COMPARISON OF THE CALCULATED AND EXPERIMENTALLY MEASURED BACKSCATTERING PHASE MATRICES OF CRYSTAL CLOUDS

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In this paper we present some results of a comparison made between the backscattering phase matrices of crystal clouds measured experimentally with the polarization lidar "Stratosfera 1M" and calculated using model polydisperse ensembles of partially oriented ice cylinders. The comparison shows that the model used describes the main experimental facts quite satisfactorily. Thus, in particular, the results on the direction of preferred orientation of particles and on the degree of their orientation calculated using the model well agree with those reconstructed from the experimental lidar data.

The extinction and scattering coefficients of a crystal cloud may strongly depend on the polarization state of radiation and on the angle, at which the radiation is incident on the cloud layer.[1,2] The matter is that crystal particles are anisometric that makes them optically anisotropic. So, if the crystal particles of a cloud have a preferred orientation in space the cloud, as a whole, is also optically anisotropic.

Earlier, in Refs. 3 and 4, we have described a possibility of acquiring information on the preferred orientation of particles' axes, as well as about the degree of such an orientation from the polarization lidar data. The direction of preferred orientation is described with an angle α that may be counted, for instance, from the x axis of a lidar polarization basis.[5] This direction may surely be determined in a geodesic coordinate system provided that the orientation of the lidar polarization basis with respect to this system is known. The degree of particles' orientation, k, is the parameter of the distribution function

$$f(\varphi, \alpha, k) = \exp\,[k \cos 2(\varphi - \alpha)]/\pi\, I_0(k), \qquad (1)$$

which is known as Mises distribution.[6] In the case of an ensemble of particles this formula describes the distribution of particle axes' orientation about some modal direction α, that is called the direction of preferred orientation. The function $I_0(k)$ in formula (1) is the modified Bessel function of the first kind and zeroth order. The distribution function (1) has specific features that at $k \to 0$ it takes the value $1/\pi$ from the interval $[-\pi/2, \pi/2]$ while at $k \to \infty$ it approaches the delta function $f \to \delta(\varphi - \alpha)$. At $k = 10$ practically all particles of the ensemble have orientation along the direction at the angle α.

If the parameters α and k may be found experimentally it is worth addressing on how this can be used for assessing the conditions for propagation of radiation and/or how the polarization of radiation may affect its extinction and scattering by such an ensemble.

The calculations of scattering phase matrices for model ensembles of crystal cloud particles with a preset orientation may enable one to find answers to these questions. Of course, in so doing one should take into account the variety of particle shapes and size-distributions in order to obtain the scattering phase matrices that are close to those of actual clouds. Fortunately, there are good grounds for making such calculations at present. Thus, for example, one may find in the literature different approaches to calculation of the scattering phase matrices of individual plates and hexagonal columns. The calculations of scattering phase matrices of polydisperse ensembles of such particles are the technical problem of mathematical modeling, though nontrivial and laborious.

This difficult task has not yet been achieved, so in the meantime one can try to reveal certain peculiarities in light scattering by the anisotropic cloud ensembles using some simplified models. In references 7 to 9 one may find the calculations of scattering phase matrices of polydisperse ensembles of cylindrical ice particles. In our opinion these matrices should allow one to reveal certain specific features of light scattering on hexagonal columns. The only question is to what extent these features are realistic. It is natural, in this connection, to try to compare the calculated properties with the experimentally measured ones.

At present we have experimental data on the backscattering phase matrices of crystal clouds and can compare them with the matrices calculated for model ensembles of oriented cylindrical particles. The calculations of the backscattering phase matrices of these ensembles were made assuming the radiation incidence angle, γ, on the cloud layer to be from 0 to 90 degrees in a ten-degree step. The angle of preferred

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orientation of particles, α, was set, in the calculations, within the interval from 0 to 180 degrees in a step of 3 degrees. The parameter k of the Mises distribution varied from 0 to 3 in a step of 0.5.

Using the procedure of search on this three dimensional grid we identify the matrix that is most close to the experimentally measured one. The criterion of closeness used is the minimum of the discrepancy

$$\delta = \left(\sum_{i=1}^{4} \sum_{j=1}^{4} (m_{ij} - m'_{ij})^2 \right)^{1/2}, \quad (2)$$

where m_{ij} and m'_{ij} are the elements of the experimental and calculated matrices compared.
The results of a comparison made among five couples of the matrices are given in the Table I. The upper row of data for each matrix presents the experimental values. The closest values of the matrix elements calculated on a three-dimensional grid (γ, k, α) are given in the second line while that obtained on a two-dimensional grid ($\gamma = 0$, k, α) in the third one. Note that in all the cases presented the experimental sounding was carried out along the zenith direction ($\gamma = 0$).

The data presented in the table show that the matrices calculated for a slant incidence ($\gamma \neq 0$) of radiation on the cloud layer agree with the experimentally measured ones better, except for the fifth matrix. However, the discrepancies vary not very strongly, except in one case. Moreover, if one averages the discrepancy value over eight matrix elements involved into the comparison its mean value is close to the experimentally assessed value of the measurement error $\delta = \pm\, 0.04$ (see Ref. 5).

TABLE I. Comparison between the experimentally measured (the upper line) and calculated (two lower lines) backscattering phase matrices.

N	γ^0	k	α^0	m_{22}	m_{33}	m_{44}	m_{12}	m_{13}	m_{34}	m_{24}	m_{23}	δ
	0	2.25	90	0.81	−0.60	−0.38	0.39	0.00	0.31	0.00	0.00	
1	30	3.0	90	0.867	−0.639	−0.507	0.433	− 0.002	0.125	− 0.004	0.000	0.31
	0	2.5	90	0.752	−0.439	−0.192	0.174	0.007	0.123	− 0.002	0.001	0.48
	0	1.0	169.8	0.62	− 0.55	− 0.15	0.00	0.00	− 0.32	− 0.12	0.00	
2	10	1.0	168.0	0.534	− 0.468	− 0.012	− 0.040	− 0.015	− 0.294	− 0.124	0.031	0.20
	0	1.0	168.0	0.513	− 0.423	0.044	− 0.005	− 0.002	− 0.297	− 0.117	0.022	0.25
	0	2.3	0	0.78	− 0.55	− 0.35	− 0.43	0.00	− 0.28	0.00	0.00	
3	30	3.0	0	0.870	− 0.636	− 0.507	− 0.433	− 0.015	− 0.125	− 0.005	− 0.00	0.37
	0	3.0	0	0.784	− 0.408	− 0.192	− 0.181	0.009	− 0.130	− 0.013	− 0.004	0.46
	0	2.7	70.1	0.32	− 0.10	0.58	0.56	0.38	− 0.27	− 0.20	0.21	
4	60	3.0	75.0	0.362	0.005	0.643	0.471	0.273	− 0.150	− 0.087	0.320	0.37
	0	2.0	75.0	0.525	− 0.367	0.107	−0.025	−0.013	− 0.238	− 0.135	0.138	1.16
	0	−	172.6	0.65	− 0.65	− 0.35	− 0.15	0.00	− 0.10	− 0.05	0.00	
5	0	1.5	177	0.716	− 0.593	− 0.310	− 0.090	− 0.011	− 0.086	− 0.034	0.006	0.13
	0	1.5	177	0.716	− 0.593	− 0.310	− 0.090	− 0.011	− 0.086	− 0.034	0.006	0.13

As to the better coincidence of the data calculated for $\gamma \neq 0$, we do not consider this fact to be an evidence of the cloud particles' tilt along a preferred direction, though we admit such a possibility, especially in the fourth case. On the whole, we would consider the parameter γ to be rather a fitting parameter that masks the difference between the actual ensemble of cloud particles and the model used. At the same time it is worth noting that the orientation parameters k and α only slightly vary with variations in γ, being in a good agreement with the experimental values, except in the case with the fifth matrix. The latter case has to be analyzed separately, since it is most likely that it is a result of the measurement errors. The matter is that in spite of a minimal discrepancy and a good agreement between the γ values the backscattering phase matrix measured experimentally does not agree well with the model ensemble of axially symmetric particles used. As follows from this model, if the equality $m_{22} = -\, m_{33}$ holds, then the equality $m_{12} = 0$ should also hold. At the same time, the experiment gives a value of this matrix element that differs from zero. It also follows from this model that at $m_{22} = -\, m_{33}$ the k parameter should take zero value, or in other words no orientation of particles occurs. However, the angle α of the direction of preferred orientation estimated from the ratio between the elements m_{12} and m_{13} equals 180° (or 0°, what is the same) and takes the value of 165.3° when estimated from the ratio of the elements m_{34} and m_{24}. In the table we give the average value. This disagreement may be removed if both m_{22} and m_{33} are increased by theamount $\delta = 0.04$ that equals the measurement error. In that case we also obtain that $k = 1.1$.

The above analysis, though very brief, clearly demonstrates that large errors may occur in the parameters of particles′ orientation when determining them from a small difference between two values or from the ratios of the small values. It is also obvious that the instrumentation we have now is capable of reliably acquiring data on the particles' orientation provided that it is well pronounced. In this connection we may state that the problem of the measurement accuracy improvement is still urgent.

It is also evident that use of quite a simple model of the cloud particles' ensemble can hardly provide for a good quantitative agreement between the calculated and measured backscattering phase matrices. In that case no reliable predictions of the scattering properties of such ensembles can be done for other scattering angles. However, one may see from the above that the discrepancies are not very strong. That, in its turn, is indicative of the model ability to correctly describe the basic features of the field of radiation scattered by an optically anisotropic cloud.

Let us now consider the case of light scattering by an ensemble of cylindrical particles with their long axes oriented randomly in a horizontal plane, paying special attention to one particular result that follows from this scheme.

Figure 1 shows surfaces that represent the function $m_{12}(\theta, \varphi)$, i.e., of the element m_{12} of the normalized backscattering phase matrix for all scattering angles at different γ angles of the radiation incidence onto a cloud layer. The top left plot shows, for a comparison, the $m_{12}(\theta, \varphi)$ surface calculated for the ensemble of particles oriented in space totally randomly.

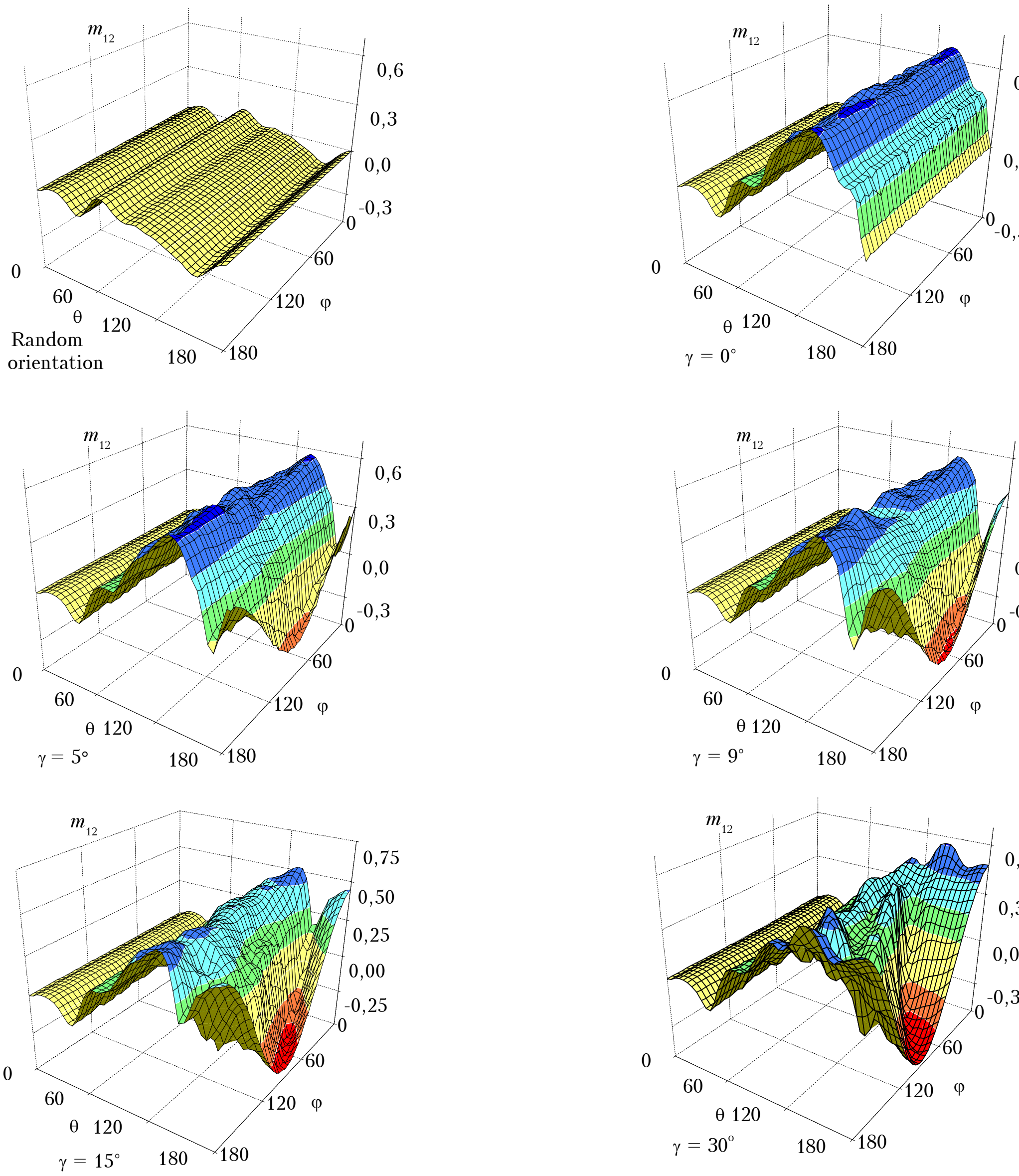

FIG. 1. Spatial angular behavior of the $m_{12}(\theta, \varphi)$ matrix element at different γ angles of the radiation incidence onto a cloud layer.

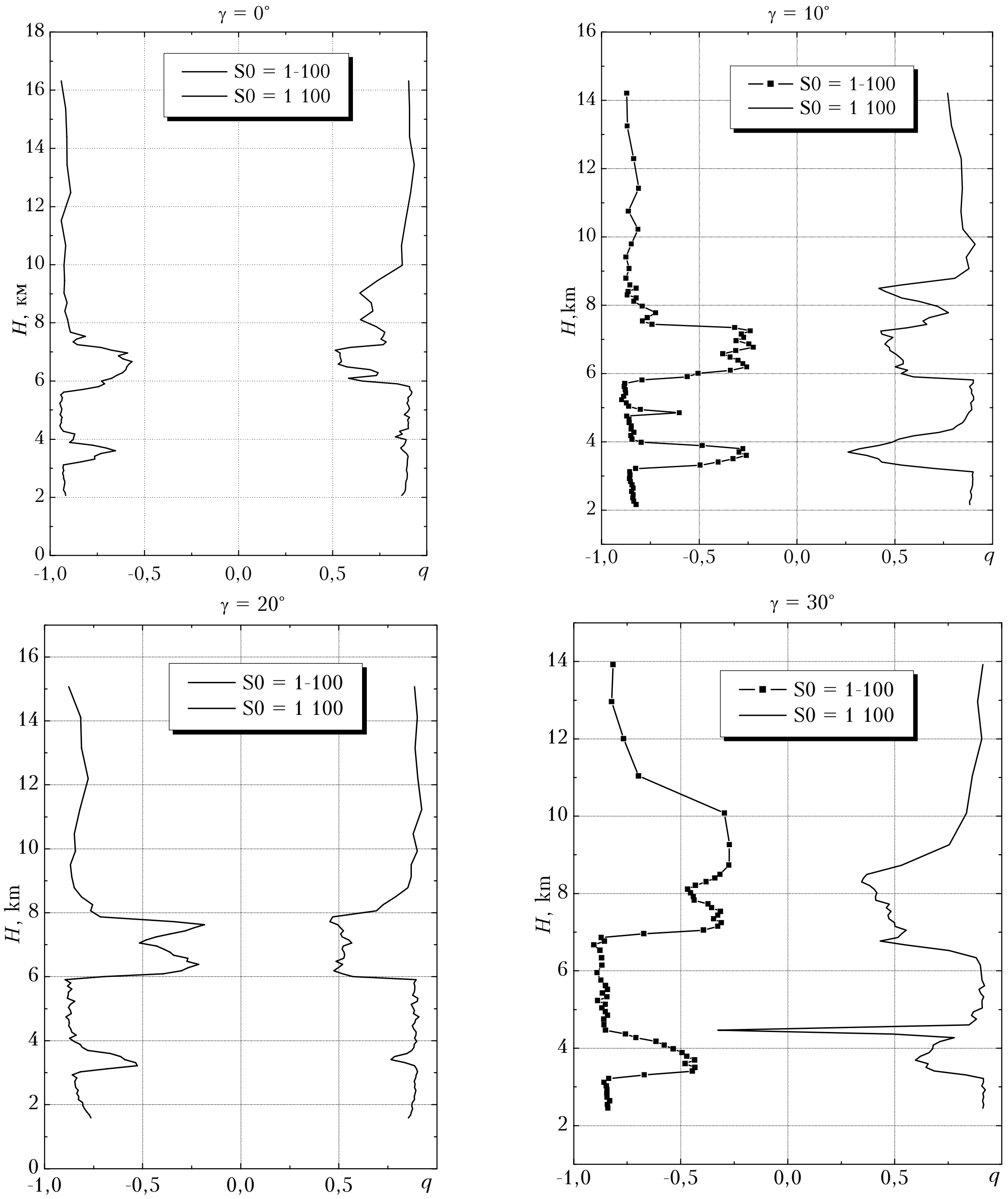

FIG. 2. Profiles of the normalized Stokes parameter q of lidar returns measured at different angles between the lidar optical axis and zenith direction. The Stokes parameters have been measured at two polarization states of sounding radiation: at the polarization (1,-1,0,0) - left curves, and (1,1,0,0) - curves to the right.

Taking into account the fact that normally lidar sensing of clouds is performed at a fixed θ angle of 180° we may compare only the curves lying in the cross-section $m_{12}(180°, \varphi)$ of this surface. But, in order to do this one should have a possibility of varying the angle φ by rotating the lidar itself or its polarization basis about its optical axis. Unfortunately, the construction of our lidar does not enable us to do this. However, we may vary the angle of the radiation incidence onto the cloud layer, what enables us to compare the points $m_{12}(\pi, 0, \gamma)$ from the family of surfaces $m_{12}(\theta, \varphi, \gamma)$.

Figure 2 shows profiles of the normalized Stokes parameter q of the backscatter measured at different angles of the radiation incidence onto the layer. Using this data one may calculate the profiles of the element m_{12}. The element $m_{12} = 0$ in the layer at heights from

5.5 to 7.5 km, when sounding along the zenith direction ($\gamma = 0$), that is indicative of the absence of a preferred orientation over the angle φ. Then, this element increases in value, up to some positive quantity, as the angle between the lidar optical axis and the zenith direction increases. This well agrees with the model used. However, this upper value of the element m_{12} is essentially lower than that predicted by the model, as one can see in Fig. 3.

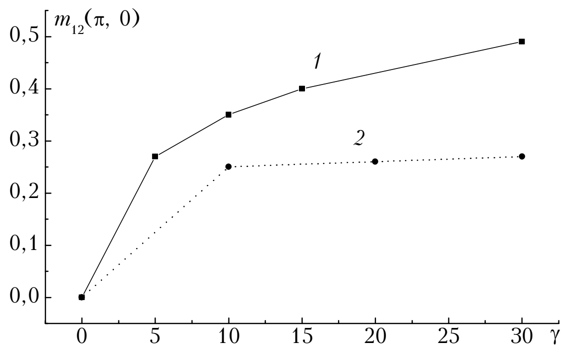

FIG. 3. The dependence of the element $m_{12}(\pi, 0)$ on the angle γ of the radiation incidence onto the cloud layer. Curve 1 shows the data calculated for an ensemble of cylindrical particles; curve 2 shows the experimental data corresponding to the case presented in Fig. 2.

It is worth noting here that the much lower experimental values of the element m_{12}, as compared to the calculated ones, could certainly be expected beforehand, since oriented particles make up only a fraction of the total number of particles in an actual ensemble of cloud particles, and it is likely that this fraction is small. The matter is that, mostly, large anisometric particles of an ensemble take some preferred orientation, while the isometric and small particles have random orientation and therefore their scattering phase matrix does not depend on the radiation incidence angle and the matrix element $m_{12}(\pi, 0)$ equals zero at any value of the angle γ.

If the model correctly describes the dependence of $m_{12}(\pi, 0)$ on γ, then the difference between the model and measured curves may be used for isolating the contributions coming to the total backscatter value from the oriented and nonoriented particles of the ensemble. Thus, the estimation made using the above experimental data shows that the contribution coming from the oriented particles of the ensemble makes about 30% of the total backscatter.

From a more general point of view the parameters of particles' orientation, as well as the dependences of the backscattering phase matrix elements on sounding angle are related to the particle size and thus may be used for particle size estimation, as shown in Ref. 10 for the case of plate particles. In our opinion this possibility should be analyzed in a more thorough way when developing models of the particle ensembles to describe actual clouds.

The simplified model we have proposed provides for a correct estimation of the parameters α and k of particles' orientation because it is based on quite general symmetry relations. Moreover, we believe that this model could be useful for making some preliminary, though rough, estimates of the effect the optical anisotropy of clouds may produce on the field of multiply scattered radiation.

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