Paraxial approximation for the problem of light beam propagation through a planar-cross stratified medium

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We consider the problem of laser beam propagation through a medium with its refractive index being a univariate function of the coordinate along the beam axis. We show in the paper that making use of the paraxial approximation enables one to reduce the task to solving the well known parabolic equation the diffusion coefficient in which depends, through the refractive index of the medium, on the longitudinal coordinate. This result is a corollary fact of the simplification of the rigorous solution and does not depend on the shape and value of the function of the refractive index in a wide range. We also derive a number of expressions that may have a practical value in using quite general outcomes of this study in practice.

Introduction

The problem on propagation of acoustic and electromagnetic waves through inhomogeneous media has been studied since long ago. The results of these studies and their use in wide-ranging applications have been summarized in a number of basic monographs.[1–4]

Since no rigorous solution of the wave equation for the field in an inhomogeneous medium can be sought, various theoretical studies do make use, as a rule, of different simplified methods. Among those, the geometrical optics approximation is apparently the best developed one. Detailed discussion of this approach can be found in Ref. 1. It is worth noting that an analytical solution to this problem can be found by use of this approximation only for some particular cases the medium inhomogeneity, see, for example Ref. 4.

If the geometrical optics approximation does not work, one must take into account diffraction (as, e.g., in the problems on laser beam propagation) and, normally, try to reduce the task to the scalar Helmholtz equation with the wave number $k = (\omega/c)n(\mathbf{r})$ that depends on coordinates. Then this equation is solved numerically. Note that such a simplification is only justified if the term (see Ref. 1)

$$\mathrm{grad}[\mathbf{E}\,\mathrm{grad}(\ln n^2)] \qquad \text{(I.1)}$$

is negligible, as in the problems with randomly inhomogeneous media,[2] for which the condition

$$n(\mathbf{r}) = n_0 + \Delta n(\mathbf{r}), \quad \Delta n(\mathbf{r})/n_0 << 1 . \qquad \text{(I.2)}$$

is almost always fulfilled quite accurately.

In this study we consider same problem on laser beam propagation, but through a continuum, whose index of refraction depends on a single coordinate, say on z, along the direction of beam propagation. Such media (sometimes these are called "greens") are interesting themselves[5] and in their particular forms like, e.g., planar-cross stratified media, a medium with a periodic structure, as well as in the simplest form of a medium with the refractive index being a stepwise function of the coordinate along the beam axis. The last case refers to practical applications dealing with focusing of laser beams into different object from transparent materials. If such an object is an anisotropic nonlinear crystal then the applicability of the results of this theoretical treatment naturally extends to the theory of generation of laser radiation harmonics, an important branch of nonlinear optics.

Strictly speaking, the main goal of this study is just to try to construct a sort of basic model that would enable one to accurately, as far as possible, account the effect of refraction and longitudinal inhomogeneity of a nonlinear crystal on the parameters of the second harmonic. In this connection, the linear fields in the isotropic media are proposed to be considered as an absolutely needed zero approximation of the above-mentioned nonlinear problem.

The calculation algorithm is as follows. First, the rigorous solution is written of the problem on laser beam propagation through a planar-cross stratified medium the boundaries between the layers in which are orthogonal to the beam axis. Then the beam is treated as a slightly diverging one (paraxial, that is the beam with a narrow angular spectrum and slowly varying amplitude). Based on this assumption the rigorous solution becomes much simpler. After that, a passage to the limit is performed by making the layer thickness to vanish. It is just this limit, which represents the general solution of the problem formulated in the paper title. In Section 2, we consider the task of focusing the laser beam into a bounded medium as an illustration that could be of a certain practical interest.

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1. General solution of the problem

Let the refractive index of the medium be dependent only on one coordinate, i.e., $n(\mathbf{r}) = n(z)$. It is also assumed that at $z < 0$ the value of the refractive index equals to a constant value n_0. Let for all $z > 0$ the function $n(\mathbf{r}) = n(z)$ be a step function, taking the value n_i within the layer $h(i-1) < z < hi$, where h is the thickness of the ith layer. The planes $z = hi$ make up the boundaries between the layers. Let us also introduce the cross vectors $\boldsymbol{\rho}_i$ in these planes to define the position of a point on the corresponding plane. The problem on radiation propagation through such a medium reduces to a recurrent sequence of solution to the problem of radiation refraction on the output boundary of each layer and to the problem of radiation propagation within the layer.

The solution to problem on optical field refraction on a plane boundary between two media is well known[1] for the case of plane waves. For any plane of plane wave propagation, the complex amplitude U of the field can be presented by the expansion over the angular spectrum of the plane waves.

$$U(z,\boldsymbol{\rho}) = \iint_{-\infty}^{+\infty} \mathrm{d}^2\boldsymbol{\kappa}\hat{U}(z,\boldsymbol{\kappa})\exp[i\boldsymbol{\kappa\rho}], \tag{1}$$

where $\hat{U}(z,\boldsymbol{\kappa})$ is the amplitude of the plane wave, $\boldsymbol{\kappa}$ is the transverse, relative to the direction of propagation, component of the wave vector.

As the field amplitude and angular spectrum of the field obey a one-to-one relation the task of seeking the relation between the amplitude of the field $U(z,\boldsymbol{\rho})$ in an arbitrary propagation plane and the boundary field $U_0(0,\boldsymbol{\rho}_0)$ incident on the medium can be reduced to seeking the ratio between the angular spectra of these fields.

Let $\hat{U}(z_{i-1},\boldsymbol{\kappa})$ be the spectrum of the field incident on the boundary $z = h(i-1)$ between the layers. Then for the spectrum just behind this boundary (within the other layer) we obtain, in a usual way that

$$\hat{U}_0(z_{i-1},\boldsymbol{\kappa}) = T(z_{i-1},\boldsymbol{\kappa})\hat{U}(z_{i-1},\boldsymbol{\kappa}) = T_{i-1}(\boldsymbol{\kappa})\hat{U}(z_{i-1},\boldsymbol{\kappa}), \tag{2}$$

where $T_{i-1}(\boldsymbol{\kappa})$ is the Fresnel refractive index of the medium for a plane wave.

The layer between the planes $z = h(i-1)$ and $z = hi$ (the ith layer) is, according to the conditions of the problem, a homogeneous medium with the refractive index n_i. Hence within this layer the field U must obey the homogeneous Helmholtz equation:

$$\Delta U + k^2 n_i^2 U = 0, \tag{3}$$

where $k = 2\pi/\lambda$ is the wave number in vacuum.

Owing to this fact and taking expression (2) as the boundary condition, one obtains that the field spectrum in the end of the ith layer is related to the field spectrum in the end of the preceding layer as follows:

$$\hat{U}(z_i,\boldsymbol{\kappa}) = T_{i-1}(\boldsymbol{\kappa})\hat{U}(z_{i-1},\boldsymbol{\kappa})\exp\left(ih\sqrt{k^2n_i^2 - \boldsymbol{\kappa}^2}\right). \tag{4}$$

The recurrence formula (4) enables one to relate the field spectrum at the exit from the Nth layer to the spectrum of the field incident on the planar-cross stratified medium. After some not very complicated transformation we have

$$\hat{U}(z_N,\boldsymbol{\kappa}) = \left(\prod_{i=1}^{N} T_{i-1}(\boldsymbol{\kappa})\right)\hat{U}(0,\boldsymbol{\kappa})\exp\left(ihk\sum_{i=1}^{N} n_i\sqrt{1-\frac{\boldsymbol{\kappa}^2}{k^2n_i^2}}\right), \tag{5}$$

where

$$\hat{U}_0(0,\boldsymbol{\kappa}) = (2\pi)^{-2}\iint_{-\infty}^{+\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, U_0(0,\boldsymbol{\rho}_0)\exp\left[-i\boldsymbol{\kappa\rho}_0\right]. \tag{6}$$

Substituting expression (5) into Eq. (1) one obtains a solution for the amplitude of the field in the end of the Nth layer:

$$U(z_N,\boldsymbol{\rho}_N) = \iint_{-\infty}^{+\infty} \mathrm{d}^2\boldsymbol{\kappa}\hat{U}(0,\boldsymbol{\kappa})\left(\prod_{i=1}^{N} T_{i-1}(\boldsymbol{\kappa})\right)\times$$
$$\times\exp\left[i\boldsymbol{\kappa\rho}_N + ihk\sum_{i=1}^{N} n_i\sqrt{1-\frac{\boldsymbol{\kappa}^2}{k^2n_i^2}}\right]. \tag{7}$$

Note that until so far we have not imposed any restrictions, however, in deriving expression (7) we did not take into account multiple reflections of the waves from the boundaries between the layers that also contribute into the forward propagated waves.

In our discussion below, we shall make use of the conditions of small angle (paraxial) approximation. Let us consider the case when the beam axis coincides with the OZ axis, i.e., the case of normal incidence of the beam onto the boundary between the media. In so doing, we shall assume that the refractive index of the medium only slightly varies within the angular spectrum of the beam. In addition, following the standard conditions of the paraxial approximation, we consider the radicand in expression (7) to be close to unity. In other words we shall make use of the following approximations:

$$T_{i-1}(\boldsymbol{\kappa}) \cong T_{i-1}(0); \tag{8}$$

$$\sqrt{1-\frac{\boldsymbol{\kappa}^2}{k^2n_i^2}} \cong 1 - \frac{\boldsymbol{\kappa}^2}{2k^2n_i^2}. \tag{9}$$

In that case solution (7) reduces to the following form

$$U(z_N,\boldsymbol{\rho}_N) \cong \left(\prod_{i=1}^{N} T_{i-1}(0)\right)\exp\left[ihk\sum_{i=1}^{N} n_i\right]\times$$
$$\times\iint_{-\infty}^{+\infty} \mathrm{d}^2\boldsymbol{\kappa}\hat{U}_0(0,\boldsymbol{\kappa})\exp\left[i\boldsymbol{\kappa\rho}_N - ih\frac{\boldsymbol{\kappa}^2}{2k}\sum_{i=1}^{N} n_i^{-1}\right]. \tag{10}$$

Expression (10) is, in fact, the solution of the problem on beam propagation through a stratified medium obtained using small angle (paraxial) approximation.

The solution obtained enables one to perform a passage to a limiting case of a stratified medium with continuous distribution of the index of refraction. Let us divide each of the layers into M sub-layers of the width $\Delta z = h/M$. By making M to tend toward infinity, we pass to a continuous medium. In this case the summation in the exponential functions evidently transforms into integrals over the range.

Consider now which form the product of the refraction indices takes due to passage to this limit. The Fresnel refractive index has the following form in the case of normal incidence of the wave onto the boundary between the madia[1]:

$$T_{i-1}(0) = \frac{2n_{i-1}}{n_{i-1} + n_i} = \frac{2n_{i-1}}{n_{i-1} + n_{i-1} + \Delta n_{i-1}} = \frac{1}{1 + 0.5\dfrac{\Delta n_{i-1}}{n_{i-1}}}. \quad (11)$$

Assuming the ith layer to be narrow enough for the following equality

$$\frac{\Delta n_{i-1}}{n_{i-1}} \cong \frac{1}{n(z_{i-1})}\frac{\mathrm{d}n}{\mathrm{d}z}h,$$

to work and dividing this layer into M sub-layers, one can write for the product of the refractive indices of the sub-layers within the ith layer the following expression

$$\lim_{M\to\infty}\prod_{j=1}^{M} T_{i-1}(0) = \lim_{M\to\infty}\left[1 + 0.5\frac{1}{n(z_{i-1})}\frac{\mathrm{d}n}{\mathrm{d}z}\frac{h}{M}\right]^{-M} =$$

$$= \exp\left[-0.5\frac{1}{n(z_{i-1})}\frac{\mathrm{d}n}{\mathrm{d}z}h\right]. \quad (12)$$

Then the product of the refractive indices of all the layers takes the following form:

$$\lim_{M\to\infty}\prod_{i=1}^{N}\prod_{j=1}^{M} T_{i-1}(0) = \exp\left[-0.5\int_0^{z_N}\mathrm{d}z\frac{1}{n(z)}\frac{\mathrm{d}n}{\mathrm{d}z}\right] =$$

$$= \exp\left[-0.5\left(\ln n(z_N) - \ln n(0)\right)\right] = \sqrt{\frac{n(0)}{n(z_N)}}. \quad (13)$$

As a consequence, the expression (10) for a continuous medium takes the form:

$$U(z,\rho) = \sqrt{\frac{n(0)}{n(z)}}\exp\left[ik\int_0^z \mathrm{d}z n(z)\right]\times$$

$$\times\iint_{-\infty}^{+\infty}\mathrm{d}^2\kappa\,\hat{U}_0(0,\kappa)\exp\left[i\kappa\rho - i\frac{\kappa^2}{2k}\int_0^z\frac{\mathrm{d}z}{n(z)}\right] =$$

$$= \sqrt{\frac{n(0)}{n(z)}}\exp\left[ik\int_0^z \mathrm{d}z n(z)\right]\times$$

$$\times\frac{k}{2\pi i\tilde{z}}\iint_{-\infty}^{+\infty}\mathrm{d}^2\rho_0\,U(0,\rho_0)\exp\left[-ik\frac{(\rho-\rho_0)^2}{2\tilde{z}}\right], \quad (14)$$

where $\tilde{z} = \displaystyle\int_0^z\frac{\mathrm{d}z}{n(z)}$.

For the case of a plane wave incident on the boundary $z = 0$ expression (14) yields the following solution

$$U(z,\rho) = \sqrt{\frac{n(0)}{n(z)}}\exp\left[ik\int_0^z \mathrm{d}t\, n(t)\right] \equiv U_e(z). \quad (15)$$

This solution exactly coincides with the so-called etalon solution of the problem on plane wave propagation through stratified media.[3] For an angular narrow beam solution (14) can be presented in the following form

$$U(z,\rho) = U_e(z)U_p(z,\rho), \quad (16)$$

where

$$U_p(z,\rho) = \frac{k}{2\pi i\tilde{z}}\iint_{-\infty}^{+\infty}\mathrm{d}^2\rho_0\,U(0,\rho_0)\exp\left[-ik\frac{(\rho-\rho_0)^2}{2\tilde{z}}\right]. \quad (17)$$

Remind that solution (16) has been derived while passing to a limit from planar-cross stratified to continuous medium. Let us now show the conditions under which this solution can be directly derived from Maxwell equations for a medium with the refractive index being a function of the only z coordinate.

Since we deal with a paraxial beam, it is quite reasonable to seek a solution of the Maxwell equation for the vector of the electric field strength, as an example, in the following form:

$$E(z,\rho) = \mathbf{e}U(z,\rho), \quad (18)$$

where $\mathbf{e}$ is the constant unit vector of polarization orthogonal to the Z axis ($\mathbf{ek} = 0$, where $\mathbf{k}$ is the unit vector along the Z axis).

Substituting expression (18) into the wave equation of the general form[1,4] and taking into account that the vector grad(n) is, in our case, directed along $\mathbf{k}$ vector (hence, term (I.1) vanishes) we obtain, after multiplying by the vector $\mathbf{e}$, the scalar Helmholtz equation:

$$\Delta U + k^2 n^2(z)U = 0. \quad (19)$$

Solution to equation (19) is sought in the following form

$$E(z,\rho) = U_e(z)U_p(\mu^2 z,\mu\rho), \quad (20)$$

where U_e and U_p are yet unknown functions; $\mu \ll 1$ is the small parameter whose value is on the order of magnitude of the beam divergence.

Substituting expression (20) into Eq. (19) and restricting ourselves to the account of only second infinitesimal order of μ we finally obtain the following equation

$$U_p\left[\nabla^2 U_e + k^2 n^2 U_e\right] + 2\nabla U_e \nabla U_p + U_e \nabla_\perp^2 U_p = 0. \quad (21)$$

Let us take for function U_e the exact solution of the problem on plane wave propagation through the medium considered here. Let the wave propagate along the Z-axis. In this case, this is the exact solution of Eq. (19). As a result, the expression in brackets in Eq. (21) vanishes and we obtain the following equation

$$2\nabla U_e \nabla U_p + U_e \nabla_\perp^2 U_p = 0. \quad (22)$$

Following Ref. 3 we present the function U_e as follows:

$$U_e(z) = \sqrt{n(0)} \exp\left[ik \int_0^z \mathrm{d}t\, n(t)\, \varphi(t)\right], \quad (23a)$$

where

$$\varphi(z) = 1 + \frac{i}{kn} \ln(n^{1/2})' + \frac{1}{2k^2} \frac{(n^{-1/2})''}{n^{3/2}} + \frac{1}{n} \sum_{\nu=3}^{\infty} \frac{\xi_\nu}{k^\nu}, \quad (23b)$$

and the view of functions ξ_ν can be found in Ref. 3. Assume that the following conditions hold

$$1 >> \frac{i}{kn^2} \frac{\partial n}{\partial z} >> \frac{1}{2k^2} \frac{(n^{-1/2})''}{n^{3/2}} >> \frac{1}{n} \sum_{\nu=3}^{\infty} \frac{\xi_\nu}{k^\nu}. \quad (24)$$

Then, taking only the zero term of the expansion (23b) we obtain the approximation

$$\nabla U_e = \mathbf{k}\,(ikn)\varphi(z) U_e \cong \mathbf{k}\,(ikn) U_e. \quad (25)$$

By substituting expression (25) into the Eq. (22) we obtain the parabolic equation

$$2ikn(z) \frac{\partial U_p}{\partial z} + \nabla_\perp^2 U_p = 0, \quad (26)$$

whose exact solution is presented by the expression (17).

Taking only two terms of expression (23b) one can see that in this case the function U_e transforms into the etalon solution (15). Thus, the solutions derived by use of both of these ways are identical if the conditions (24) hold.

2. Focusing of the beam into a planar-cross stratified medium

Actually, the general solution of thus formulated problem we have already derived in Section 1 of this paper in the form of the integral (17). Thus now we only have to concretize the boundary conditions (to set the functions $U(0, \rho_0)$ and the function $n(z)$).

Let the boundary condition at $z = 0$ be as follows

$$U(0, \rho) = A_0 \exp\left[-\frac{\rho_0^2}{a^2} - ik\frac{\rho_0^2}{2f}\right], \quad (27)$$

where A_0, a, f are the real constants; $k = 2\pi/\lambda$.

The condition (27) defines a Gaussian beam with the radius a, which is focused with a thin lens of the focal length f. This formula has been derived in the paraxial (parabolic) approximation for the plane $z = 0$. We have chosen this particular case only for simplicity, because the exact values of the integrals in expression (17) can be calculated.

Let us consider a medium the refractive index of which can be presented for all $z > 0$ in the following form:

$$n(z) = \begin{cases} 1, \text{ if } z < z_N,\ z > z_k, \\ n(z), \text{ if } z_N \le z \le z_k, \end{cases} \quad (28)$$

where $n(z) \ge 1$; $z_k = z_N + L$.

In other words, we deal, in this case, with an inhomogeneous medium, which in fact is an infinite layer of thickness L confined between the planes $z = z_N$ and $z = z_k$. The medium outside this "inhomogeneous layer" is assumed vacuum.

It is worth noting here that the above-mentioned initial conditions of the problem are quite typical in the theory of frequency conversion of laser radiation. The only exception is that in this theory a nonlinear anisotropic crystal of the length L is taken as an inhomogeneous medium.

As the exact account of the crystal inhomogeneity, including the longitudinal one, is of great practical interest,[6] the results we present in this section refer just to the problems of nonlinear optics as we have already mentioned in the Introduction.

Substituting expression (27) into Eq. (17) and taking the integrals one obtains the expression for a Gaussian beam focused at any point of the half-space $z > 0$. This solution differs from the known[4] derived for the case of homogeneous media (for this reason we do not give it here) only by that now we have to use $k = 2\pi/\lambda$ in place of $k = 2\pi n/\lambda$ and replace z by

$$\tilde{z} = \int_0^z \frac{\mathrm{d}t}{n(t)}. \quad (29)$$

Taking into account expressions (28) and (29) we obtain

$$\tilde{z} = \begin{cases} z\,, \text{ if } z < z_N, \\ z_N + \varphi(\Delta), \quad \text{if } z_N \le z \le z_k, \\ z - L + \varphi(L), \quad \text{if } z > z_k, \end{cases} \quad (30)$$

where

$$\varphi(\Delta) = \int_{z_N}^{z} \frac{dt}{n(t)} = \int_{0}^{z-z_N} \frac{dt}{n(t+z_N)} = \int_{0}^{\Delta} \frac{dt}{n'(t)};\ 0 \le \Delta \le L$$

is the length of the path the beam travels in the inhomogeneous medium, $n'(t) = n(t + z_N)$.

It is quite easy to determine, using Eq. (30), the dependence of the Gaussian beam parameters on the particular view of the function $n'(t)$, so we do not consider thus calculated results. We thought it to be pertinent to give here some expressions that may be useful in treating the tasks of focusing laser beams into crystals.

Note, first that, at least for a Gaussian beam, the longitudinal inhomogeneities of the medium do not affect its minimum radius $a_{\min}$, which is defined by the known relationship[4]

$$a_{\min} = \frac{aD_f}{\sqrt{1+D_f^2}}, \tag{31}$$

where $D_f = 2f/ka^2$.

At the same time, the position of the caustic ($z = z_p$) quite strongly depends on the view of the function $n'(t)$ chosen. Let us consider this circumstance in a more detail.

According to Ref. 4 the distance $z_p = z_{p0}$ from the lens to the caustic of a Gaussian beam in vacuum is determined by the following expression[4]

$$z_{p0} = \frac{f}{1+D_f^2}. \tag{32}$$

In our case, we obtain using expression (30) the following equation for z_p value

$$\int_0^{z_p} \frac{dt}{n(t)} = z_{p0}. \tag{33}$$

To illustrate the equation (33) let us consider the simplest situation with $n'(t) = n_0$. From Eq. (33) one can obtain

$$z_p = \begin{cases} z_{p0}, \text{ if } z_{p0} < z_N, \\ z_N + (z_{p0} - z_N)n_0, \text{ if } z_N \le z_{p0} \le z_N + L, \\ z_{p0} + L(1 - 1/n_0), \text{ if } z_{p0} > z_N + L. \end{cases} \tag{34}$$

Note that the result in the form of expression (34) can easily be obtained by treating refraction of separate geometric optics rays of the paraxial beam at the plane boundary between two inhomogeneous media.

In making some practical calculations one often needs for a solution of the following, in a certain sense inverse, problem. Assume that position of the caustic is set initially ($-\infty \le \Delta_f \le \infty$), from the entrance to the inhomogeneous medium. The question is to be addressed on the distance, at which the layer of an inhomogeneous medium must be placed with respect to the lens (that is at which z_N) for the caustic to be formed at a preset place. To answer this question, let us present z_p in the form

$$z_p = z_N + \Delta_f,$$

and then substitute this formula into Eq. (33). After that one obtains by making use of expression (30) the result sought

$$z_N = \begin{cases} z_{p0} - \Delta_f, \text{ if } \Delta_f < 0, \\ z_{p0} - \varphi(\Delta_f), \text{ if } 0 \le \Delta_f \le L, \\ z_{p0} + L - \Delta_f - \varphi(L), \text{ if } \Delta_f > L. \end{cases} \tag{35}$$

The length of the beam waist L_p is an important parameter of a focused Gaussian beam. Let it be defined as follows:

$$L_p = -L_1 + L_2, \tag{36}$$

where L_1 and L_2 are the distances from the waist plane, at which beam diameter exceeds the $a_{\min}$ value by $m > 1$ times. If z_p has been found from Eq. (33) then for L_1 and L_2 we obtain (of course for a Gaussian beam) the following equations:

$$a\sqrt{\left(1 - \frac{z_{1,2}}{f}\right)^2 + \left(\frac{2z_{1,2}}{ka^2}\right)^2} = ma_{\min}, \tag{37}$$

where

$$z_1 = \int_0^{z_p+L_1} \frac{dt}{n(t)}, \quad z_2 = \int_0^{z_p+L_2} \frac{dt}{n(t)}.$$

In the particular case of $n'(t) = n_0$ and $L_p \le L$ one obtains from expression (37) that

$$L_2 = -L_1 = n_0 z_{p0} D_f \sqrt{m^2 - 1}. \tag{38}$$

In the case of an arbitrary $n(z)$ function the solution of equation (37) can easily be obtained by numerical methods. We shall not demonstrate such calculations here, while noting that in the general case the shape of the caustic of the Gaussian beam will not be symmetric with respect to the plane of waist.

Conclusion

In our opinion, the most interesting, among those presented in the paper, is the following one. There exists a certain set of the initial conditions (quite likely it is a unique one), which allows the problem on laser beam propagation through an inhomogeneous medium to be reduced to solution of a parabolic equation. It is quite important that the accuracy of simple enough analytical expressions describing the final result does not worsen even if the conditions (I.2) do not hold for sure. Let us summarize again the circumstances, which determine the applicability limits of the approach proposed.

1. The refractive index of the medium depends only on one coordinate, z.

2. The derivatives of the refractive index are limited by the conditions (24).

3. Laser beam propagates along the Z-axis.

4. The radiation has a narrow angular spectrum (paraxial beam).

5. Light refraction at the boundary between two media is considered only for normal incidence.

Violation of any of these requirements reduces to nothing the validity of the tricks used which we tried to follow up in the paper. On the other side, all the above-mentioned requirements themselves are quite typical and this enables us to hope that the outcome of this study may have certain practical value. In this context, we think that the main outcome of the work presented is the possibility shown of generalizing the results obtained by solving parabolic equation for the field in a homogeneous medium to the case considered in this paper.

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