Numerical simulation of laser radiation propagation in rain

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The model of phase screen for numerical simulation of laser propagation in rain is suggested. Since the optical radiation scattering on rain drops is considered within the diffraction approximation for the scattering phase function, the spectrum of correlation function of water drop dielectric permeability fluctuations in the atmosphere has a Gaussian form. The simulation results are compared with known theoretical estimates. The averaging effect of the receiver on measured parameters of laser radiation is investigated.

Introduction

Laser beams, propagating in the atmosphere, are distorted due to fluctuations of the medium dielectric permeability. These fluctuations are concerned not only with turbulent inhomogeneity of air density, but with the discrete component of the atmosphere, i.e., aerosol particles, fog, atmospheric precipitations.[1–4] Laser radiation scattering on rain drops results in beam broadening and intensity fluctuations inside it, like in the turbulent atmosphere.[4] In addition, radiation attenuates due to absorption by discrete scatterers.[4,5]

Statistical characteristics of the laser beam filed propagating in rain have been experimentally and theoretically investigated in Refs. 4–12. Laser beam propagation in rain, like in the turbulent atmosphere, can be described to parabolic approximation.[13]

In this work, we suggest a phase screen model,[14–16] considering scattering on discrete scatterers in a turbulent medium. Based on the exponential law of water-drop size distribution, accounting for the rain rate dependence,[1] equations are obtained for the correlation function spectrum of fluctuations of the effective dielectric permeability of discrete atmospheric component in rain. For the turbulent component, the Karman model is accepted, taking into account the influence of the inner scale of inhomogeneities.[17] The results of numerical simulation of laser beam propagation along a path in turbulent atmosphere with rain are presented and compared with known theoretical estimates. The averaging effect of the receiving aperture on the measured parameters of radiation, passing through the atmospheric layer with rain, is studied.

1. Phase screen model for describing radiation scattering in rain

Sizes of rain drops a_p are larger than the wavelength λ, $a_\text{p} \gg \lambda$; hence, the only forward scattering can be taken into account,[18] and the parabolic equation

$$2ik\frac{\partial U(\mathbf{r})}{\partial z} + \Delta_\perp U(\mathbf{r}) + k^2\left(\varepsilon_T(\mathbf{r}) + \varepsilon_\text{p}(\mathbf{r})\right)U(\mathbf{r}) = 0 \quad (1)$$

can be used to describe laser radiation propagation in rain.[4,10,17] Here $U(\mathbf{r})$ is the complex amplitude of the field $E(\mathbf{r}) = U(\mathbf{r})e^{ikz}$; $\mathbf{r} = (x, y, z)$, z is the coordinate towards propagation; $k = 2\pi/\lambda$ is the wave number; $(x, y) \equiv \boldsymbol{\rho}$ are the transversal coordinates;

$$\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2};\ \varepsilon_T(\mathbf{r}) = \langle\varepsilon_T(\mathbf{r})\rangle + \tilde{\varepsilon}_T(\mathbf{r}) \simeq 1 + \tilde{\varepsilon}_T(\mathbf{r})$$

is the dielectric permeability of the atmosphere without precipitations; $\tilde{\varepsilon}_T(\mathbf{r})$ is its turbulent fluctuations; $\varepsilon_\text{p}(\mathbf{r}) = \langle\varepsilon_\text{p}(\mathbf{r})\rangle + \tilde{\varepsilon}_\text{p}(\mathbf{r})$ is the dielectric permeability of atmosphere with water drops, $\langle\varepsilon_\text{p}(\mathbf{r})\rangle$ is its mean value, and $\tilde{\varepsilon}_\text{p}(\mathbf{r})$ is its fluctuations caused by random distribution of drops within air volume and of the drop number over radii and sizes.

When describing rains, the Low–Parsons size distribution[1–4] of the drop numbers versus rain rate is widely used:

$$p(a_\text{p}, J) = N_0(J)\exp[-\Lambda(J)a_\text{p}],$$
$$N_0(J) = 4.382 \cdot 10^6 J^{0.112}\ \text{m}^{-4};$$
$$\Lambda(J) = 5932J^{-0.182}\ \text{m}^{-1}, \quad (2)$$

where J is the rain rate, mm/h; a_p is the drop radius, m. In view of this distribution, the spectrum of correlation function of fluctuations $\tilde{\varepsilon}_\text{p}(\mathbf{r})$ depends on the rain rate, i.e., $\Phi_{\tilde{\varepsilon}_\text{p}} = \Phi_{\tilde{\varepsilon}_\text{p}}(\boldsymbol{\kappa}, J)$, and has the form[10]

$$\Phi_{\tilde{\varepsilon}_\text{p}}(\boldsymbol{\kappa}, J) = \frac{2}{\pi k^4}\int_0^\infty \text{d}a_\text{p}\, p(a_\text{p}, J)\left|f_0(\boldsymbol{\kappa}, a_\text{p})\right|^2, \quad (3)$$

where $\boldsymbol{\kappa}$ are the spectral coordinates; $f_0(\boldsymbol{\kappa}, a_\text{p})$ is the amplitude of wave scattering on an individual particle of a_p radius. Since water drops are larger in comparison with λ, the diffraction component of the scattering amplitude can represent it[2,3]; and equation (3) can be approximated to the square-law exponent

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$$\Phi_{\tilde{\varepsilon}_p}(\kappa, J) = A_p C_p^2(J)\exp\left[-\kappa^2 a_m^2/4\right], \quad (4)$$

where $C_p^2(J)$ is the constant similar to the structural characteristic of turbulent fluctuations of the dielectric permeability C_ε^2:

$$C_p^2(J) \cong 1.28 \cdot 10^{-12} J^{1.822} k^{-2}. \quad (5)$$

In this approximation, the real part $\mathrm{Re}\,\varepsilon(\mathbf{r}) = \varepsilon_T(\mathbf{r})$ turns out to be connected with turbulent fluctuations, and the imaginary one $\mathrm{Im}\,\varepsilon(\mathbf{r}) = \varepsilon_p(\mathbf{r})$ – with discrete scatterers (rain drops).

The parameter a_m in Eq. (4) is the scale of inhomogeneities of a continuum medium equivalent to a discrete scattering medium with the characteristic drop scale a_m. It is possible to show (Fig. 1) that approximation (4) is the best square-exponent approximation of spectrum (3), where the volume median radius of rain drops[1,4] (Fig. 1, curve *4*) is chosen as the characteristic scale a_m:

$$a_m = 6.19 \cdot 10^{-4} J^{0.182}. \quad (6)$$

The spectra with mean (curve *2*) and mean-square radii (curve *3*) are shown in Fig. 1 for comparison.

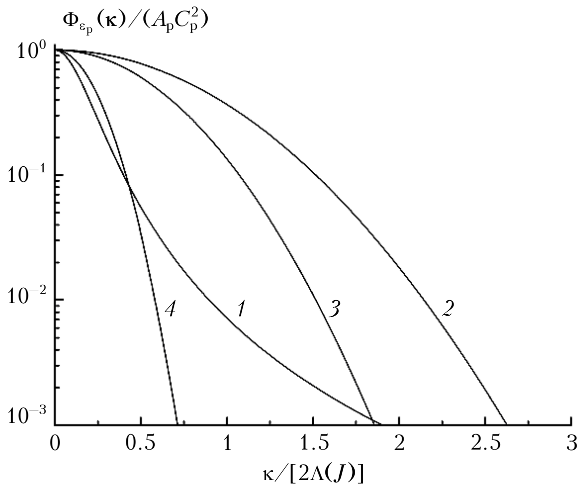

Fig. 1. The normalized spectrum of correlation function of fluctuations of effective dielectric permeability of a discrete scattering medium with the Low–Parsons size distribution of particles: exact value (*1*) (Eq. (3)); approximation for mean (*2*), mean-square (*3*), and median (Eq. (6)) (*4*) drop radii.

The mean value $\langle\varepsilon_p(\mathbf{r})\rangle$ determines the total radiation attenuation in rain

$$\tau = kz\langle\varepsilon_p(\mathbf{r})\rangle \cong 2.638 \cdot 10^{-4} zJ^{0.658}. \quad (7)$$

The constant A_p in Eq. (4) can be defined as the correlation factor between $C_p^2(J)$ and the variance of plane wave intensity fluctuations $\sigma_I^2(\tau)$ to the first approximation of the method of smooth perturbations (MSP), $\tau \ll 1$:

$$A_p = \frac{\tau(L,J)}{\pi^2 k^3 C_p^2 f_2(a_m)}, \quad f_2(a_m) = \frac{4L}{ka_m^2} - \arctan\left(\frac{4L}{ka_m^2}\right), \quad (8)$$

where L is the length of laser radiation propagation path in the atmosphere with precipitations. Thus, $C_p^2(J)$ correlates with the amplitude of spectrum (4) with the rain rate J, while A_p – with conditions on propagation path.

Representing atmospheric precipitations as an equivalent continuum medium with the characteristic scale a_m, where radiation scattering is accompanied by its attenuation, laser beam propagation in turbulent atmosphere with precipitations can be considered as similar to those in a continuum medium with three characteristic scales (outer and inner scales of turbulent inhomogeneities; as well as rain drop scale a_m); and the splitting technique[15] can be applied to Eq. (1). Within this technique, the length Δz of each path layer should satisfy the condition

$$\tau(\Delta z) \ll 1. \quad (9)$$

In the middle of each layer, a thin screen is located, determining distortions of the field, passed through the layer

$$U(\Delta z, \rho) = U(0, \rho)\exp\left\{\frac{ik}{2}\int_0^{\Delta z}\frac{i\tau(z)}{kz}\mathrm{d}z\right\} e^{i\Psi_t(\rho) - \Psi_p(\rho)}, \quad (10)$$

where $U(0,\rho)$ is the incident field; $U(\Delta z, \rho)$ is the field passed through the layer; $\Psi_t(\rho)$ is the random phase incursion when scattering on turbulent inhomogeneities; $e^{-\Psi_p(\rho)}$ is the random variation of field amplitude caused by a discrete scatterer. The quantity

$$\exp\left\{-\frac{k}{2}\int_0^{\Delta z}\frac{\tau(z)\mathrm{d}z}{kz}\right\} = \exp\left\{-\frac{\tau(\Delta z)}{2}\right\} \quad (11)$$

defines the field attenuation by the medium discrete scattering component. Thus, the influence of the discrete component of a random medium on a laser beam, passed through the layer Δz, is defined by the amplitude screen

$$e^{-\tau(\Delta z)/2 - \Psi_p(\rho)}. \quad (12)$$

As is seen from Eqs. (10) and (12), a thin screen is to be amplitude-phase. However, the solution of the equation for the second moment of the field $\Gamma_2(z, \rho_1, \rho_2)$ with replacing the factor $e^{-\Psi_p(\rho)}$ by the phase screen $e^{i\tilde{\Psi}_p(\rho)}$ ($\tilde{\Psi}_p(\rho)$ is the random phase incursion, acquired by the wave while passing through the layer, coincides with the rigorous solution of this equation to the Markov approximation.[12] Using this fact, consider only the field phase fluctuations, performing the above factor replacing in Eq. (12).

Since $\tau(\Delta z)$ is independent of transversal coordinates, total attenuation of a laser beam passed a path of L in length can be defined by the factor

$e^{-\tau(L)/2}$ after executing all the steps Δz along the path, consisting in sequential multiplication of the field by the phase screen and free field diffraction between screens.

A two-dimensional $N \times N$ phase screen is generated by the equation

$$\Psi(j\Delta x, l\Delta y, J) = \frac{1}{N} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} \sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} [\xi_{nm} + i\zeta_{nm}] \times$$
$$\times \exp\left[2\pi i \left(\frac{jn + lm}{N}\right)\right] \sqrt{\frac{\Phi_\Psi\left(\frac{n}{N\Delta x}, \frac{m}{N\Delta y}, J\right)}{\Delta x \Delta y}}, \quad (13)$$

where Δx and Δy are the distances between nodes; (ξ_{nm}, ζ_{nm}) are the independent random sequence with zero mean and unit variance. The spectrum of correlation phase function $\Phi_\Psi(\boldsymbol{\kappa}_\perp, J)$ is related with the spectrum of correlation function of dielectric permeability fluctuations as[14–16]:

$$\Phi_\Psi(\boldsymbol{\kappa}_\perp, J) = \frac{k^2 \Delta z}{4} \Phi_{\tilde{\varepsilon}}(\boldsymbol{\kappa}, 0, J). \quad (14)$$

Here $\Phi_{\tilde{\varepsilon}}(\boldsymbol{\kappa}, 0, J)$ is the total spectrum of correlation function of fluctuations of the air dielectric permeability and water drops,

$$\Phi_{\tilde{\varepsilon}}(\boldsymbol{\kappa}_\perp, \kappa_z = 0, J) =$$
$$= \Phi_{\tilde{\varepsilon}}(\boldsymbol{\kappa}_\perp, \kappa_z = 0) + \Phi_{\tilde{\varepsilon}_p}(\boldsymbol{\kappa}_\perp, \kappa_z = 0, J), \quad (15)$$

$\Phi_{\tilde{\varepsilon}_T}(\boldsymbol{\kappa}_\perp, \kappa_z = 0)$ is the Karman spectrum.[17]

Now determine step and size of the numerical grid in a transverse plane, the length Δz of the path layer L for a given radiation parameters of the source, and such medium parameters as radiation wavelength λ, rain rate J, turbulence parameter C_ε^2. The beam radius a_0 and wave front curvature F are to be chosen so that to satisfy the listed below conditions. First, the grid size in the transverse plane is to exceed the beam diameter and the drop diffraction radius at far end of a path of the length $z = L$:

$$N\Delta x \geq \max\left(2ha_0, \frac{h\lambda z}{a_\text{m}(J)}\right), \quad (16)$$

the parameter h is an integer-number resolution common for all scales. The relation between beam and drop radii is defined in comparison with the Fresnel radius:

$$ha_\text{m}(J) \geq \sqrt{z/k}, \quad a_0 \geq h\sqrt{z/k}. \quad (17)$$

Finally, the transverse resolution is determined from the relation between characteristic scales of medium dielectric permeability fluctuations and the wave front curvature of the beam field F:

$$\Delta x \leq \min\left(\rho_T(C_\varepsilon^2, z)/h, a_\text{m}/h, \frac{1}{8}\sqrt{\frac{\pi\lambda F}{N}}\right), \quad (18)$$

where $\rho_T(C_\varepsilon^2, z) = (0.3 C_\varepsilon^2 k^2 z)^{-3/5}$ is the coherence radius of a plane wave in a turbulent medium. According to Eq. (17), the beam radius is larger than the drop one; hence, there is no need in using the variable a_0 in Eq. (18) to determine the grid step Δx.

2. Simulation results

The above algorithm was tested for a plane wave (Fig. 2) and a Gaussian beam (Fig. 3) in the mode of weak fluctuations of the radiation intensity. The variance of plane wave intensity fluctuations $\sigma_{I,\text{pw}}^2$ at small optical depths $\tau \ll 1$ is determined by τ (see Fig. 2)[7,9,11,12]:

$$\sigma_{I,\text{pw}}^2 \simeq \tau, \quad \tau \ll 1. \quad (19)$$

At $\tau > 1$, $\sigma_{I,\text{pw}}^2$ is comparable with unity and tends to 1 at the further increase of the optical depth $\tau \gg 1$ (see Fig. 2)[10,12]:

$$\sigma_{I,\text{pw}}^2 \simeq 1, \quad \tau \gg 1. \quad (20)$$

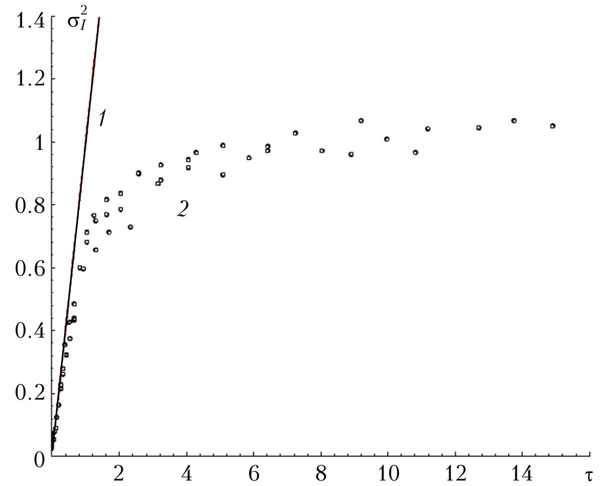

Fig. 2. Relative variance of plan wave intensity fluctuations: calculation to the first approximation of MSP (*1*); simulation (*2*).

An addition effect of the turbulence on the variance of rain-propagating radiation intensity fluctuations manifests itself as displacement of the above τ dependence along X-axis. For a rain-propagating Gaussian beam, the variance of intensity fluctuations on its axis to the first approximation of MSP is determined by the equation[9,12]:

$$\sigma_I^2(z) \simeq \frac{\tau k a_0 a_\text{m}}{2z} \sqrt{(1-\mu)^2 + \Omega_0^{-2}} \times$$
$$\times \arctan\left[\frac{2z}{k a_0 a_\text{m}^2} \left\{(1-\mu)^2 + \Omega_0^{-2}\right\}^{-1/2}\right], \quad (21)$$

where $\mu = z/F$; $\Omega_0 = \frac{ka_0^2}{z}$ is the Fresnel number. As is seen from Fig. 3, the simulation results coincide with the variance σ_I^2 calculated by Eq. (21) within the domain $\tau < 0.5$.

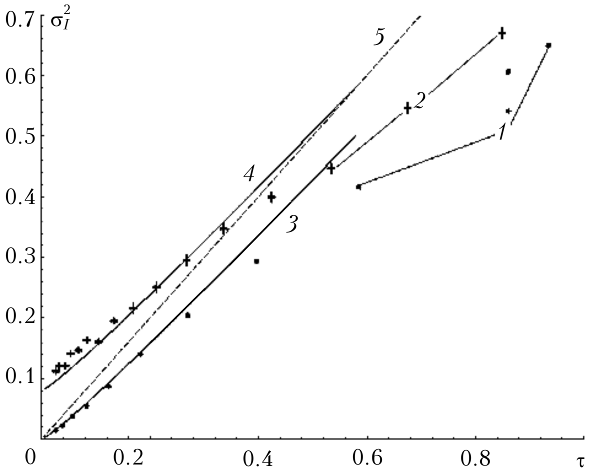

Fig. 3. Relative variance of intensity fluctuations of the collimated beam $a_0 = 3$ cm (the Fresnel number $\Omega_0 = 57$) in the atmosphere with precipitations: rain in the homogeneous atmosphere (simulation and calculation to MSP approximation) (*1* and *3*); turbulent atmosphere with precipitations, plane wave scintillation index $\beta^2_{0,\mathrm{pw}} = 0.31C_\varepsilon^2 k^{7/6}z^{11/6} = 0.1$ (simulation and MSP) (*2* and *4*); rain in the homogeneous atmosphere, plane wave, MSP (*5*).

In practice, when measuring the variances of intensity fluctuations, the size of the receiver aperture a_r and different diaphragms, used in the experiment, make an averaging effect on the measured parameters (mean intensity, variance of intensity fluctuations) because the signal in the receiver is, in fact, the power

$$P = \int_S I(\rho)\,\mathrm{d}(\rho), \tag{22}$$

where S is the receiver's area, and the division of P by the receiver area yields the intensity averaged over S area

$$\tilde{I} = \int_S I(\rho)\,\mathrm{d}(\rho)/S. \tag{23}$$

Therefore, the parameter, measured in the experiment, can be, for example, the variance of radiation flux fluctuations

$$\sigma_{\tilde{I}}^2 = \left\langle \tilde{I}^2 \right\rangle / \left\langle \tilde{I} \right\rangle^2 - 1 \tag{24}$$

instead of the variance of intensity fluctuations.

Figure 4 shows the variances of received radiation flux fluctuations as functions of the receiver apertures ($a_\mathrm{r} = 0.04$, 0.24, and 0.95 mm). Note, that $\sigma_{\tilde{I}}^2$ decreases by 2–3 times at $\tau \approx 0.1$ already at $a_\mathrm{r} \approx 0.5 \div 0.7$ mm.

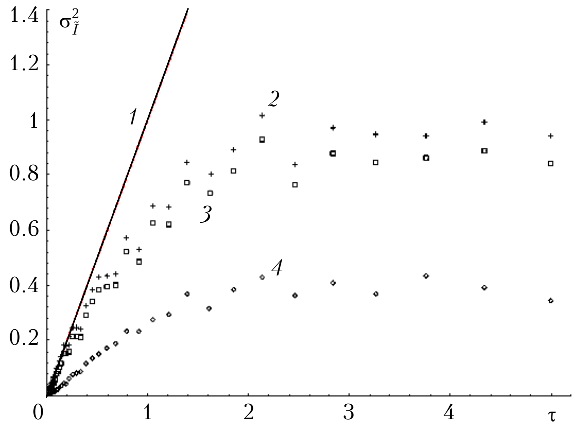

Fig. 4. Variance of the received radiation flux fluctuations (plane wave) as a function of optical depth and receiver size to the MSP approximation (*1*) and at $a_\mathrm{r} = 0.04$ (*2*), 0.24 (*3*), and 0.95 mm (*4*).

Hence, for the chosen receiver size, there is a limiting value of optical depth, for which $\sigma_{\tilde{I}}^2$ is still measurable (see Fig. 4). For $a_\mathrm{r} = 0.95$ mm, the variance of intensity fluctuations tends not to 1, like at $a_\mathrm{r} = 0.4$ mm, but to a level significantly less than unity.

Conclusion

The model of phase screen is suggested and verified for numerical simulation of laser propagation in the atmosphere with precipitations. The model is based on the diffraction approximation of the scattering phase function; the phase screen has the Gaussian profile of the correlation function of effective dielectric permeability fluctuations. The spectrum scale is determined by the volume median radius of water drops.

The averaging effect of the receiver aperture on the variation of intensity fluctuations has been studied. The results of numerical simulations of laser beam propagation in rain and turbulent atmosphere with precipitations by the method of phase screen are presented. The simulation results agree with the well-known theoretical estimates of the variance of beam intensity fluctuations to the MSP approximation for both components of atmosphere with precipitations.

It is shown that for the chosen receiver size there is a limiting value of the optical depth, for which the variation of intensity fluctuations is still measurable.

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