$ along the upper half-plane $(y > 0)$, the azimuth angle changes smoothly from π to 0, and when moving along the lower half-plane $(y < 0)$ the angle changes from 0 to 2π. The difference of the azimuth angles characterizing the transposition points changes smoothly from 0 to 2π. Since the field phase at the ith transposition point is $\varphi_i = \pm V_\mathrm{d}\, az_i$, in the model (2) the phase difference between two transposition points $\varphi(\mathbf{r}, t) - \varphi(\mathbf{r}', t - \tau)$ can be set equal to

$$\varphi(\mathbf{r}, t) - \varphi(\mathbf{r}', t - \tau) \equiv \varphi_{i+1} - \varphi_i = \pm V_\mathrm{d}\, (az_{i+1} - az_i).$$

It is obvious, first, that this difference is the same for pairs of transposition points located on the rays coming from the origin of coordinates. Second, at such a pass around the origin of coordinates, the phase difference of the initial wave at the transposition points varies from 0 to $\pm 2\pi V_\mathrm{d}$. Then we can conclude that at this pass around the beam center the characteristic peculiarities of formation of optical structures repeat only so much times changing each other. The number of changes is unambiguously connected with the order of the spiral dislocation V_d. Thus, having determined the number of changes of structure peculiarities, we can find the value of V_d.

Since the mentioned phase difference varies smoothly, the structure elements identified as belonging to the same type are usually observed within some sectors, rather than on infinitely narrow rays. Formation of sectors of rays in actual structures is favored by diffusion of molecules of the nonlinear medium (or beam diffraction). Therefore, when describing structures theoretically, it makes sense to deal with the concept of sectors as well.

So, the following statement based on the above reasoning and our computer experiments is valid for some particular case. *If the optical field experiences specular reflection relative to the axis passing through the beam center in the feedback unit of the NRI and there is no delay, then proper choose of the nonlinearity parameter (for example,* $K = 3.5$*) and the diffusion coefficient (for example,* $D_e = 0.01$*) can provide formation of the structure consisting of sector pairs in the beam cross section; the intensity of this structure varies from one sector to another; therewith, at azimuthal pass (in any half-plane) from* $[0, \pi)$ *the number of its sharp changes is equal to the order of the spiral dislocation:* $N_f = V_\mathrm{d}$.

To illustrate the theses from Section 2, we will consider the results of computer experiments.

3. Demonstration of peculiarities in formation of optical structures when simulating processes in NRI

The periodicity of mapping of the set of fields with different V_d into the set of optical structures in the laser beam cross section for the case of field turning in the feedback unit is demonstrated in Figs. 3 and 4.

Figure 3 shows the distributions of the nonlinear phase incursion $U(\mathbf{r}, t)$ and the parameter proportional to the intensity $A^2_{\text{in NM}}(\mathbf{r}, t)$ in the beam cross section for the field turning through the angle 120° (i.e., $M = 1$, $m = 3$), diffusion coefficient 0.001, delay close to 0.1 τ_n (but at $\varphi_{0fb} = 0$ and $L = 0$), and the parameter of nonlinearity $K = 3.5$.

As is seen from Fig. 3, the period of mapping of the SD order into the set of structures is equal to three, i.e., structures, for example, at the SD order equal to 0, 0 + 3, 0 + 7 · 3, are similar. Dissimilarity of structures in their inner details in Fig. 3 is caused by reasons of computational origin: lack of coincidence between the type of symmetry of field map in the feedback unit (rotational symmetry axis of the third order) and the type of symmetry of a rectangular grid used in the computational scheme of the finite-difference method.[11–13]

It should be noted that at the SD order equal to $1 + 3i$, the presence of delay leads to generation of rotating (in the cross plane) structures. In other cases, only static structures are formed at so short delay. This fact gives a simple criterion for recognition of spiral dislocations with $V_d = 1 + 3i$: periodic character of a signal when recording the intensity of a part of the beam at the NRI exit. Recall, that rotation of structures is impossible in a ring interferometer without nonlinearity. The SD order can be also determined from the averaged (over cross section) intensity of the whole beam at the NRI exit.

Figure 4 shows two types of similar structures formed at field turning through 180° (i.e. $M = 1$, $m = 2$) and the parameter of nonlinearity $K = 2.2$ (all other parameters are the same as in Fig. 3). At even orders of spiral dislocation, a spatially bistable structure[5,6] is formed, and at odd orders, the structure is monostable (the light intensities in the transposition points are the same). The period of mapping in Fig. 4 is equal to two, what is in agreement with the first thesis of Section 2.

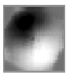
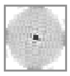
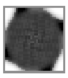
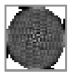
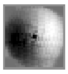
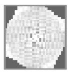
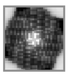
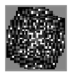
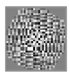
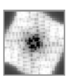
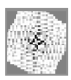
Phase incursion $U(\mathbf{r}, t)$		Square amplitude at the entrance to nonlinear medium NM/(1 − R)		V_d	Mean square amplitude at the entrance to nonlinear medium NM/(1 − R)
	max = 3.4906 min = 1.9239		max = 1.0000 min = 0.5000	0	0.7038
	max = 4.3795 min = 2.8255		max = 1.3876 min = 0.6571	1	1.0266
	max = 5.1051 min = 3.5097		max = 1.4994 min = 1.0000	2	1.3324
	max = 3.4906 min = 1.9231		max = 1.0000 min = 0.5002	0 + 3 = 3	0.7044
	max = 4.3991 min = 2.8135		max = 1.4191 min = 0.5078	1 + 3 = 4	1.3324
	max = 5.1020 min = 3.5097		max = 1.5000 min = 0.6885	2 + 3 = 5	1.2784
	max = 3.7124 min = 1.9758		max = 1.4783 min = 0.5000	0 + 3 · 7 = 21	0.7401
	max = 4.1347 min = 3.0387		max = 1.4986 min = 0.5001	1 + 3 · 7 = 22	1.028
	max = 5.0173 min = 3.0275		max = 1.5000 min = 0.5000	2 + 3 · 7 = 23	1.2784

Fig. 3. Observed structures at the diffusion coefficient $D_e = 0.001$, delay $t_e/\tau_n = 0.1$, coefficient of nonlinearity $K = 3.5$, and field turn $\Delta = 120°$ (approximation of high loss).

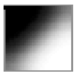
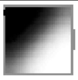
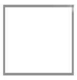
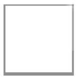
Phase incursion $U(\mathbf{r}, t)$		Square amplitude at the entrance to nonlinear medium NM/(1 − R)		V_d	Mean square amplitude at the entrance to nonlinear medium NM/(1 − R)
	max = 2.3917 min = 1.3953		max = 1.0873 min = 0.6341	0	0.8561
	max = 3.2882 min = 3.2882		max = 1.4946 min = 1.4946	1	1.4946

Fig. 4. Observed structures at $D_e = 0.001$, $t_e = 0$, $K = 2.2$, $\Delta = 180°$ (approximation of high loss).

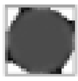
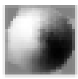
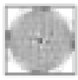
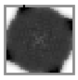
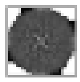
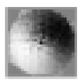
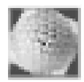
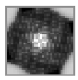
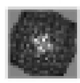
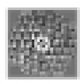
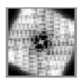
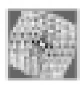
Phase incursion $U(\mathbf{r}, t)$		Square amplitude at the entrance to nonlinear medium NM/(1 − R)		V_d
	max = 3.4919 min = 2.1440		max = 1.0000 min = 0.5626	0
	max = 4.6147 min = 2.4430		max = 1.4403 min = 0.5504	1
	max = 5.3639 min = 3.5112		max = 1.6176 min = 1.0000	2
	max = 3.4919 min = 2.1415		max = 1.0000 min = 0.5625	0 + 3 = 3
	max = 4.5974 min = 2.4517		max = 1.4660 min = 0.5357	1 + 3 = 4
	max = 5.5358 min = 3.5108		max = 1.7260 min = 0.6937	2 + 3 = 5
	max = 3.8316 min = 2.1874		max = 1.5056 min = 0.5021	0 + 3 · 7 = 21
	max = 4.5294 min = 2.9678		max = 1.6759 min = 0.4628	1 + 3 · 7 = 22
	max = 5.4238 min = 3.4579		max = 1.7754 min = 0.5154	2 + 3 · 7 = 23

Fig. 5. Observed structures at $D_e = 0.001$, $t_e = 0$, $K = 3.5$, $\Delta = 120°$ (multipass interferometer).

According to Eq. (5), the phase delay in the feedback unit is

$$\varphi_{fb} = \varphi_{0fb} - \{\pm 2\pi\, [(LM) \bmod m]\}.$$

The computer experiments carried out at varying φ_{fb}, when the value of L was varied and $\varphi_{0fb} = 0$, confirmed the proposed rule of numeration of the structures (6), i.e., $N_s = V_d \oplus_m L$, and the equivalent influence of V_d and L on the dynamics of the structures.

Figures 5 and 6 have been obtained at the same parameters of the interferometer and input radiation as Figs. 3 and 4, but the structures shown in them correspond to the model of multipass interferometer described in Ref. 8. Comparison between Fig. 3 and Fig. 5, as well as between Fig. 4 and Fig. 6, shows a qualitative similarity of the structures obtained at the same SD orders. This promises that the structures in the multipass interferometer can be numerated by the same or similar rule as in the interferometer in the approximation of high loss or single pass.

Figure 7 corresponds to the case of specular reflection in the feedback unit. It shows the structures of the nonlinear phase incursion consisting of pairs of sectors whose intensity varies from sector to sector. It is seen from Fig. 7 that at azimuthal pass (in any half-plane) from $[0, \pi)$ the number of sharp intensity changes is equal to the SD order. The validity of the second thesis of Section 2 is thus demonstrated.

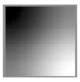
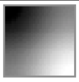
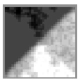
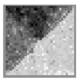
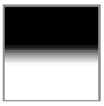
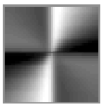
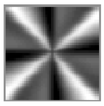
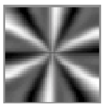
Phase incursion $U(\mathbf{r}, t)$		Square amplitude at the entrance to nonlinear medium NM/$(1 - R)$		V_d
	max = 1.8404 min = 1.8399		max = 0.8365 min = 0.8365	0
	max = 3.5923 min = 3.5923		max = 1.6329 min = 1.6329	1

Fig 6. Observed structures at $D_e = 0.001$, $t_e = 0$, $K = 2.2$, $\Delta = 180°$ (multipass interferometer).


max = 3.1852
min = 1.7517
$V_d = 0$


max = 4.8909
min = 1.9609
$V_d = 1$


max = 4.8626
min = 2.1033
$V_d = 2$


max = 4.7417
min = 2.1082
$V_d = 3$

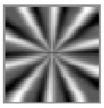

max = 4.5793
min = 2.2575
$V_d = 5$

Fig. 7. Structures formed under the NRI exposure to vortex field with different dislocation order V_d at $K = 3.5$ with specular reflection of field in feedback unit.

Conclusion

The analysis and imitation experiments have shown a possibility to identify the order of spiral dislocation of vortex field at the input to the nonlinear ring interferometer by the shape of structures of nonlinear phase incursion (and intensity), and by their dynamics in the model (2) of the processes in the NRI. In the case of field rotation in the NRI feedback unit through some angle, it was found that the dislocation order V_d and delay (phase delay) of the field in the feedback unit have the equivalent effect on formation of optical structures. This equivalence is formalized in the rule of structure numeration (6).

The obtained result allows us to suggest the following applications of the NRI: (1) to identify the order of spiral dislocations in optical vortex (by the shape of structures in the beam cross section), (2) to compensate for the vortex influence (by fitting L, which meets the condition (6) at given N_s and V_d) on the behavior and result of formation of optical structures in the interferometer, (3) to be used as a basis of an arithmetic and logic unit executing the operation of the modulo sum of m.

The potential advantage of NRI application to identification of the SD order is the possibility to obtain situations, in which the dynamics of processes in the NRI (or formed structures) changes qualitatively at varying V_d. This can simplify significantly the process of identification of the SD order V_d.

In the case of specular reflection of the field in the feedback unit, the order V_d can be determined from the number of changes of structure peculiarities at azimuthal pass around a singular point.

Acknowledgments

We would like to express our gratitude to V.P. Aksenov and O.V. Tikhomirova for useful criticism when discussing the results and E.A. Sosnin who directed our attention to Refs. 3 and 4.

References

1. V.P. Korolenko, Soros Obraz. Zh., No. 6, 94–99 (1998).
2. V.P. Aksenov, V.V. Kolosov, V.A. Tartakovskii, and B.V. Fortes, Atmos. Oceanic Opt. **12**, No. 10, 912–918 (1999).
3. M. Mansuripur and E. Wrignt, Optics & Photonics News **10**, No. 2, 40–44 (1999).
4. C.O. Weiss et al., Appl. Phys. **B 68**, 151–168 (1999).
5. S.A. Akhmanov and M.A. Vorontsov, in: *Nonlinear Waves: Dynamics and Evolution* (Nauka, Moscow, 1989), pp. 228–237.
6. S.A. Akhmanov and M.A. Vorontsov, eds., *New Physical Principles of Optical Processing of Information* (Nauka, Moscow, 1990), pp. 263–326.
7. V.P. Lukin, Atmos. Oceanic Opt. **8**, No. 1, 145–150 (1995).
8. I.V. Izmailov, A.L. Magazinnikov, and B.N. Poizner, Izv. Vyssh. Uchebn. Zaved., Fizika, No. 2, 29–35 (2000).
9. S.A. Akhmanov and S.Yu. Nikitin, *Physical Optics. Student's Book* (Moscow State University Press, Moscow, 1998), 656 pp.
10. A.I. Arshinov, R.R. Mudarisov, and B.N. Poizner, Izv. Vyssh. Uchebn. Zaved., Fizika, No. 6, 77–81 (1995).
11. R. Rihtmaier and K. Morton, *Difference Methods for Solution of Boundary-Value Problems* [Russian translation] (Mir, Moscow, 1972), 418 pp.
12. S.K. Godunov and V.S. Ryaben'kii, *Difference Schemes* (Nauka, Moscow, 1977), 439 pp.
13. A.A. Samarskii, *Introduction to Numerical Methods* (Nauka, Moscow, 1987), 288 pp.