# INTERPRETATION OF THE ANOMALOUS BACKSCATTERING EFFECT

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*Received April 13, 1990*

*We derive a formula, based on the physical optics method, for the lidar backscattering coefficient which describes signals specularly reflected from oriented ice plates. This formula relates the anomalously high amplitudes of the reflected signals to the basic parameters of a polydisperse crystal medium. We also analyze the possibility of estimating the mean size of the crystals using measurements with a single-frequency lidar.*

When sensing natural crystalline clouds by a monostatic lidar, experimenters quite often have to overcome certain problems in detecting the backscattered signal. In particular, when clouds are scanned by a lidar, some of them scatter back to the receiver such unexpectedly strong pulses that it may either be overexposed or totally destroyed. Such strong backscattered signals were first observed during laboratory experiments.[1] Strong backscattering was subsequently detected many times in the course of lidar sensing of natural crystalline clouds.[2–5,7]

At present, following the generally accepted terminology, such strong backscattering is called the "anomalous backscattering effect." The backscattering coefficients corresponding to such anomalous signals sometimes reach several or several tens of inverse kilometers.[4,7] The anomalous backscattering is always found to result from specular reflection of the signal from a system of oriented plate crystals. Although the mechanism of this backscattering has been known for a long time, satisfactory theoretical estimates of high-amplitude lidar reflection are still lacking. This circumstance has stimulated us to propose our own model for interpreting anomalous backscattering.

Using the physical optics approach, we have obtained analytical formulas for the extinction and backscattering cross sections of polarized optical radiation by a round plate.[8] These are used below to derive the extinction and backscattering coefficients for polarized lidar radiation interacting with a system of oriented plate crystals. In particular, we present analytical formulas for a combination of the backscattering coefficients $\beta_{\pi j}$ ($j$ = 1, 2, 3, and 4),[9] each of which is proportional to the respective Stokes vector parameter of the reflected signal. Numerical computations show that internal reflections of an electromagnetic wave in a crystal cannot noticeably affect the backscattering coefficients, so that the latter may be represented in a simpler form. The backscattering coefficient $\beta_{\pi 1}$ thus may be expressed as being proportional to the total intensity of the return signal

$$\beta_{\pi_1} = A_1 \frac{k^2}{\pi} \int\limits_0^\infty N(a)\left[\frac{1+\cos 2\beta}{2}\cdot \pi a^2 \cos\beta G(\beta)\right]^2 da, \quad (1)$$

$$A_1 = \frac{|R_\parallel|^2 + |R_\perp|^2}{2} + \frac{I_2}{I_1}\frac{|R_\parallel|^2 - |R_\perp|^2}{2}\cos 2\gamma - \frac{I_3}{I_1}\frac{|R_\parallel|^2 - |R_\perp|^2}{2}\sin 2\gamma, \quad (2)$$

$$G(\beta) = \frac{2J_1(ka\sin 2\beta\cos\beta)}{ka\sin 2\beta\cos\beta}. \quad (3)$$

Here $I_1$, $I_2$, and $I_3$ are the first three Stokes vector parameters, which characterize the polarization state of the incident radiation; β is the narrow angle between the direction of sensing and the normal to the lower plate face; γ is the angle by which the components of the incident electromagnetic wave are rotated with respect to the incidence plane; $k = 2\pi/\lambda$ is the wave number; $J_1(t)$ is the Bessel function of first order; and, $N(a)$ is the plate radius ($a$) distribution function. The Fresnel reflectances $R_\parallel$ and $R_\perp$ are given by the relations

$$R_\parallel = \frac{\tilde{n}\cos\beta - \cos\theta}{\tilde{n}\cos\beta + \cos\theta}, \quad R_\perp = \frac{\cos\beta - \tilde{n}\cos\theta}{\cos\beta + \tilde{n}\cos\theta}, \quad (4)$$

where $\tilde{n} = n + i\kappa$ is the complex refractive index of the plate crystals; θ is the complex refraction angle described by Snell's law: $\sin\theta = \sin\beta / \tilde{n}$.

Assume that the lidar pulses are directed toward the zenith and the plate crystals be horizontally oriented. The angle β should then be set equal to zero, significantly simplifying the above formulas. In particular, for $A_1$ and $G(\beta)$ we obtain

$$A_1 = \left|\frac{\tilde{n}-1}{\tilde{n}+1}\right|^2, \quad \lim_{\beta\,\infty} G(\beta) = 1. \quad (5)$$

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Taking Eqs. (5) into account, the expression for the backscattering coefficient becomes

$$\beta_{\pi_1}\Big|_{\beta=0} = \beta_a = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2 \frac{k^2}{\pi} \pi^2 \int_0^\infty N(a) a^4 da . \tag{6}$$

For definiteness, we assume that the function $N(a)$ follows a $\gamma$-distribution,[10] i.e.,

$$N(a) = N \cdot \frac{\mu^{\mu+1}}{\Gamma(\mu + 1)} \cdot \frac{1}{a_m} \left[ \frac{a}{a_m} \right]^\mu e^{-\mu(a/a_m)} . \tag{7}$$

This assumption does not detract from the generality of our problem, since the $\gamma$-distribution adequately describes the actual size distribution of the crystals. Relation (7) includes the following parameters: the number density of plate crystals, $N$; the plate radius at which the distribution $N(a)$ is maximum, $a_m$; and, $\mu$ is a dimensionless parameter characterizing the sharpness of the given maximum. Note that 'elation (7) yields an analytical expression for $N(a)$, so that further simplifications become possible in Eq. (6), which result in the following algebraic expression for the anomalous backscattering

$$\beta_a = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2 N \frac{k^2}{\pi} (\pi a_m^2)^2 \prod_{j=1}^{4} \left[ 1 + \frac{j}{\mu} \right] . \tag{8}$$

To analyze the experimental data, relation (8) must be transformed so that it is expressed in terms of the average plate radius $\bar{a}$. If the plates size distribution function is available, this parameter may be found from the well-known relation

$$\bar{a} \int_0^\infty N(a)\, da = \int_0^\infty a N(a)\, da. \tag{9}$$

If the function $N(a)$ follows the $\gamma$-distribution, a formula for $\bar{a}$ may be obtained from Eq. (9)

$$\bar{a} = a_m \left[ 1 + \frac{1}{\mu} \right]. \tag{10}$$

Then, relation (8) for the anomalous backscattering coefficient $\beta_a$ becomes

$$\beta_a = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2 N \frac{k^2}{\pi} (\pi \bar{a}^2)^2 \prod_{j=1}^{3} \left[ 1 + \frac{j}{(\mu + 1)} \right] . \tag{11}$$

Thus, the amplitude of the specularly reflected signal may be estimated if the average radius and the particle number density in the scattering volume are known.

Table I lists the anomalous backscattering coefficients $\beta_a$ computed for several real values of the parameters $\bar{a}$ and $N$ for natural crystalline clouds.

*TABLE I. Calculational results for the anomalous backscattering coefficient $\beta_a$ ($km^{-1}$) for a system of oriented plates (computed for the refractive index $\tilde{n} = 1.31 + i \cdot 10^{-3}$, wavelength $\lambda$ = 0.694 μm).*

| μ | $\bar{a}$, μm | | | | |
|---|---|---|---|---|---|
| | 37 | 100 | 150 | 200 | 250 |
| | $N$, $l^{-1}$ | | | | |
| | 0.8 | 25 | 20 | 15 | 10 |
| 1 | $5.22 \cdot 10^1$ | $8.69 \cdot 10^4$ | $3.52 \cdot 10^5$ | $8.35 \cdot 10^5$ | $1.36 \cdot 10^6$ |
| 2 | $3.09 \cdot 10^1$ | $5.15 \cdot 10^4$ | $2.09 \cdot 10^5$ | $4.95 \cdot 10^5$ | $8.05 \cdot 10^5$ |
| 3 | $2.28 \cdot 10^1$ | $3.80 \cdot 10^4$ | $1.54 \cdot 10^5$ | $3.65 \cdot 10^5$ | $5.94 \cdot 10^5$ |
| 4 | $1.87 \cdot 10^1$ | $3.12 \cdot 10^4$ | $1.26 \cdot 10^5$ | $2.99 \cdot 10^5$ | $4.87 \cdot 10^5$ |
| 5 | $1.62 \cdot 10^1$ | $2.71 \cdot 10^4$ | $1.10 \cdot 10^5$ | $2.60 \cdot 10^5$ | $4.23 \cdot 10^5$ |
| 6 | $1.46 \cdot 10^1$ | $2.43 \cdot 10^4$ | $9.86 \cdot 10^4$ | $2.34 \cdot 10^5$ | $3.80 \cdot 10^5$ |
| 7 | $1.35 \cdot 10^1$ | $2.24 \cdot 10^4$ | $9.08 \cdot 10^4$ | $2.15 \cdot 10^5$ | $3.50 \cdot 10^5$ |
| 8 | $1.26 \cdot 10^1$ | $2.10 \cdot 10^4$ | $8.50 \cdot 10^4$ | $2.02 \cdot 10^5$ | $3.28 \cdot 10^5$ |
| 9 | $1.19 \cdot 10^1$ | $1.99 \cdot 10^4$ | $8.06 \cdot 10^4$ | $1.91 \cdot 10^5$ | $3.11 \cdot 10^5$ |
| 10 | $1.14 \cdot 10^1$ | $1.90 \cdot 10^4$ | $7.71 \cdot 10^4$ | $1.83 \cdot 10^5$ | $2.97 \cdot 10^5$ |
| 11 | $1.10 \cdot 10^1$ | $1.83 \cdot 10^4$ | $7.42 \cdot 10^4$ | $1.76 \cdot 10^5$ | $2.86 \cdot 10^5$ |

These values of the anomalously high backscattering coefficients are quite sufficient to explain photodetector overexposure in most of the experiments. As of the present time only comparatively low values of $\beta_a$ have been experimentally recorded, reflecting the low concentrations of particles in the scattering volumes and their small sizes. In particular, the experimentally measured values of $\beta_a$ reached 17 $km^{-1}$ (Ref. 4) for $N$ = 0.8 $l^{-1}$ and $\bar{a}$ = 36 μm. The first column in Table I shows the calculated coefficient $\beta_a$ for the same values of the parameters $N$ and $\bar{a}$ of the crystal medium. It can be easily seen that the theoretical and experimental data are in both qualitative and quantitative agreement.

The crystal number density $N$ enters into formula (11) for $\beta_a$ linearly, so that this coefficient may be directly estimated for crystal ensembles with arbitrary number densities from the data of Table I. In particular, the values of $\beta_a = 0.110$ km$^{-1}$ and $\beta_a = 0.522$ km$^{-1}$, which are 100 times less than the extreme values from the first column of Table I, correspond to the crystal number densities in the scattering volume ($N = 0.008$ l$^{-1}$) being 100 times lower. Similar estimates may be given for crystals of a different average size (see Table I). The results already outlined provide a sufficient basis to conclude that even a few relatively small but horizontally oriented ($\bar{a} = 37$ μm) ice, plates (eight of them per cubic meter) would produce a high-amplitude backscattered signal. Moreover, such crystals may remain visually unnoticeable, but the lidar pulses reflected from these mirror-like objects, received by the photodetector, would have high amplitudes. Such a situation arises quite often during actual atmospheric sensing. For example, the specialists at the Institute of Atmospheric Optics of the Siberian Branch of the USSR Academy of Sciences have recorded backscattered lidar signals backscattered from an altitude of 4 km, which correspond to backscattering coefficients of up to 0.3 km$^{-1}$, during vertical sounding of the cloudless clear atmosphere at $\lambda = 0.532$ μm. Note that an altitude of 4 km corresponds to the lower level of mid-layer cloudiness, and in the case under discussion, it was the bottom of the still unformed crystalline cloud.

*TABLE II. Calculational results for the anomalous backscattering coefficient $\beta_a$(km$^{-1}$) of oriented plates for circular polarization of the incident wave ($\tilde{n} = 1.31 + i\cdot 10^{-3}$, $\mu = 5$).*

| $\beta°$ | $\bar{a}$, μm | | | | |
|---|---|---|---|---|---|
| | 37 | 100 | 150 | 200 | 250 |
| | $N$, l$^{-1}$ | | | | |
| | 0.8 | 25 | 20 | 15 | 10 |
| | $\lambda = 0.694$ μm | | | | |
| 0 | $1.62\cdot10^{1}$ | $2.71\cdot10^{4}$ | $1.10\cdot10^{5}$ | $2.60\cdot10^{5}$ | $4.23\cdot10^{5}$ |
| 0.01 | $1.61\cdot10^{1}$ | $2.51\cdot10^{4}$ | $9.25\cdot10^{4}$ | $1.93\cdot10^{5}$ | $2.68\cdot10^{5}$ |
| 0.02 | $1.56\cdot10^{1}$ | $2.01\cdot10^{4}$ | $5.75\cdot10^{4}$ | $8.70\cdot10^{4}$ | $8.46\cdot10^{4}$ |
| 0.05 | $1.26\cdot10^{1}$ | $5.42\cdot10^{3}$ | $5.82\cdot10^{3}$ | $4.84\cdot10^{3}$ | $3.80\cdot10^{3}$ |
| 0.1 | 6.27 | $5.04\cdot10^{2}$ | $5.65\cdot10^{2}$ | $5.65\cdot10^{2}$ | $4.71\cdot10^{2}$ |
| 0.2 | $9.06\cdot10^{-1}$ | $5.89\cdot10^{1}$ | $7.05\cdot10^{1}$ | $7.04\cdot10^{1}$ | $5.86\cdot10^{1}$ |
| 0.5 | $4.46\cdot10^{-2}$ | 3.75 | 4.49 | 4.49 | 3.74 |
| 1 | $5.56\cdot10^{-3}$ | $4.68\cdot10^{-1}$ | $5.61\cdot10^{-1}$ | $5.61\cdot10^{-1}$ | $4.68\cdot10^{-1}$ |
| 2 | $6.93\cdot10^{-4}$ | $5.85\cdot10^{-2}$ | $7.02\cdot10^{-2}$ | $7.02\cdot10^{-2}$ | $5.85\cdot10^{-2}$ |
| 5 | $4.45\cdot10^{-5}$ | $3.76\cdot10^{-3}$ | $4.51\cdot10^{-3}$ | $4.51\cdot10^{-3}$ | $3.76\cdot10^{-3}$ |
| | $\lambda = 10.6$ μm | | | | |
| 0 | $6.95\cdot10^{-2}$ | $1.16\cdot10^{2}$ | $4.70\cdot10^{2}$ | $1.11\cdot10^{3}$ | $1.81\cdot10^{3}$ |
| 0.1 | $6.92\cdot10^{-2}$ | $1.12\cdot10^{2}$ | $4.37\cdot10^{2}$ | $9.78\cdot10^{2}$ | $1.48\cdot10^{3}$ |
| 0.5 | $6.22\cdot10^{-2}$ | $5.42\cdot10^{1}$ | $9.86\cdot10^{1}$ | $9.65\cdot10^{1}$ | $6.96\cdot10^{1}$ |
| 1 | $4.52\cdot10^{-2}$ | $1.01\cdot10^{1}$ | 9.32 | 8.67 | 7.19 |
| 5 | $6.93\cdot10^{-4}$ | $5.76\cdot10^{-2}$ | $6.90\cdot10^{-2}$ | $6.89\cdot10^{-2}$ | $5.74\cdot10^{-2}$ |
| 10 | $8.65\cdot10^{-5}$ | $7.26\cdot10^{-3}$ | $8.71\cdot10^{-3}$ | $8.71\cdot10^{-3}$ | $7.26\cdot10^{-3}$ |
| 20 | $1.13\cdot10^{-5}$ | $9.58\cdot10^{-4}$ | $1.15\cdot10^{-3}$ | $1.15\cdot10^{-3}$ | $9.58\cdot10^{-4}$ |
| 30 | $3.80\cdot10^{-6}$ | $3.21\cdot10^{-4}$ | $3.85\cdot10^{-4}$ | $3.85\cdot10^{-4}$ | $3.20\cdot10^{-4}$ |

The amplitude of the anomalously backscattered signal typically drops when the beam axis is shifted off zenith. To study this feature, we calculated the backscattering coefficient $\beta_{\pi 1}$ for various deviation angles $\beta$ of the lidar beam axis from the vertical. The coefficient $A_1$ in relation (1) describing $\beta_{\pi 1}$ was given in the form

$$A_1 = \frac{|R_{\parallel}|^2 + |R_{\perp}|^2}{2}, \quad (12)$$

corresponding to the circular polarization ($I_2 = I_3 = 0$) of the incident wave. The numerical data presented in Table II demonstrate that even a minor deviation of the lidar beam axis may result in very significant changes in the backscattering coefficient. For example, during sensing of ice plate crystals in the visible spectral range ($\lambda = 0.697$ μm) deviation of the lidar beam axis from the vertical by only 1° results in a decrease of the reflected signal amplitude by 4–6 orders of magnitude. Similar changes of the backscattered signal in the IR range ($\lambda = 10.6$ μm) are somewhat slower. Indeed, the same change of the backscattered amplitude by 4–6 orders of magnitude here corresponds to the scanning angles of 10–20.

Note that at small values of the angle β the steepenss of the $\beta_{\pi 1}(\beta)$ dependence (its rate of decay) is uniquely determined by the average radius $\overline{\alpha}$ of the plate crystals. This result makes it possible to estimate the average size of the cloud crystals from relative changes in the backscattered signal when the beam is scanned about the vertical.

Difficulties of implementation and flaws in the semiempirical interpretation schemes prevent a detailed analysis of the results on the anomalous backscattering, even though they may reflect quite vividly the microstructure of crystalline clouds. Platt et al.[7] have suggested that certain data on the microstructure could be obtained from the depolarization ratio. However, in the first approximation, this ratio does not contain information on the microstructure of plate crystals.[9] Therefore we believe that it is still more promising to estimate such cloud parameters as the concentration of crystals and their average radius from an analysis of the anomalous backscattering coefficient. Moreover, the missing information may be deduced from the known physical properties of the crystal medium. For example, an interrelation has been found[6] between the cloud temperature and its crystal concentration and their average size. This circumstance significantly broadens the possibilities of remote sensing of the cloud microstructure. As for the depolarization ratio, it may be used for retrieving the crystals' refractive index.

Remote sensing of a polydisperse system is traditionally associated with the operation of a multifrequency lidar. Within the framework of these concepts we suggested in Ref. 9 that comprehensive data on a crystalline cloud might be obtained from simultaneous measurements by polarization and multifrequency lidars. It is assumed that first the refractive index and the crystal orientation are retrieved from the polarization lidar data. Then the problem may be posed of retrieving the average size of the crystals from the multifrequency lidar signal. We have demonstrated in the present paper that the system of oriented crystals scatters light in a special way, so that an angular scanning lidar system may be used for remote studies of the crystalline cloud microstructure at a single frequency. In other words, a single-frequency polarization lidar will essentially yield all of the information on the principal parameters of a crystalline cloud.

To realize the full potential of a polarization lidar, its measurement routine should combine two opposing features. On the one hand, to estimate the microstructure parameters of a crystalline cloud, backscattered signals of anomalously high amplitudes should be recorded. On the other hand, one must bear in mind that the depolarization ratio has the larger information! content in relation to the refractive index, the further the lidar axis is from the vertical, i.e., the lower is the backscattered signal. In addition, the possibility should be envisaged of scanning the beam during the experiment, which leads to additional problems in obtaining all the information on a crystalline cloud from polarization lidar measurements. However, all of the difficulties of employing a monostatic, polarization lidar for comprehensive sensing of crystalline clouds are eliminated when one turns to a bistatic sensing scheme. The bistatic polarization lidar sensor is capable of producing a high-amplitude polarization signal, which, contains all of the information on the optical and microphysical properties of the studied natural cloud.

The authors express their sincere gratitude to V.S. Shamanaev for providing experimental data.

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